U.P.B. Sci. Bull., Series D, Vol. 79, Iss. 4, 2017 ISSN 1454-2358

MATHEMATICAL MODEL FOR MICROLAUNCHER, PERFORMANCES EVALUATION

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The paper presents some aspects regarding the mathematical model and performance evaluation for a four stages microlauncher. This work uses three separate models dedicated for each flight phase. For the ascending phase, we will use a three degrees of freedom model in quasi–velocity frame. For the ballistic phase we will use a Kepler model, and for the orbital injection a Gauss perturbing model. The results analyzed will be in quasi-velocity frame but also some orbital parameters will be presented. Using these models, the microlauncher performances will be evaluated. The novelty of the paper consists in orbital injection approach, with optimal maneuver description.

Keywords: mathematical model, microlauncher performances, orbital injection.

Nomenclature

ψ_0 - Azimuth angle; φ - Geocentric latitude; λ - Geocentric longitude; γ - Climb angle; χ - Path track angle; α - Attack angle; β^* - Sideslip angle; μ - Aerodynamic bank angle; $\mathbf{\Omega}_V^*$ - Rotation velocity of the quasi-velocity frame; $\mathbf{\Omega}_p$ - Earth spin; D - Drag force; L - Lift force; N - Lateral force; $\mathbf{T}$ - Thrust force; m - Mass; t - Time; $\mathbf{V}$ - Velocity; V_x, V_y, V_z - Velocity components in start frame; $O_P X_P Y_p Z_p$ - Earth frame; $O_L X_L Y_L Z_L$ - Local frame; $O_S X_S Y_S Z_S$ - Start frame; r - The distance between rocket and Earth center: R_p - Earth radius; $\mathbf{g}$ - Gravitational acceleration; e - Eccentricity; a -Semi-major axis; θ - True anomaly; ψ - Eccentric anomaly; p - The orbit parameter.

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1. Introduction

Today it is most often that micro or nano satellites are carried into space as "an additional payload" or the so called "piggyback" missions. It is too costly to dedicate a separated mission that involves a relatively large launcher to a satellite whose mass is much smaller than the designed mass of the launcher. Therefore, the necessity of designing launchers for satellites weighing up to 100 kg is justified. ESA has basically three launchers for LEO orbit: VEGA, for satellites with a mass ranging from 0.3 to 2.5 tons; Soyuz, for satellites with a mass of 2.8 – 4.8 tons; Ariane for satellites with a mass of 16 - 21 tons. If a country from Europe would like to launch a satellite with a mass of 100 kg, or a few smaller ones for a dedicated mission, they have to buy the launch from Russia, USA, China or India. Since the number of such satellites will be increasing in the near future, Europe and ESA should develop a small rocket launcher to close the gap in the existing family of European launchers and allow an easier and independent access to space for European micro or nano satellites. Starting from this necessity, Romania under ESA coordination promoted a pilot-project consisting in the analysis of the possibility to achieve microlauncher in zonal cooperation - ML. To approach this problem and in general for evaluating the launching capabilities it is necessary to elaborate an adequate mathematical model that ensures the evaluation of the launcher's capability to inject the payload on different circular orbits. The mathematical model presented below seeks to answer these needs. The model was split into two sub-models. The first one is dedicated to finding the optimal flight parameters in the ascending phase; the second one is used to evaluate the launcher's evolution during the orbital injection phase. Because at this stage we are interested in evaluating the technical possibility of building a microlauncher starting from imposed performances and to make a preliminary dimensional evaluation (preliminary design), our models approaches both phases, until the circular orbit is reached. Having in mind these ideas regarding the needs, at this stage from the ML model, we will describe the frames used, the coordinate transformations, the motion equations and the guidance relations necessary to define the launcher's motion for flight phases.

2. Coordinate systems

First, we will define the coordinate systems specific for the motion of the microlauncher.

A. The Earth Frame

This inertial coordinate system is originated in the center of the Earth, being loosed from Earth and does not participate in its diurnal rotation (Earth spin). The axis X_p is in the equatorial plane along vernal axis. Axis Z_p is along polar axis, toward North Pole. The axis Y_p completes a right frame being in the equatorial plane.

B. The Local Frame

This coordinate system has the origin in the starting position, being earthbound, and participating in the diurnal rotation (Earth spin). The axis Y_L is the position along the vector $\mathbf{r}$ at the start moment. The axis Z_L is parallel with the equatorial plane, being oriented to the East. The axis X_L arising is forming with the first two axes a right trihedral (1).

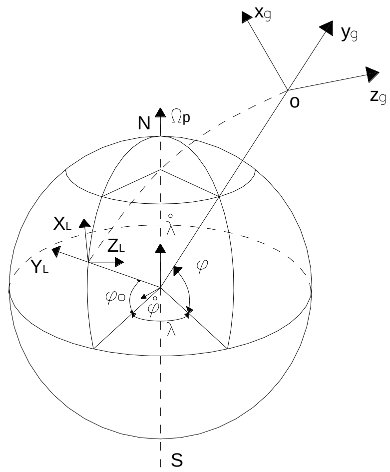

Fig. 1 The Geocentric and Geographical Frames

C. The Start Frame

This coordinate system has the origin in the starting position, being earthbound and participating in the diurnal rotation (Earth spin). The axis Y_S is the position along the vector $\mathbf{r}$ at the start moment. The axis X_S is oriented toward launch direction and makes an azimuth angle ψ_0 related to the X_L axis. The axis Z_S, is forming with the first two axes a right trihedral, being oriented to the right related launch plane.

D. The Geographical Mobile Frame

This coordinate system has the origin in the center of mass of the launcher, being earthbound and participating in the diurnal rotation (1). The axis y_g is the position along the vector $\mathbf{r}$. The axis z_g is parallel with the equatorial plane, being oriented towards the East. The axis x_g is forming with first two axes a right trihedral. The geographical mobile frame overlaps the local frame at the start moment.

E. The Geocentric Spherical Frame

This coordinate system is originated in the center of the Earth, being earthbound and participating in its diurnal rotation (Earth spin). The launcher position can be described using spherical coordinates λ, φ, r, as can be seen in 1.

F. The Quasi-Velocity Frame

This coordinate system has the origin in the center of mass of the launcher. Similarly to the velocity frame, the quasi-velocity frame has the axis x_a^* along the velocity vector, but the axis y_a^* it is in vertical plane. The axis z_a^* is forming with the first two axes a right trihedral (Fig. 2). Next we will use this trihedral to write the dynamic translation motion equations of the center of the mass.

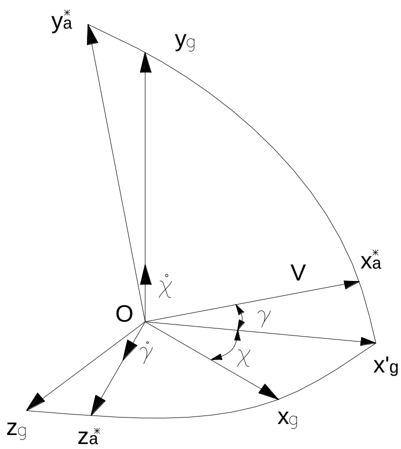

Fig. 2 The rotations between the geographical frame and quasi-velocity frame

3. The Gravitational acceleration

If we consider so call "J2" model, the gravity is expressed by two terms: first denoted g_{Ar} [3], oriented along radius , and the second, denoted $g_{A\omega}$ along the polar axis $N-S$. These terms, containing only the gravitational component without centrifugal contribution, which will be added later

$$g_{Ar} = \frac{a_{00}}{r^2} - \frac{3}{2}\frac{a_{20}}{r^4}(5\sin^2\varphi - 1)... \quad g_{A\omega} = \frac{g_{A2}}{\cos\varphi} = 3\frac{a_{20}}{r^4}\sin\varphi... \tag{1}$$

where:

$$a_{00} = 3{,}9861679 \cdot 10^{14}; \qquad \frac{3}{2}a_{20} = 26{,}32785 \cdot 10^{24}. \tag{2}$$

4. The equations of motion in quasi – velocity frame

Because quasi-velocity frame is not an inertial frame, the dynamic equation of motion in quasi-velocity frame has following form [1], [4], [8]:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{\Omega}_V^* \times \mathbf{V} = \frac{\mathbf{N}}{m} + \mathbf{g} + \mathbf{a}_C, \tag{3}$$

where we have grouped the aerodynamic and thrust forces.

$$\mathbf{N} = \mathbf{F} + \mathbf{T}, \tag{4}$$

The Coriolis acceleration is:

$$\mathbf{a}_c = -2\mathbf{\Omega}_p \times \mathbf{V} \tag{5}$$

The local derivative of the velocity in quasi-velocity frame is $\partial \mathbf{V}/\partial t$. $\mathbf{\Omega}_V^*$ is the rotation velocity of the quasi-velocity frame related to the local frame, which can be express as vectors:

$$\mathbf{\Omega}_V^* = \dot{\boldsymbol{\gamma}} + \dot{\boldsymbol{\chi}} + \dot{\boldsymbol{\varphi}} + \dot{\boldsymbol{\lambda}} \tag{6}$$

The derivatives of latitude and longitude angles along geographical frame are:

$$\dot{\boldsymbol{\lambda}} = \dot{\lambda}\left(\mathbf{i}_g \cos\varphi + \mathbf{j}_g \sin\varphi\right); \; \dot{\boldsymbol{\varphi}} = -\mathbf{k}_g \dot{\varphi} \tag{7}$$

where $\mathbf{i_g}, \mathbf{j_g}, \mathbf{k_g}$ are unitary vectors in geographical frame.

If we make the projection along quasi-velocity frame we get:

$$\begin{gathered} \dot{\boldsymbol{\lambda}} = \dot{\lambda}\begin{bmatrix} \mathbf{i}_a\left(\cos\varphi\cos\chi\cos\gamma + \sin\varphi\sin\gamma\right) + \\ \mathbf{j}_a\left(-\cos\varphi\cos\chi\sin\gamma + \sin\varphi\cos\gamma\right) + \\ \mathbf{k}_a\left(\cos\varphi\sin\chi\right) \end{bmatrix} \\ \dot{\boldsymbol{\varphi}} = \dot{\varphi}\left[\mathbf{i}_a \sin\chi\cos\gamma - \mathbf{j}_a \sin\chi\sin\gamma - \mathbf{k}_a \cos\chi\right] \end{gathered} \tag{8}$$

The derivatives of the climb angle and the air-path track angle are:

$$\dot{\boldsymbol{\gamma}} = \dot{\gamma}\mathbf{k}_a; \; \dot{\boldsymbol{\chi}} = \dot{\chi}\left(\mathbf{i}_a \sin\gamma + \mathbf{j}_a \cos\gamma\right) \tag{9}$$

In this case, the components of the angular velocity vector along quasi-velocity frame become:

$$\begin{aligned} \omega_l^* &= \dot{\lambda}\left(\cos\varphi\cos\chi\cos\gamma + \sin\varphi\sin\gamma\right) + \\ &\quad + \dot{\varphi}\sin\chi\cos\gamma + \dot{\chi}\sin\gamma \\ \omega_m^* &= \dot{\lambda}\left(-\cos\varphi\cos\chi\sin\gamma + \sin\varphi\cos\gamma\right) - \\ &\quad - \dot{\varphi}\sin\chi\sin\gamma + \dot{\chi}\cos\gamma \\ \omega_n^* &= \dot{\lambda}\cos\varphi\sin\chi - \dot{\varphi}\cos\chi + \dot{\gamma} \end{aligned} \tag{10}$$

Taking in consideration that the vector $\mathbf{\Omega}_p$ has the same orientation as the vector $\dot{\boldsymbol{\lambda}}$, we can write:

$$\mathbf{\Omega}_p = \Omega_p \begin{bmatrix} \mathbf{i}_a(\cos\varphi\cos\chi\cos\gamma + \sin\varphi\sin\gamma) + \\ \mathbf{j}_a(\sin\varphi\cos\gamma - \cos\varphi\cos\chi\sin\gamma) + \\ \mathbf{k}_a(\cos\varphi\sin\chi) \end{bmatrix}, \tag{11}$$

where the Coriolis acceleration components in quasi-velocity frame are:

$$a_{cx} = 0;\ a_{cy} = -2V\Omega_{pz} = -2V\Omega_p\cos\varphi\sin\chi;$$
$$a_{cz} = 2V\Omega_{py} = 2V\Omega_p(\sin\varphi\cos\gamma - \cos\varphi\cos\chi\sin\gamma) \tag{12}$$

The gravitational acceleration previously introduced, is expressed by two terms, one term denoted g_r and oriented along radius r and the other term g_ω parallel with polar axis $N-S$. These two terms contain gravitational components and also centrifugal components given by the Earth's spin.

$$g_r = g_{Ar} - \Omega_p^2 r;\quad g_\omega = g_{A\omega} + \Omega_p^2 r\sin\varphi, \tag{13}$$

where g_{Ar} and $g_{A\omega}$ are given by relations (1) , (2), depending on the range.

Next, we will project the terms given by relation (13) along quasi-velocity frame. For this we need to keep in mind that the term g_r is along the angular velocity vector $\dot{\boldsymbol{\chi}}$, given by relation (8), and the term g_ω is along angular velocity vector $\dot{\boldsymbol{\lambda}}$, given by relation (8) but contrary to it. In this case we yield:

$$g_x = -g_r\sin\gamma - g_\omega(\cos\varphi\cos\chi\cos\gamma + \sin\varphi\sin\gamma);$$
$$g_y = -g_r\cos\gamma - g_\omega(-\cos\varphi\cos\chi\sin\gamma + \sin\varphi\cos\gamma); \tag{14}$$
$$g_z = -g_\omega\cos\varphi\sin\chi.$$

Summarizing, starting from relation (3), we obtain the dynamic equation which describes the motion of the center of mass of the launcher in quasi-velocity frame [1][4]:

$$\dot V = \frac{N_x}{m} - g_r\sin\gamma - g_\omega(\cos\varphi\cos\chi\cos\gamma + \sin\varphi\sin\gamma)$$
$$\dot\gamma = \frac{N_y}{mV} - \frac{g_r}{V}\cos\gamma - \frac{g_\omega}{V}(-\cos\varphi\cos\chi\sin\gamma + \sin\varphi\cos\gamma) +$$
$$+\frac{V}{r}\cos\gamma - 2\Omega_p\cos\varphi\sin\chi \tag{15}$$
$$\dot\chi = -\frac{N_z}{mV\cos\gamma} + \frac{g_\omega\cos\varphi\sin\chi}{V\cos\gamma} + \frac{V}{r}\tan\varphi\sin\chi\cos\gamma +$$
$$+2\Omega_p(\cos\varphi\cos\chi\tan\gamma - \sin\varphi)$$

complemented with kinematic equations:

$$\dot r = V\sin\gamma\,.\ \dot\varphi = \frac{V}{r}\cos\chi\cos\gamma;\ \dot\lambda = -\frac{V\sin\chi\cos\gamma}{r\cos\varphi}; \tag{16}$$

where N_x, N_y, N_z are projection of the applied forces along quasi-velocity frame.

Supposing that due control contribution the oscillations around center of mass are damped, and the thrust vector is aligned with body axis, and more the aerodynamic bank angle is null $\mu = 0$, the components of the applied forces become:

$$N_x = D + T\cos\alpha\cos\beta^*;\ \ N_y = L + T\sin\alpha;\ \ \ N_z = N - T\cos\alpha\sin\beta^* \tag{17}$$

where α, β^* are the aerodynamic angles and $D; L; N$ -are the aerodynamic force components in velocity frame;

Based on aerodynamic angles and launcher geometry, we can evaluate aerodynamic translation coefficients:

$$C_D = C_{D0} + \frac{1}{2}\frac{\partial^2 C_D}{\partial\alpha^2}\alpha^2;\ \ C_L = C_{L\alpha}\alpha;\ \ \ C_N = C_{N\beta}\beta^{\bullet} \tag{18}$$

Taking in consideration the hypothesis regarding alignment between thrust vector and launcher axis, the orientation of the thrust vector and velocity vector is descripts by aerodynamic angles α, β^* where can be considerate as command parameters of the system that can control the climb angle γ respectively air -path track angle χ by relation:

$$\alpha = -k_1(\gamma - \gamma_d) \quad \ \beta^* = -k_1(\chi - \chi_d) \tag{19}$$

where the reference sizes are: γ_d; χ_d

5. Evolution in orbital phase, orbital injection

In order to evaluate the orbital phase and the orbital injection we use as inertial frame, the Earth frame. First, we obtain the velocity in geographic frame related to inertial frame, by adding Earth rotation:

$$V_{xg} = V\cos\gamma\cos\chi;\ V_{yg} = V\sin\gamma;\ V_{zg} = -V\cos\gamma\sin\chi + r\Omega_p\cos\varphi \tag{20}$$

Also we modify longitude taking in consideration Earth rotation $\lambda = \lambda + \Omega_p t$, obtaining launcher coordinates in Earth frame:

$$X_p = r\cos\varphi\cos\lambda;\ \ Y_p = r\cos\varphi\sin\lambda;\ \ Z_p = r\sin\varphi \tag{21}$$

In order to obtain velocity components in Earth frame we make a rotation from geographic frame to Earth frame:

$$\begin{bmatrix} V_{xp} & V_{yp} & V_{zp} \end{bmatrix}^T = \mathbf{B}_G \begin{bmatrix} V_{xg} & V_{yg} & V_{zg} \end{bmatrix}^T \tag{22}$$

where rotation matrix is:

$$\mathbf{B}_G = \begin{bmatrix} -\sin\varphi\cos\lambda & \cos\varphi\cos\lambda & -\sin\lambda \\ -\sin\varphi\sin\lambda & \cos\varphi\sin\lambda & \cos\lambda \\ \cos\varphi & \sin\varphi & 0 \end{bmatrix}$$

From velocity components we can obtain the velocity module in inertial frame:

$$v=\sqrt{V_{xp}^2+V_{yp}^2+V_{zp}^2} \tag{23}$$

Next, we are interested in the kinetic moment for unitary mass, and its components in Earth frame:

$$\mathbf{h}=\mathbf{r}\times\mathbf{v}=h_x\mathbf{I}+h_y\mathbf{J}+h_z\mathbf{K}, \tag{24}$$

from where:

$$h_x=Y_PV_{zp}-Z_pV_{yp};\; h_y=Z_pV_{xp}-X_pV_{zp};\; h_z=X_pV_{yp}-Y_PV_{xp}, \tag{25}$$

Also we can obtain the kinetic moment module:

$$h=\sqrt{h_x^2+h_y^2+h_z^2}; \quad , \tag{26}$$

Having the velocity (23), we can obtain the energy for a body with unitary mass:

$$E=v^2/2-a_{00}/r, \tag{27}$$

and geometric elements for orbit: e - eccentricity and a - semi major axis:

$$e=1+2Eh^2a_{00}^{-2}; \quad a=p\zeta^{-2} \tag{28}$$

where parameter p is given by: $p=h^2a_{00}^{-1}$ and $\zeta^2=1-e^2$

Taking in consideration the expressions of the orbit:

$$r=a(1-e\cos\psi)\;;\; r=\frac{p}{1+e\cos\theta} \tag{29}$$

we can obtain ψ - Eccentric anomaly, θ - True anomaly, and by time Kepler equation M -Mean anomaly

$$\cos\psi=\frac{a-r}{ae};\; \cos\theta=\frac{p-r}{er};\; M=\psi-e\sin\psi \tag{30}$$

In order to obtain a circular orbit, from Gauss perturbing equations [3], [6], [7] we can extract the eccentricity equation:

$$\dot{e}=\frac{\zeta(\zeta^2-f^2)a_T\cos\delta_2}{neaf}\left(\frac{e\zeta}{\zeta^2-f^2}\sin\psi\sin\delta_1+\cos\delta_1\right), \tag{31}$$

where: $f=1-e\cos\psi$

a_T - The acceleration derived from thrust.

δ_1 - The angular deflection of the thrust vector, relative to the perpendicular direction on $\mathbf{r}$ in the orbit plane;

δ_2 - The angular deflection of the thrust vector outside the orbit plane.

If we want an optimal maneuver to minimize in minimum time the eccentricity and achieve a circular orbit, we impose condition:

$$\frac{\partial \dot{e}}{\partial \delta_1} = \frac{\zeta(\zeta^2 - f^2)a_T \cos\delta_2}{neaf}\left(\frac{e\zeta}{\zeta^2 - f^2}\sin\psi\cos\delta_1 - \sin\delta_1\right) = 0 \tag{32}$$

and obtain optimal value for the thrust angular deflection:

$$\tan\delta_1 = \frac{e\zeta}{\zeta^2 - f^2}\sin\psi \tag{33}$$

Next we evaluate the sign of relation (31) for the angular deflection (33) in order to obtain an eccentricity minimization.

If we substitute in relation (31) the angular deflection form (33) and consider the second angular deflection null ($\delta_2 = 0$) , will result:

$$\dot{e} = \frac{\zeta(\zeta^2 - f^2)a_T \cos\delta_1}{neaf}\left[\left(\frac{e\zeta}{\zeta^2 - f^2}\sin\psi\right)^2 + 1\right] \tag{34}$$

Taking in consideration that:

$$f = 1 - e\cos\psi \geq 0\,, \tag{35}$$

and if we restricted the angular deflection to:

$$-\pi/2 \geq \delta_1 \geq \pi/2 \tag{36}$$

will result:

$$\cos\delta_1 \geq 0\,, \tag{37}$$

thus, the sign of relation (34) is given by:

$$\zeta^2 - f^2 = 1 - e^2 - (1 - e\cos\psi)^2 = -e^2 - e^2\cos^2\psi + 2e\cos\psi \tag{38}$$

If we want to decrease the eccentricity, we impose the condition:

$$\cos^2\psi - 2e^{-1}\cos\psi + 1 > 0 \tag{39}$$

which means:

$$-1 \leq \cos\psi \leq (1-\zeta)e^{-1} \tag{40}$$

Based on these results (33) we can impose optimal pitch and yaw command for injection in circular orbit:

$$\alpha = -(\gamma - \gamma_d) \quad \beta^* = -(\chi - \chi_d) \tag{41}$$

where the reference sizes are: $\gamma_d = \delta_1;\ \chi_d = \delta_2$

6. Optimizing the ascending phase

We start by describing - for a four stage launcher with the first three solid rocket motor, the typical ascending phase. Lift off is considerate from $t_0 = 0$ up to $t_1 = 2s$, when the climb angle is $\gamma_d = 90^\circ$ and the ML evolution is vertically. At $t_2 = 7s$ the climb angle is $\gamma_d = \gamma_1$ and maintains this value up to $t_3 = t_2 + \Delta_1$. Between t_3 and t_4 (the ignition of third stage), the climb angle has no constrains, being in the gravity turn phase. At the ignition of third stage, t_4, the climb angle is constrained to take the value $\gamma_d = \gamma_2$ during third stage till t_5. During the coasting Δ_3 between third stage and fourth stage and after ignition of the fourth stage, we have a gravity turn maneuver. The gravity turn maneuver continue for a duration Δ_4 after fourth stage ignition till t_6 when orbital maneuver starting, For ML, the burning duration of the first stage is $t_{a1} = 76\ s$ and the burnout duration of stage 2 and 3 are the same $t_{a2} \cong t_{a3} \cong 50\ s$. Between the burnout of the second stage and the ignition of the third stage we have a coasting phase with a duration Δ_2. Between the burnout of the third stage and the ignition of the fourth stage (AVUM) we have a second coasting phase with a duration Δ_3. The fairing jettison is in synchronies with the separation of the stages 2 – 3. Summarizing, the ascending phase of ML depends on six independent parameters, Δ_1, Δ_2, Δ_3, Δ_4, γ_1, γ_2, which can be the subject of optimization during ascending phase. The strategy adopted consist that for different initial azimuth angle ψ_0 (orbit inclination) and different payload mass (MPL), to obtains an circular orbit with maximum altitude and minimum effort by optimization: Δ_1, Δ_2, Δ_3, Δ_4, γ_1, γ_2, Taking in consideration that, for ascending and ballistic phase we choose as performance index:

$$J = -\varepsilon_1 a + \int_0^{t_f} \varepsilon_2 a_y^2 - \varepsilon_3 D dt , \tag{42}$$

where ε_k are the weights, $a_y = \dot{\gamma} V$ and minimize them by using random number generators,

The optimization method allows us to obtain at the end of orbital phase a circular orbit with maximum altitude and minimum maneuvering effort for different orbit inclinations and different payload mass, which means the ML performances.

7. Input data for ML model

The input data used are taken from paper [2].

Table 1 **Mass Characteristics**

Configuration	Mass m [tons]	
	Initial	Final
Stage I + II + III + AVUM + P/L+FER	34.5	10.6
Stage II + III + AVUM + P/L+FER	8.4	2.6
Stage III + AVUM + P/L	2.0	0.6
AVUM + P/L	0.48	0.36
P/L	0.1	0.1

The input data used are taken from paper [2].

Table 1 shows ML mass characteristics.
Main geometrical sizes at ML start are: $l = 16.6m$ $d = 1.9m$

Table 2 Thrust Characteristics

Stage	Specific impulse (*) [s]	Propellant mass [tons]	Burnout duration [s]	Se Output section [m2]
I	280	23.9	76.5	1.13
II	290	5.81	47.9	0.956
III	295	1.41	50.1	0.441
IV	340	0.12	100	0.11

(*) – Vcuum conditions, total extension (pe=0)

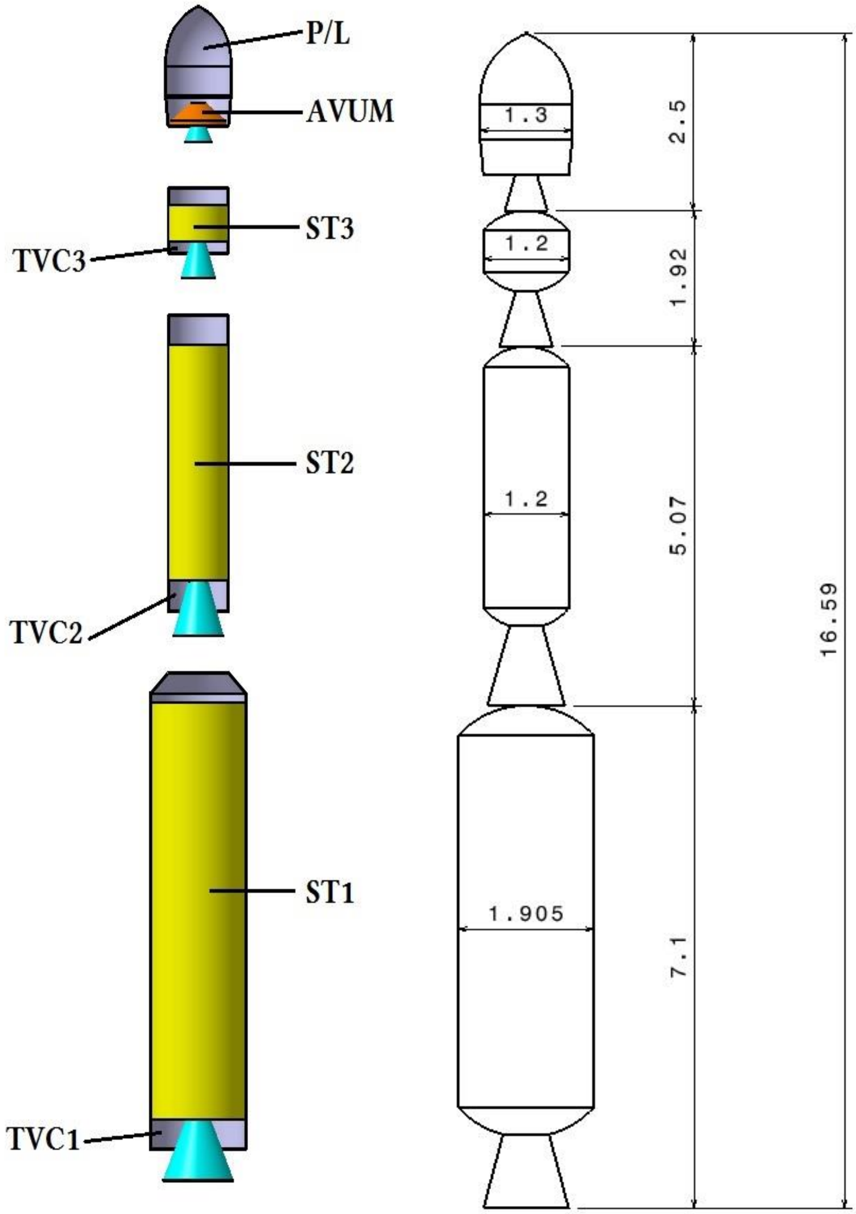

Fig. 3 ML Configuration

In Fig. 3 we have: P/L Payload; AVUM - Attitude and Vernier Upper Module; ST - Stage; TVC - Thrust Vector Control.

8. Test case

As test case, we choose an polar orbit, with the following initial conditions: Geographic orientation: Azimuth angle $\psi_0 = 0°$ (towards the North); Geocentric latitude $\varphi = 0°$ (Equatorial latitude); Altitude: $h_0 = 1m$; Initial velocity $V_0 = 1[m/s]$; Initial climb angle $\gamma_0 = 90°$. Payload mass $MPL = 100\,[kg]$. Corresponding to minimal value of performance index (42), we obtain: $\Delta_1 = 9[s]$, $\Delta_2 = 761[s]$ $\Delta_3 = 88[s]$ $\Delta_4 = 26[s], \gamma_1 = 72.2°$, $\gamma_2 = 2.1°$ which leads to a circular orbits with altitude $h_p = 1778\,[km]$. Using these parameters, we have defined a circular orbit described in next item.

9. Results

Fig. 4 shows the relative velocity, which means the ratio between absolute velocity in inertial frame (21) and velocity corresponding to a circular orbit. We can observe that after injection phase relative velocity remain at unit value. In the same diagram is shown the altitude, which after injection remains at a constant value.

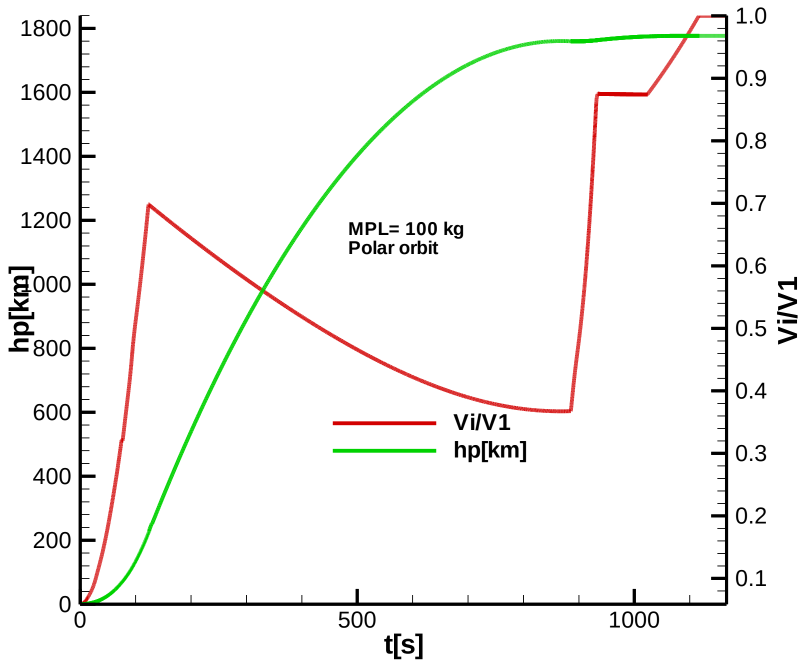

Fig. 4 Vi/V1 Relative Velocity and hp – Altitude

Fig. 5 shows attack angle of the thrust vector/ body axe related velocity during ascending and orbital phase. From this diagram, one can observe large values of the attack angle during the third stage flight and during final injection maneuver. Fortunately, these maneuvers are produced outside the atmosphere, which does not increase the aerodynamic load of the launcher. The same diagram shows the climb angle γ which is controlled by the thrust attack angle. One can observe that it starts at $\gamma_0 = 90°$, followed by the imposed value $\gamma_1 = 72.2°$ and after orbital injection remains at zero value.

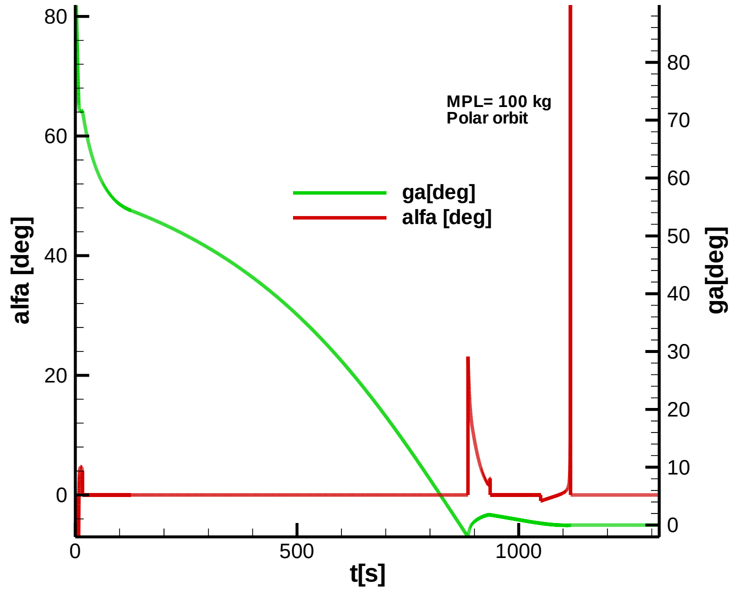

Fig. 5 DTN-Deflection angle and ga- climb angle

For the same test case, Fig. 6 shows the acceleration in quasi - velocity frame. One can observe a_x as the result of thrust of each stage along velocity vector and also acceleration a_y normal on velocity in orbital plane. The acceleration a_z normal on orbital plane are insignificant.

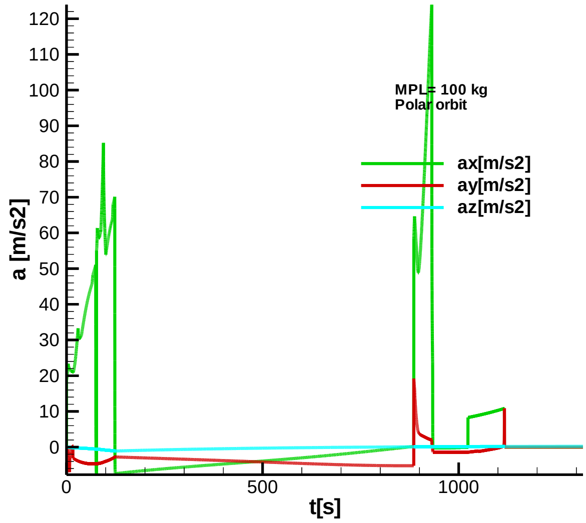

Fig. 6 Acceleration in Quasi Velocity frame

Fig. 7 shows two orbital parameters, eccentricity and semi major axis during ascending and ballistic phase.

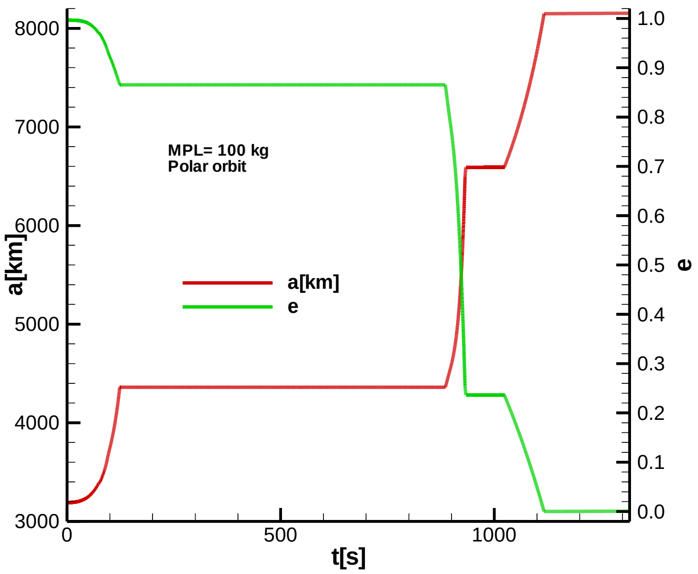

Fig. 7 e- Eccentricity and a- Semi-major axis

One can observe that eccentricity decreases until zero value, and after injection phase remains at zero value. In the same time semi major axis increases simultaneous with the velocity enhancing and remains constant after orbital injection.

Fig. 8 shows the orbit anomalies: θ-true anomaly, ψ-eccentric anomaly and M mean anomaly. One can observe that after orbital injection all three anomalies have the same values, specific situation for circular orbit. Also we can observe that eccentric anomaly ψ satisfy relation (40) necessary to decrease eccentricity through optimal maneuver (33).

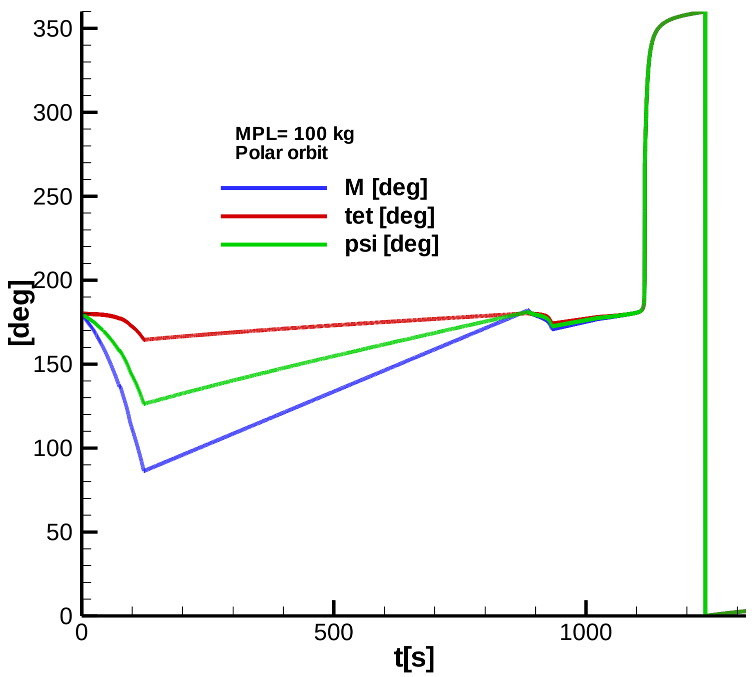

Fig. 8 True anomaly - tet, Eccentric anomaly – psi and Mean anomaly - M

Fig. 9 shows maximum altitude orbit as a function of orbit inclination and payload mass which means ML performances.

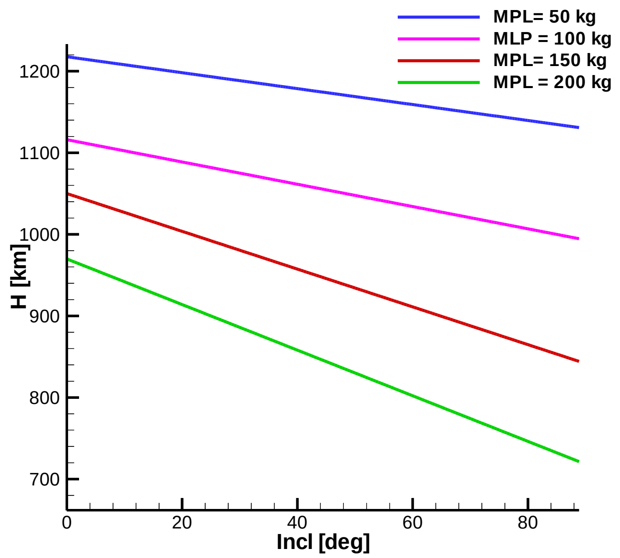

Fig. 9 Performances H-Altitude orbit as function of Incl-inclination and payload mass

10. Conclusions

As we said at the beginning, the paper has as objective the building of a simple mathematical model able to evaluate launcher's performances. In order to solve this item, we separated the launcher's evolution into two phases, the first phase being the ascending phase until the fourth stage of it is in optimal position to make orbital injection and the second phase when the upper stage performs orbital injection. For each phase, we developed a separate calculus model. For the ascending phase we developed a 3DOF model which describes the functionality of the launcher in the quasi-velocity frame in accordance with the work [3]. For the second phase, we used a sample model based on Kepler's theory [7], which allows us, to evaluate orbital parameters, and use Gauss orbital perturbed equation [7] in order to obtain optimal injection maneuver. Despite different model used for each flight phases, for unitary approach we use actually only 3DOF model in quasi-velocity frame, by transform the command from orbital frame in quasi – velocity frame. Considering that small launchers are targeted at a circular orbit, we built a performance index based on maximum semi major-axis and minimum maneuvering effort, which allows the defining of the characteristics parameter of a trajectory which is able to obtain a circular orbit with maximum altitude. The test case build and the results obtained prove the correctness of the model developed, including the strategy adopted for optimizing the accessional phase. Considering other case, with deferent initial condition, we used the model developed to evaluate the entire field of ML performance. Solution adopted for ML mission design must take in consideration that the accuracy of the desired orbit must not depend on the technical possibility to realize angular parameters of ascending phase and also on the accuracy of the prediction of the thrust characteristics of the solid rocket motor. The accuracy of the desired orbit depends directly on the upper stage, which makes the injection for transferring the payload to the desired orbit.

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