SOME REMARKS ABOUT THE ABSTRACT FAMILIES OF FUZZY LANGUAGES

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Se demonstrează că familiile abstracte de limbaje fuzzy sunt închise atât faţă de aplicaţia GSM (generalized sequential machines) fuzzy ε – free , cât şi faţă de aplicaţia GSM fuzzy inversă.

One proves that the abstract families of fuzzy language are closed under both the ε – free fuzzy GSM (generalized sequential machines) application and the inverse fuzzy GSM application , respectively.

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Introduction

The abstract families of fuzzy languages were defined earlier [1], by analogy with the abstract families of languages [2]. A family of fuzzy languages is an abstract family of fuzzy languages ($AFFL$) if and only if it contains a non-empty language and it is closed under the following operations: union, ε – free Kleene closure, ε – free fuzzy homomorphism, inverse fuzzy homomorphism and intersection with regular fuzzy languages. The families of regular fuzzy languages and of the context-free fuzzy languages, respectively, are examples of $AFFL$ [1].

In Ref. [3] we introduced the fuzzy generalized sequential machines ($FGSM$) as an extension of the generalized sequential machines , that is , by assigning to each state a certain grade with which it may be initial or final state , respectively , as well as grades of application to the productions . Then, we have studied the property of closure of the families of regular fuzzy languages under the ε – free $FGSM$ application.

In this work we investigate the more general question of the closure properties of the $AFFL$ under $FGSM$ applications.

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Closure properties of the AFFL

Theorem 1: Any $AFFL$ is closed under the ε – free $FGSM$ application.

Proof. Let $FGSM$ be an ε – free fuzzy generalized sequential machine

$$FGSM = (S, V_I, V_0, \mu, \pi, \eta)$$

where

$$\mu : S \times V_I \times V_0^+ \times S \to [0,1]$$
$$\pi : S \to [0,1]$$
$$\eta : S \to [0,1]$$

Let $\mathcal{L}$ be an $AFFL$. To prove the theorem , we choose an arbitrary language L from $\mathcal{L}$ and show that $FGSM(L)$ belongs to $\mathcal{L}$. We introduce the following auxiliary alphabet :

$$V_1 = \left\{ \left[s_i, a, x, s_j \right] \middle| \; s_i a \to x s_j \; , \; s_i, s_j \in S \; , \; a \in V_I \; , \; x \in V_0^+ \right\}$$

and define a binary relation T on V_1 as follows :

$$T\left(\lfloor s_i, a, x, s_j \rfloor, \lfloor s_i', a', x', s_j' \rfloor \right) \quad \text{holds iff } s_j = s_i'$$

Consider now the fuzzy grammar with type 3 rules (Ref. [4]):

$$FG_3 = (V_N, V_T, P, T, J, \delta) \quad \text{where}$$
$$V_N = \{ T, X_1, X_2, ..., X_k \}$$
$$V_T = V_1$$
$$P = P_1 \cup P_2 \cup P_3 \cup P_4$$
$$J = J_1 \cup J_2 \cup J_3 \cup J_4$$

The sets P_i, J_i with $i = 1,2,3,4$ are given as :

(1) P_1 is the set of nonterminal initial rules of the form

$$\text{(r)} \quad T \to \lfloor s_i, a, x, s_j \rfloor X_j \quad \delta(r)$$

for $1 \le j \le k$ where $\delta(r) = \min\lfloor \pi(s_i), \mu(s_i, a, x, s_j) \rfloor$. J_1 is the set of labels corresponding to these rules .

(2) P_2 is the set of nonterminal rules of the form

$$\text{(r)} \quad X_i \to \lfloor s_i', a', x', s_j' \rfloor X_j \quad \delta(r)$$

for $1 \le i, j \le k$ and $T\left(\lfloor s_i, a, x, s_j \rfloor, \lfloor s_i', a', x', s_j' \rfloor \right)$, with $\delta(r) = \mu(s_i', a', x', s_j')$. J_2 is the set of labels corresponding to the new rules .

(3) P_3 is the set of terminal rules of the form

$$\text{(r)} \quad X_i \to \lfloor s_i', a', x', s_j' \rfloor \quad \delta(r)$$

for $1 \le i \le k$ and $\delta(r) = \min\lfloor \mu(s_i', a', x', s_j'), \eta(s_j') \rfloor$. J_3 is the set of labels of these rules .

(4) P_4 is the set of terminal initial rules of the form

$$\text{(r)} \quad T \to \lfloor s_i, a, x, s_j \rfloor \quad \delta(r)$$

where $\delta(r) = \min \lfloor \pi(s_i), \mu(s_i, a, x, s_j), \eta(s_j) \rfloor$. J_4 is the set of labels of these rules . We note by $R = L(FG_3)$ the regular fuzzy language generated by the grammar FG_3 defined above. One observes that the words $t_1 t_2 ... t_n \in R$ are of the form

$$[s_0, a_1, x_1, s_1] \ [s_1, a_2, x_2, s_2] \dots [s_{n-1}, a_n, x_n, s_n]$$

If s_0 is the initial state with the grade $\pi(s_0) = \pi_0$, s_n is the final state with the grade $\eta(s_n) = \eta_n$ and the productions $s_{i-1}a_i \to x_i s_i$ apply with the grade $\mu(s_{i-1}, a_i, x_i, s_i) = \mu_i$ for $1 \le i \le n$, then the grade of the membership of the word $t_1 t_2 ... t_n \in V_1{*}$ to the set R is given by

$$\delta_R(t_1 t_2 ... t_n) = \max_D \min [\pi_0, \mu_1, \mu_2, ..., \mu_n, \eta_n] \tag{1}$$

where the maximum is taken over all the fuzzy derivation chains D from T to $t_1 t_2 ... t_n$. Next , we introduce two ε-free fuzzy homomorphisms [1] in the following way :

$$h_1 : V_1 \times V_I \to [0,1] \text{ and } h_2 : V_1 \times V_0^+ \to [0,1] \text{ and}$$

$$h_1([s_i, a, x, s_j], b) = \begin{cases} 1 & \text{if } b = a \\ 0 & \text{if } b \ne a \end{cases}$$

$$h_2([s_i, a, x, s_j], y) = \begin{cases} 1 & \text{if } y = x \\ 0 & \text{if } y \ne x \end{cases}$$

One observes that the homomorphism h_2 is ε-free , since the $FGSM$ application was assumed ε-free. Then the following equality of fuzzy sets has to be proved :

$$FGSM(L) = h_2(h_1^{-1}(L) \cap R) \tag{2}$$

Let $y \in FGSM(L)$ with grade $\gamma(y)$, $y = x_1 x_2 ... x_n$ and $x_i \in V_0^+$ for $1 \le i \le n$. Then, there is $x \in L$ with grade $\alpha(x)$, $x = a_1 a_2 ... a_n$ and $a_i \in V_I$ for $1 \le i \le n$, such that $y \in FGSM(x)$ with the grade $\beta(x, y)$. From here it results that for any s_0, initial state with grade π_0, there is s_n, final state with the grade η_n, such that :

$$M \quad : \quad s_0 a_1 a_2 ... a_n \underset{\mu_1}{\Rightarrow} x_1 s_1 a_2 ... a_n \underset{\mu_2}{\Rightarrow} ... \underset{\mu_{n-1}}{\Rightarrow} x_1 x_2 ... x_{n-1} s_{n-1} a_n \underset{\mu_n}{\Rightarrow} x_1 x_2 ... x_n s_n$$

where $\mu_i = \mu(s_{i-1}, a_i, x_i, s_i)$ for $1 \le i \le n$. It then results :

$$\beta(x, y) = \max_M \min [\pi_0, \mu_1, \mu_2, ..., \mu_n, \eta_n]$$

where the maximum is taken over all the chains of moves M which translate x in y.

The grade of the membership of y to $FGSM(L)$ is given by

$$\gamma(y) = \min[\alpha(x), \beta(x,y)] \text{ or}$$

$$\gamma(y) = \min\left[\alpha(x), \max_{M} \min\left[\pi_0, \mu_1, \mu_2, ..., \mu_n, \eta_n\right]\right]$$

Since $x \in L$ with the grade $\alpha(x)$ it results that

$$t_1 t_2 ... t_n = h_1^{-1}(a_1 a_2 ... a_n) \in h_1^{-1}(L)$$

with the same grade $\alpha(x)$. Then, the grade of the membership of the word $t_1 t_2 ... t_n$ to $h_1^{-1}(L) \cap R$ is given as

$$\rho_{h_1^{-1}(L)\cap R}(t_1 t_2 ... t_n) = \min[\alpha(x), \delta_R(t_1 t_2 ... t_n)]$$

which, according to eq. (1) can be written as

$$\rho_{h_1^{-1}(L)\cap R}(t_1 t_2 ... t_n) = \min\left[\alpha(x), \max_{D} \min[\pi_0, \mu_1, ..., \mu_n, \eta_n]\right]$$

Then, $y = x_1 x_2 .. x_n = h_2(t_1 t_2 ... t_n) \in h_2(h_1^{-1}(L) \cap R)$ with the same grade with which $t_1 t_2 ... t_n \in h_1^{-1}(L) \cap R$, therefore

$$\nu_{h_2(h_1^{-1}(L)\cap R)}(y) = \min\left[\alpha(x), \max_{D} \min[\pi_0, \mu_1, ..., \mu_n, \eta_n]\right]$$

We have thus shown that $y \in h_2(h_1^{-1}(L) \cap R)$ with the same grade with which $y \in FGSM(L)$, wherefrom it results the inclusion

$$FGSM(L) \subseteq h_2(h_1^{-1}(L) \cap R)$$

The inverse inclusion can be proved in a similar way , therefore the equality (2) is true. Since $\mathcal{L}$ is an $AFFL$, by using its closure under the ε-free fuzzy homomorphism , inverse fuzzy homomorphism and intersection with fuzzy regular languages , it results that $FGSM(L) \in \mathcal{L}$, which proves the theorem.

Next, we investigate the closure property of the $AFFL$ with respect to the inverse $FGSM$ application.

Theorem 2: Any $AFFL$ is closed under the inverse $FGSM$ application

Proof. Let $FGSM$ be a fuzzy generalized sequential machine:

$$FGSM = (S, V_I, V_0, \mu, \pi, \eta)$$

where

$$\mu : S \times V_I \times V_0^* \times S \to [0,1]$$

$$\pi : S \to [0,1]$$

$$\eta : S \to [0,1]$$

and let $\mathcal{L}$ be an $AFFL$. We consider an arbitrary language L from $\mathcal{L}$ and must show that $FGSM^{-1}(L)$ also belongs to $\mathcal{L}$.

Let us consider a rewriting system RW obtained from the $FGSM$ by inverting all the productions [2]. Then, the set of the productions from RW consists of all the productions of the form:

$$xs_j \to s_i a \quad , \quad s_i, s_j \in S \ , \ a \in V_I \ \text{ and } \ x \in V_0^*$$

such that $s_i a \to xs_j$ is a production of the $FGSM$.

We introduce an auxiliary alphabet :

$$V_1 = \left\{ [x, s_j, s_i, a] \,\middle|\, xs_j \to s_i a \in RW \right\}$$

and define a binary relation T on V_1:

$$T([x, s_j, s_i, a], [x', s'_j, s'_i, a']) \ \text{ holds iff } \ s_i = s'_j$$

The regular fuzzy language R over V_1 is defined in the same way as in the previous proof and the words $t_1 t_2 ... t_n \in R$ will be of the form :

$$[x_n, s_n, s_{n-1}, a_n] \ [x_{n-1}, s_{n-1}, s_{n-2}, a_{n-1}] \ldots [x_1, s_1, s_0, a_1]$$

We introduce two fuzzy homomorphisms :

$$h_1 : V_1 \times V_0^* \to [0,1] \ \text{ and } \ h_2 : V_1 \times V_I \to [0,1]$$

defined as follows

$$h_1([x, s_j, s_i, a], y) = \begin{cases} 1 & if \ y = x \\ 0 & if \ y \neq x \end{cases}$$

$$h_2([x, s_j, s_i, a], b) = \begin{cases} 1 & if \ b = a \\ 0 & if \ b \neq a \end{cases}$$

One observes that h_2 is an ε-free homomorphism. The following equality of fuzzy sets takes place :

$$FGSM^{-1}(L) = h_2\left(h_1^{-1}(L) \cap R\right)$$

Since $\mathcal{L}$ is an $AFFL$ it results that $FGSM^{-1}(L) \in \mathcal{L}$, and the theorem is proved.

Conclusions

The abstract families of fuzzy languages $(AFFL)$ were defined [1] as an extension of the abstract families of languages [2]. In the present work, we have shown that the $AFFL$ have additional closure properties, namely, under the ε – free fuzzy generalized sequential machine ($FGSM$) application and under the inverse $FGSM$ application.

REFERENCES

1. *I. Bucurescu , A. Pascu*, "Abstract families of fuzzy languages" , Rev. Roum. Linguistique **33** (1988), 3-9
2. *A. Salomaa*, "Formal languages", New York and London, Academic Press, 1973
3. *I. Bucurescu, A. Pascu*, "Fuzzy generalized sequential machines ", Bull. Math. Soc. Sci. Math. Roum. **36(84)** nr.2 (1992), 99-102
4. *M. Mizumoto*, "Fuzzy automata and fuzzy grammars", Ph.D. Thesis, Faculty of Engineering Science, Osaka University, Osaka, Japan, 1971.