KINEMATICS OF THE SPATIAL 3-*UPU* PARALLEL ROBOT

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Recursive matrix relations for kinematics analysis of a parallel manipulator, namely the universal-prismatic-universal (3-UPU) robot, are established in this paper. Knowing the translational motion of the platform, the inverse kinematics problem is solved based on the connectivity relations. Finally, some simulation graphs for the input displacements, velocities and accelerations are obtained.

Keywords: Connectivity relations; kinematics, parallel robot

1. Introduction

Parallel robots are closed-loop structures presenting very good potential in terms of accuracy, stiffness and ability to manipulate large loads. One of the main bodies of the mechanism is fixed and is called *the base*, while the other is regarded as movable and hence is called *the moving platform* of the manipulator. Generally, the number of actuators is typically equal to the number of degrees of freedom and each leg is controlled at or near the fixed base [1].

Compared with traditional serial manipulators, the following are the potential advantages of parallel architectures: higher kinematical accuracy, lighter weight and better structural stiffness, stable capacity and suitable position of actuator's arrangement, low manufacturing cost and better payload carrying ability. Accuracy and precision in the direction of the tasks are essential since the positioning errors of the tool could end in costly damage [2].

Important efforts have been devoted to the kinematics and dynamic investigations of parallel robots. Among these, the class of manipulators known as Stewart-Gough platform, used in flight simulators, focused great attention (Stewart [3]; Di Gregorio and Parenti Castelli [4]). The prototype of the Delta parallel robot developed by Clavel [5] at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper [6] at the University of Maryland, as well as the Star parallel manipulator (Hervé and Sparacino [7]), are equipped with three motors, which train on the mobile platform in a three-degrees-of-freedom general translational motion. Angeles [8], Wang and Gosselin [9] analysed the kinematics, dynamics and singularity loci of Agile Wrist spherical robot with three revolute

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actuators. In the previous works of Li and Xu [10], [11], the 3-PRC and 3-PUU spatial parallel kinematical machines with relatively simple structure were presented with their kinematics solved in details.

In the present paper, a recursive matrix method, already implemented in the inverse kinematics of parallel robots, is applied to the inverse analysis of a spatial 3-DOF mechanism. It has been proved that the number of equations and computational operations reduces significantly by using a set of matrices for kinematics modelling.

2. Kinematics analysis

The 3-*U<u>P</u>U* architecture parallel manipulators are already well known in the mechanism community. The manipulator consists of a fixed base $A_1B_1C_1$, a circular mobile platform $A_5B_5C_5$ and three extensible legs with identical kinematical structure. Each limb connects the fixed base to the moving platform by two universal (*U*) joints interconnected through a prismatic (<u>*P*</u>) joint made up of a cylinder and a piston. Hydraulic or pneumatic systems can be used to vary the lengths of the prismatic joints and to control the location of the platform (Fig. 1).

Since each *U* joint consists of two intersecting revolute (*R*) joints, each leg is equivalent to a *RR<u>P</u>RR* kinematical chain. But, the mechanism can be arranged to achieve only translational motions with certain conditions satisfied, i.e., in each kinematical chain the axis of the first revolute joint is parallel to that of the last one and the two intermediate joint axes are parallel to one another. There are three active mobile prismatic joints and six passive universal joints. The first leg *A* is typically contained within the Ox_0z_0 vertical plane, whereas the remaining legs B, C make the angles $\alpha_B = 120^0, \alpha_C = -120^0$ respectively, with the first leg (Fig. 2).

For the purpose of analysis, we assign a fixed Cartesian coordinate system $Ox_0y_0z_0(T_0)$ at the centred point *O* of the fixed base platform and a mobile frame $Gx_Gy_Gz_G$ on the mobile platform at its centre *G*. The angle ν between Ox_0 and Gx_G axes is defined as the *twist* angle of the robot.

The moving platform is initially located at a *central configuration*, where the platform is not translated with respect to the fixed base and the origin *O* of the fixed frame is located at an elevation $OG = h$ above the mass centre *G*.

To simplify the graphical image of the kinematical scheme of the mechanism, in what follows we will represent the intermediate reference systems by only two axes, so as is used in most of robotics papers [1], [2], [8]. It is noted that the relative rotation with angle $\varphi_{k,k-1}$ or the relative translation of the body T_k with the displacement $\lambda_{k,k-1}$ must always be pointed along the direction of the z_k axis.

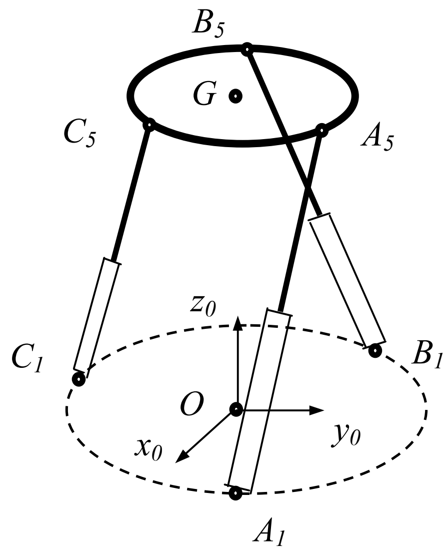

Fig. 1 Symmetric spatial 3-*U<u>P</u>U* parallel robot

The first active leg *A*, for example, consists of the cross of a fixed Hooke joint linked at the frame $A_1x_1^Ay_1^Az_1^A$, characterised by absolute angle of rotation φ_{10}^A, angular velocity $\omega_{10}^A=\dot{\varphi}_{10}^A$ and the angular acceleration $\varepsilon_{10}^A=\ddot{\varphi}_{10}^A$, connected at a moving cylinder $A_2x_2^Ay_2^Az_2^A$ of length l_2, which has a relative rotation around $A_2z_2^A$ axis with the angle φ_{21}^A, so that $\omega_{21}^A=\dot{\varphi}_{21}^A, \varepsilon_{21}^A=\ddot{\varphi}_{21}^A$. An actuated prismatic joint is as well as a piston of length l_3 linked to the $A_3x_3^Ay_3^Az_3^A$ frame, having a relative displacement λ_{32}^A, velocity $v_{32}^A=\dot{\lambda}_{32}^A$ and acceleration $\gamma_{32}^A=\ddot{\lambda}_{32}^A$. Finally, a second universal joint $A_4x_4^Ay_4^Az_4^A$ having the angular velocity $\omega_{43}^A=\dot{\varphi}_{43}^A$ and the angular acceleration $\varepsilon_{43}^A=\ddot{\varphi}_{43}^A$ is introduced at the edge of a moving platform, which can be schematised as a circle of radius r in a relative rotation around $A_5z_5^A$ axis with angular velocity $\omega_{54}^A=\dot{\varphi}_{54}^A$ and angular acceleration $\varepsilon_{54}^A=\ddot{\varphi}_{54}^A$.

At the central configuration, we also consider that the three sliders are initially starting from the same position $l_1=h/\sin\beta-l_2$ and that the angles of orientation of universal joints are given by

$$\alpha_A=0,\ \alpha_B=\frac{2\pi}{3},\ \alpha_C=-\frac{2\pi}{3},\ \nu=\frac{\pi}{6} \tag{1}$$

$$(l_0-r\cos\nu)\tan\delta=r\sin\nu\ ,\ r\sin\nu\tan\beta=h\sin\delta\ ,$$

where δ and β are two constant angles of rotation around the axes z_1^A and z_2^A, respectively.

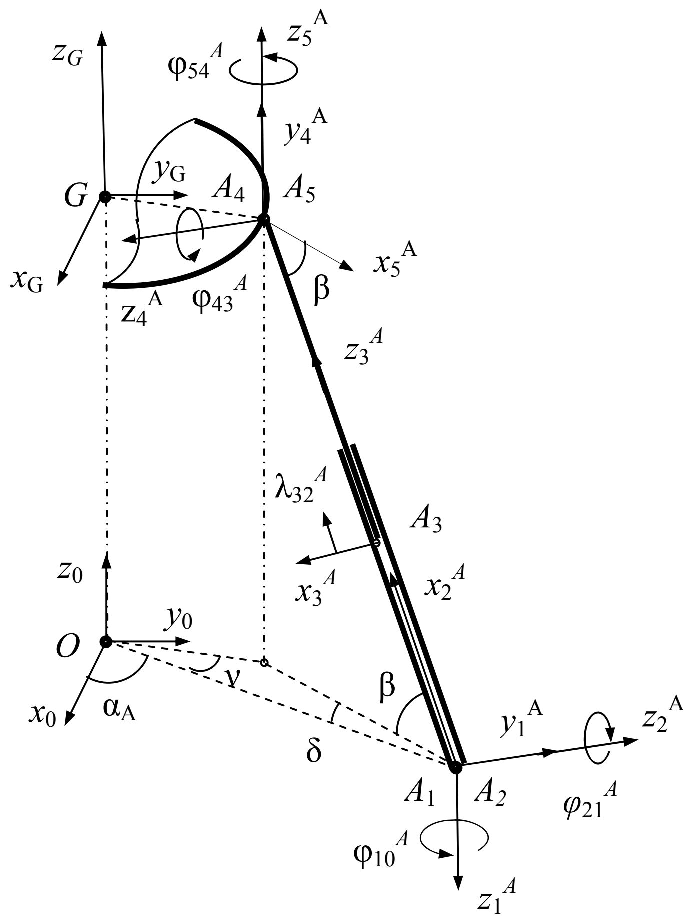

Fig. 2 Kinematical scheme of first leg A of parallel mechanism

Starting from the reference origin O and pursuing along three independent legs $OA_0A_1A_2A_3A_4$, $OB_0B_1B_2B_3B_4$, $OC_0C_1C_2C_3C_4$, we obtain following transformation matrices

$$p_{10} = p_{10}^{\varphi} a_{\delta} \theta_3 a_{\alpha}^{i},\ p_{21} = p_{21}^{\varphi} a_{\beta} \theta_1^T,\ p_{32} = \theta_2,\ p_{43} = p_{43}^{\varphi} a_{\beta} \theta_2,\ p_{54} = p_{54}^{\varphi} \theta_1^T \qquad (2)$$

$$p_{20} = p_{21} p_{10},\ p_{30} = p_{32} p_{20},\ p_{40} = p_{43} p_{30},\ p_{50} = p_{54} p_{40} \quad (p = a, b, c), \quad (i = A, B, C),$$

where we denote the matrices [12]:

$$a_{\alpha}^{i} = rot(z, \alpha_i),\ a_{\delta} = rot(z, \delta),\ a_{\beta} = rot(z, \beta)$$

$$\theta_1 = rot(x, \pi/2),\ \theta_2 = rot(y, \pi/2),\ \theta_3 = rot(y, \pi),\ p_{k,k-1}^{\varphi} = rot(z, \varphi_{k,k-1}^{i}). \qquad (3)$$

The angles $\varphi_{10}^{A}, \varphi_{21}^{A}$, for example, characterise the sequence of rotations for the first universal joint A_1.

In the inverse geometric problem, the position of the mechanism is completely given through the coordinate x_0^G, y_0^G, z_0^G of the mass centre *G*. Consider, for example, that during three seconds the moving platform remains in the same orientation and the motion of the centre G along a *rectilinear trajectory* is expressed in the fixed frame $Ox_0y_0z_0$ through the following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{h - z_0^G}{z_0^{G*}} = 1 - \cos\frac{\pi}{3}t\,, \tag{4}$$

where the values $2x_0^{G*}, 2y_0^{G*}, 2z_0^{G*}$ denote the final position of the moving platform.

Nine independent variables $\varphi_{10}^{A},\ \varphi_{21}^{A},\ \lambda_{32}^{A},\ \varphi_{10}^{B},\ \varphi_{21}^{B},\ \lambda_{32}^{B},\ \varphi_{10}^{C},\ \varphi_{21}^{C},\ \lambda_{32}^{C}$ will be determined by vector-loop equations

$$\vec{r}_{10}^{\,A} + \sum_{k=1}^{4} a_{k0}^{T}\vec{r}_{k+1,k}^{\,A} - \vec{r}_{G}^{\,A_5} = \vec{r}_{10}^{\,B} + \sum_{k=1}^{4} b_{k0}^{T}\vec{r}_{k+1,k}^{\,B} - \vec{r}_{G}^{\,B_5} = \vec{r}_{10}^{\,C} + \sum_{k=1}^{4} c_{k0}^{T}\vec{r}_{k+1,k}^{\,C} - \vec{r}_{\mathrm{G}}^{\,C_5} = \vec{r}_{0}^{\,G}\,, \tag{5}$$

where

$$\begin{aligned}
&\vec{r}_{10}^{\,i} = l_0 a_{1\alpha}^{iT}\vec{u}_1,\ \vec{r}_{21}^{\,i} = \vec{0},\ \vec{r}_{32}^{\,i} = (l_1 + \lambda_{32}^{i})\vec{u}_1,\ \vec{r}_{43}^{\,i} = l_3\vec{u}_3,\ \vec{r}_{54}^{\,i} = \vec{0}\\
&\vec{r}_{G}^{\,i_5} = [r\cos(\alpha_i + \nu) \quad r\sin(\alpha_i + \nu) \quad 0]^T,\ \ (i = A, B, C).
\end{aligned} \tag{6}$$

From the vector equations (5) we obtain the inverse geometric solution for the spatial manipulator:

$$\begin{aligned}
&(l_1 + l_3 + \lambda_{32}^{i})\sin(\varphi_{10}^{i} + \delta)\cos(\varphi_{21}^{i} + \beta) = r\sin\nu - x_0^G\sin\alpha_i + y_0^G\cos\alpha_i\\
&-(l_1 + l_3 + \lambda_{32}^{i})\sin(\varphi_{10}^{i} + \delta)\cos(\varphi_{21}^{i} + \beta) = r\cos\nu + x_0^G\cos\alpha_i + y_0^G\sin\alpha_i - l_0\\
&(l_1 + l_3 + \lambda_{32}^{i})\sin(\varphi_{21}^{i} + \beta) = z_0^G\,.
\end{aligned} \tag{7}$$

The translation conditions concerning the absolute orientation of the moving platform are given by the following identities

$$\overset{\circ}{p}{}_{50}^{T}\, p_{50} = R = I\,,\ (p = a, b, c),\quad (i = A, B, C) \tag{8}$$

$$\overset{\circ}{p}_{50} = p_{50}(t = 0) = \theta_1^T a_\beta \theta_2 \theta_2 a_\beta \theta_1^T a_\delta \theta_3 a_\alpha^i\,,$$

where $R = I$ is the diagonal identity matrix. From these conditions we obtain the relations between the angles of rotation

$$\varphi_{43}^{i} = \varphi_{21}^{i},\ \varphi_{54}^{j} = \varphi_{10}^{i},\ (i = A, B, C)\,. \tag{9}$$

The motion of the component elements of the leg A, for example, are characterized by the relative velocities of the joints

$$\vec{v}_{32}^{A} = \dot{\lambda}_{32}^{A}\vec{u}_3,\ \vec{v}_{k,k-1}^{A} = \vec{0} \tag{10}$$

and by the following relative angular velocities

$$\vec{\omega}_{32}^{A} = \vec{0},\ \vec{\omega}_{k,k-1}^{A} = \dot{\varphi}_{k,k-1}^{A}\vec{u}_3\,, \tag{11}$$

which are *associated* to skew-symmetric matrices

$$\tilde{\omega}_{32}^{A} = \tilde{0},\ \tilde{\omega}_{k,k-1}^{A} = \dot{\varphi}_{k,k-1}^{A}\tilde{u}_3 \quad (k = 1, 2, 4, 5)\,. \tag{12}$$

From the geometrical constraints (5), we obtain the *matrix conditions of connectivity* and, finally, the relative velocities $v_{10}^{A}, \omega_{21}^{A}, \omega_{32}^{A}$ of the first leg A [13]:

$$\omega_{10}^{A}\vec{u}_j^{\,T} a_{10}^{T}\tilde{u}_3 a_{21}^{T}\{\vec{r}_{32}^{\,A} + a_{32}^{T}\vec{r}_{43}^{\,A}\} + \omega_{21}^{A}\vec{u}_j^{\,T} a_{20}^{T}\tilde{u}_3\{\vec{r}_{32}^{\,A} + a_{32}^{T}\vec{r}_{43}^{\,A}\} + v_{32}^{A}\vec{u}_j^{\,T} a_{20}^{T}\vec{u}_1 = \vec{u}_j^{\,T}\dot{\vec{r}}_0^{\,G}\,,$$

$$(j = 1, 2, 3)\,. \tag{13}$$

If the other two kinematical chains of the robot are pursued, analogous relations can be easily obtained.

To describe the kinematical state of each link with respect to the fixed frame, we compute the angular velocity $\vec{\omega}_{k0}^{A}$ and the linear velocity $\vec{v}_{k0}^{A}$ in terms of the vectors of the preceding body, using a recursive manner:

$$\vec{\omega}_{k0}^{A} = a_{k,k-1}\vec{\omega}_{k-1,0}^{A} + \dot{\varphi}_{k,k-1}^{A}\vec{u}_3\,,\ \ \vec{v}_{k0}^{A} = a_{k,k-1}\vec{v}_{k-1,0}^{A} + a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}\vec{r}_{k,k-1}^{\,A} + \dot{\lambda}_{k,k-1}^{A}\vec{u}_3\,. \tag{14}$$

Rearranging, the above nine constraint equations (7) can be written as three independent relations

$$(r\sin\nu - x_0^G\sin\alpha_i + y_0^G\cos\alpha_i)^2 + (r\cos\nu + x_0^G\cos\alpha_i + y_0^G\sin\alpha_i - l_0)^2 + (z_0^G)^2 =$$

$$= (l_1 + l_3 + \lambda_{32}^{i})^2 \quad (i = A, B, C)\,, \tag{15}$$

concerning the coordinates x_0^G, y_0^G, z_0^G and the displacements $\lambda_{32}^A, \lambda_{32}^B, \lambda_{32}^C$ only. The derivative with respect to the time of conditions (15) leads to the matrix equation

$$J_1\dot{\vec{\lambda}}_{10} = J_2[\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{z}_0^G]^T\,, \tag{16}$$

where two significant matrices J_1 and J_2 are, respectively,

$$J_1 = diag\,\{\delta_A \quad \delta_B \quad \delta_C\}\,,\ J_2 = \begin{bmatrix} \beta_1^A & \beta_2^A & \beta_3^A \\ \beta_1^B & \beta_2^B & \beta_3^B \\ \beta_1^C & \beta_2^C & \beta_3^C \end{bmatrix}, \tag{17}$$

with the notations

$$\delta_i = l_1 + l_3 + \lambda_{32}^i \qquad (i = A, B, C)$$

$$\beta_1^i = x_0^G + r\cos(\alpha_i + \nu) - l_0 \cos\alpha_i \,,\; \beta_2^i = y_0^G + r\sin(\alpha_i + \nu) - l_0 \sin\alpha_i \,,\; \beta_3^i = z_0^G \,. \qquad (18)$$

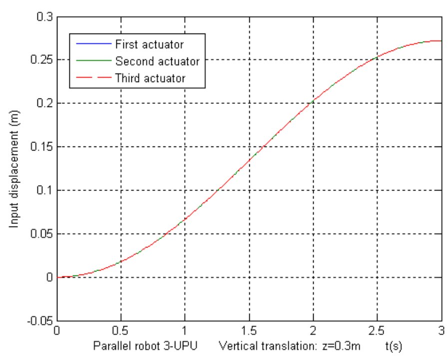

Fig. 3 Input displacements λ_{32}^i of the three sliders

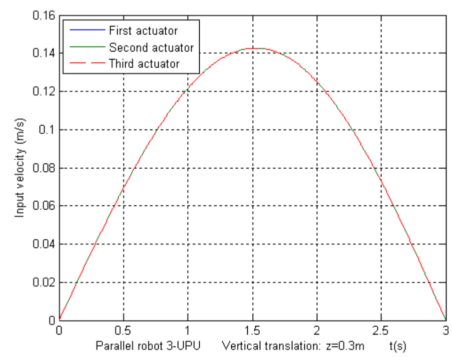

Fig. 4 Input velocities v_{32}^i of the three sliders

The particular configurations of the three kinds of singularities for the closed-loop kinematical chains can be determined through the analysis of two Jacobian matrices J_1 and J_2 [14], [15], [16].

The angular accelerations ε_{10}^A, ε_{21}^A and the relative acceleration γ_{32}^A of leg A are expressed by new conditions of connectivity [17]:

$$\begin{aligned}
&\varepsilon_{10}^A \vec{u}_j^{\,T} a_{10}^T \tilde{u}_3 a_{21}^T \{\vec{r}_{32}^{\,A} + a_{32}^T \vec{r}_{43}^{\,A}\} + \varepsilon_{21}^A \vec{u}_j^{\,T} a_{20}^T \tilde{u}_3 \{\vec{r}_{32}^{\,A} + a_{32}^T \vec{r}_{43}^{\,A}\} + \gamma_{32}^A \vec{u}_j^{\,T} a_{20}^T \vec{u}_1 = \vec{u}_j^{\,T} \ddot{\vec{r}}_0^{\,G} - \\
&- \omega_{10}^A \omega_{10}^A \vec{u}_j^{\,T} a_{10}^T \tilde{u}_3 \tilde{u}_3 a_{21}^T \{\vec{r}_{32}^{\,A} + a_{32}^T \vec{r}_{43}^{\,A}\} - \omega_{21}^A \omega_{21}^A \vec{u}_j^{\,T} a_{20}^T \tilde{u}_3 \tilde{u}_3 \{\vec{r}_{32}^{\,A} + a_{32}^T \vec{r}_{43}^{\,A}\} - \qquad (19)\\
&- 2\omega_{10}^A \omega_{21}^A \vec{u}_j^{\,T} a_{10}^T \tilde{u}_3 a_{21}^T \tilde{u}_3 \{\vec{r}_{32}^{\,A} + a_{32}^T \vec{r}_{43}^{\,A}\} - 2\omega_{10}^A v_{32}^A \vec{u}_j^{\,T} a_{10}^T \tilde{u}_3 a_{21}^T \vec{u}_1 - 2\omega_{21}^A v_{32}^A \vec{u}_j^{\,T} a_{20}^T \tilde{u}_3 \vec{u}_1 \,, \\
&\qquad (j = 1, 2, 3)\,.
\end{aligned}$$

Computing the derivatives with respect to the time of equations (14), we obtain a recursive form of accelerations $\vec{\varepsilon}_{k0}^{\,A}$ and $\vec{\gamma}_{k0}^{\,A}$:

$$\vec{\varepsilon}_{k0}^{\,A} = a_{k,k-1} \vec{\varepsilon}_{k-1,0}^{\,A} + \ddot{\varphi}_{k,k-1}^A \vec{u}_3 + \dot{\varphi}_{k,k-1}^A a_{k,k-1} \tilde{\omega}_{k-1,0}^A a_{k,k-1}^T \vec{u}_3 \,, \qquad (20)$$

$$\vec{\gamma}_{k0}^{\,A} = a_{k,k-1} \vec{\gamma}_{k-1,0}^{\,A} + a_{k,k-1} \{\tilde{\omega}_{k-1,0}^A \tilde{\omega}_{k-1,0}^A + \tilde{\varepsilon}_{k-1,0}^A\} \vec{r}_{k,k-1}^{\,A} + 2\dot{\lambda}_{k,k-1}^A a_{k,k-1} \tilde{\omega}_{k-1,0}^A a_{k,k-1}^T \vec{u}_3 + \ddot{\lambda}_{k,k-1}^A \vec{u}_3$$

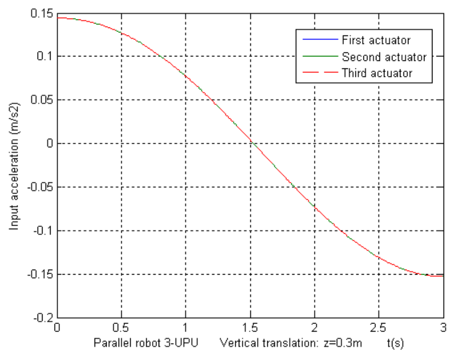

Fig. 5 Input accelerations γ_{32}^{i} of the three sliders

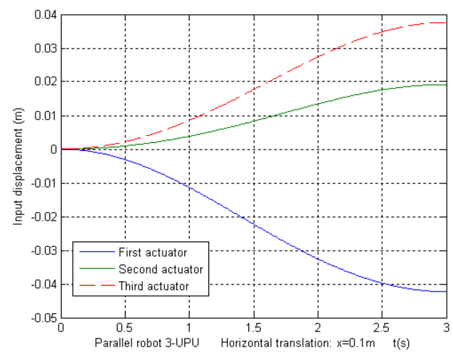

Fig. 6 Input displacements λ_{32}^{i} of the three sliders

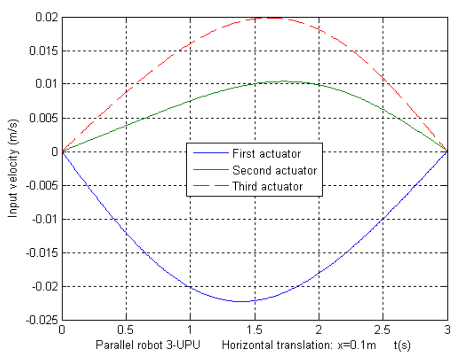

Fig. 7 Input velocities v_{32}^{i} of the three sliders

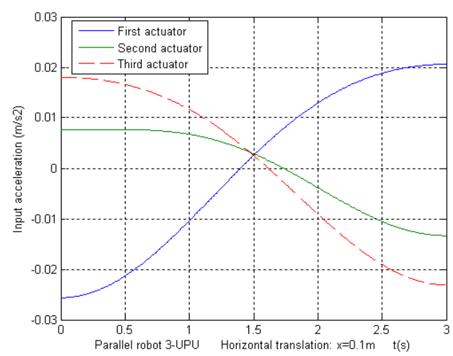

Fig. 8 Input accelerations γ_{32}^{i} of the three sliders

As an application let us consider a 3-*U<u>P</u>U* parallel manipulator which has the following architectural characteristics

$$x_0^{G*} = 0.05\,m\ ,\ y_0^{G*} = 0.05\,m\ ,\ z_0^{G*} = 0.15\,m$$

$$r = 0.2\,m\ ,\ OA_1 = l_0 = 0.6\ m,\ A_3A_4 = l_3 = 0.6\,m,\ h = 0.8\ m,\ \Delta t = 3\ s\ .$$

Using MATLAB software, a computer program was developed to solve the kinematics of the 3-*U<u>P</u>U* parallel robot. To develop the algorithm, it is assumed that the platform starts at rest from a central configuration and moves pursuing successively rectilinear translations.

Two examples are solved to illustrate the algorithm. For the first example, the platform moves along the *vertical direction* z_0 with variable acceleration while all the other positional parameters are held equal to zero. The time-histories for the input displacements λ_{32}^{i} (Fig. 3), relative velocities v_{32}^{i} (Fig. 4) and relative accelerations γ_{32}^{i} (Fig. 5) are carried out for a period of $\Delta t = 3$ seconds in terms of analytical equations (4).

For the case when the platform's centre G moves along a *rectilinear horizontal trajectory* without any rotation of the platform, the graphs are illustrated in Fig. 6, Fig. 7 and Fig. 8.

3. Conclusions

Some exact relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in the present paper. The simulation certifies that one of the major advantages of the current matrix recursive formulation is the accuracy and a smaller processing time for the numerical computation.

Choosing the appropriate serial kinematical circuits connecting many moving platforms, the present method can be easily applied in forward and inverse mechanics of various types of parallel mechanisms, complex manipulators of higher degrees of freedom and particularly *hybrid structures*, with increased number of components of the mechanisms.

R E F E R E N C E S

[1] *Tsai, L-W.*, Robot analysis: the mechanics of serial and parallel manipulator, Wiley, 1999

[2] *Merlet, J-P.*, Parallel robots, Kluwer Academic, 2000

[3] *Stewart, D.*, A Platform with Six Degrees of Freedom, Proc. Inst. Mech. Eng., 1, 15, 180, pp. 371-378, 1965

[4] *Di Gregorio, R.*, *Parenti Castelli, V.,* Dynamics of a class of parallel wrists, ASME Journal of Mechanical Design, 126**,** 3, pp. 436-441, 2004

[5] *Clavel, R.*, Delta: a fast robot with parallel geometry, Proceedings of 18th International Symposium on Industrial Robots, Lausanne, pp. 91-100, 1988

[6] *Tsai, L-W., Stamper, R.,* A parallel manipulator with only translational degrees of freedom, ASME Design Engineering Technical Conferences, Irvine, CA, 1996

[7] *Hervé, J-M., Sparacino, F.*, Star. A New Concept in Robotics, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, pp.176-183, 1992

[8] *Angeles, J.,* Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms, Springer, 2002

[9] *Wang, J., Gosselin, C.*, A new approach for the dynamic analysis of parallel manipulators, Multibody System Dynamics, Springer, 2, 3, pp. 317-334, 1998

[10] *Li, Y., Xu, Q.*, Dynamic modeling and robust control of a 3-PRC translational parallel kinematic machine, Robotics and Computer-Integrated Manufacturing, Elsevier, 25, pp. 630-640, 2009

[11] *Li, Y., Xu, Q.*, Stiffness analysis for a 3-PUU parallel kinematic machine, Mechanism and Machine Theory, Elsevier, 43, pp.186-200, 2008

[12] *Staicu, S.*, Modèle dynamique en robotique, UPB Scientific Bulletin, Series D: Mechanical Engineering, 61, 3-4, pp. 5-19, 1999

[13] *Staicu, S.,* Dynamics analysis of the Star parallel manipulator, Robotics and Autonomous Systems, Elsevier, 57, 11, pp. 1057-1064, 2009

[14] *Liu, X-J., Zin, Z-L., Gao, F.,* Optimum design of 3-DOF spherical parallel manipulators with respect to the conditioning and stiffness indices, Mechanism and Machine Theory, Elsevier, 35, 9, pp. 1257-1267, 2000

[15] *Xi, F., Zhang, D., Mechefske, C.M., Lang, S.Y.T.*, Global kinetostatic modelling of tripod-based parallel kinematic machine, Mechanism and Machine Theory, Elsevier, 39, 4, pp. 357-377, 2001

[16] *Bonev, I., Zlatanov, D., Gosselin, C.,* Singularity analysis of 3-DOF planar parallel mechanisms via screw theory, ASME Journal of Mechanical Design, 25, 3, pp. 573-581, 2003

[17] *Staicu, S.,* Dynamics of the 6-6 Stewart parallel manipulator, Robotics and Computer-Integrated Manufacturing, Elsevier, 27, 1, pp. 212-220, 2011