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CHAOTIC DYNAMICS IN ECONOPHYSICS: MODELING THE MARKET FORCES

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În această lucrare prezentăm generarea unor serii numerice prin metoda "scării diavolului generalizate" (GDSC). Ponderile utilizate in costrucția GDSC țin cont de cele trei categorii de factori care influențeaza piața cursului de schimb, anume factorii economici, politici, şi psihologici; un al patrulea factor luat în considerație, suplimentar fiind dat de "condiția de închidere", care din punct de vedere matematic exprimă normarea probabilității, iar din punct de vedere economic echilibrul pe termen lung. Seriile sintetizate sunt comparate cu seria reală a cursului de schimb leu-dolar american (ROL-USD) din perioada ianuarie 1990-decembrie 2007, luâd în considerare parametrii caracteristici dinamicilor neliniare: dimensiunea de corelație, exponenții Hurst şi Lyapunov, şi deviația de la structura monofractală. Valorile ponderilor din GDSC sunt alocate în conformitate cu caracteriticile celor trei sub-perioade reprezentative ale evoluției sistemului economic românesc. Rezultatele indică faptul că seriile pot fi simulate în acest fel, dar sunt necesare studii suplimentare privind alocarea numerică a ponderilor.

In this work we simulate real time series by using the synthesizing method of the „generalized devil staircase" (GDSC). The weights are reflecting the forces that currently influence the exchange rate markets, namely the economic factors, the political situation, and the environmental psychology driving the economic behaviour; as a fourth factor we use the closure condition representing both the mathematical condition of probability normalization and the long run economic equilibrium. The simulated series are compared with the real one of the exchange rate between the Romanian currency „leu" and the United States Dollar (ROL-USD) over the interval January 1990-December 2007 by considering the quantities that usually characterize the non linear dynamics: the correlation dimension, the Hurst and Lyapunov exponents, and a factor pointing out the deviation from monofractality. The numerical values for the weights are assigned according to the partition in three distinct subintervals corresponding to the main evolutionary features of the Romanian economy. We

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obtained promising results but supplemental studies are necessary for a better allocation of the numerical weights.

Keywords: market forces, time series synthesis, devil staircase, ROL-USD exchange rate.

1. Introduction

Market functionality is basically imprinting the features of any economic system and therefore is one of the most often analyzed issues in the last decades. The yearly Index of Economic Freedom (IEF) is providing a useful picture of more than 160 countries against a list of 50 independent variables divided into 10 broad factors of economic freedom [1]. To what extent the market preserves its functional autonomy and could accept the government intervention is a subject of discussion. Real economies are essentially non-linear and their evolution, extremely sensitive to the initial conditions is often characterized by strange attractors which yield a balance between the role of the government tending to preserve the economic stability and social fairness, and the undertakers' autonomy whose ultimate goals are profit and growth. Econophysics [2] mainly consist of physico-mathematical models that apply to the markets. The concept of self similarity has extended to read out functions emerging as time series from the economic systems, the exchange rate being one of the most available parameter [3]. The ultimate goal of having reliable forecasting models fascinates and engages a lot of research for analyzing local conditions that help understanding markets co-integration and how general principles of pricing are influenced in order to be incorporated in useful risk management theories [4, 5]. In this respect the problem of replicating the real series seems to be a difficult task of limited success [6] and only in the area of theoretical self-similar shapes like the Cantor set, Koch's curve or Mandelbrot's angular generators [7].

Here, we propose a new method suggested by an obvious qualitative resemblance of two shapes, namely the exchange rate Japanese Yen-USD (Fig.1a) and synthesized GDSC (Fig.1b) reproduced after C. Arizmendi [8].

We focus upon the synthesis of the exchange rate time series of the daily sampled exchange rate between ROL and USD over more than 18 years split in three distinctive periods according to the existing studies on the recent history of the Romanian economy [9]: 1990-1997 (inertia and passive transition), 1998-2002 (active transition), and 2003-2007 (quasi-stable).

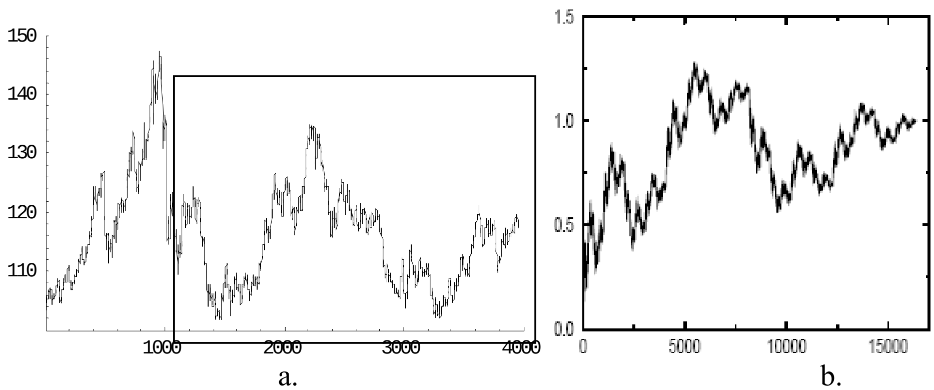


Fig.1 Yen-USD exchange rate between Jan. 1996- Dec. 2006 (a), and GDSC synthesis (b). The rectangle on the left side encloses a fragment resembling the simulated GDSC on the right side

2. Theoretical background

2.1 The generalized devil staircase

The functional f representing GDSC [10] is recursively constructed in the same manner as the Cantor sequence, but for four segments: each interval at each step j of the construction is divided into four subintervals of the same length on which we distribute respectively the weights p_1, p_2, p_3, and p_4 such as

$$f\left(2^{-2(k+1)}, j\right) = f\left(2^{-2k}, j\right) + p_j\,, j{=}1{,}2{,}3{,}4. \qquad (1)$$

The constraints on the values of the weights are:

i) The closure condition, according to which the sum equals the unity

$$p_1 + p_2 + p_3 + p_4 = 1. \qquad (2)$$

The upper limit for each term is also set to unity and negative values are allowed

$$|p_j|<1. \qquad (2')$$

ii) The initial condition

$$f\left(2^{-2}, j\right) = 0 \text{ for all } j = 1,2,3,4\,. \qquad (3)$$

For sure, the GDSC generating method imposes some intrinsic limits for the fractal features of the series relying in the built-in, point-to-point variation given by Eq.(1) with conditions (2) and (2') (e.g. four variables, one bonding relation, limited ranges). Therefore we also expect some limitations in the ranging values of the parameters we chose to characterize the non linear dynamics of the series.

2.2 The market forces and the exchange rate

Supply and demand for any given currency, and thus its value, are influenced by several elements. These elements generally fall into three categories

[11]: *i)* economic factors, *ii)* political conditions, and *iii)* market psychology. The closure condition (2) is acting as a balancing force consistent with the long run basic economic equation. Practically, the national consumption can never exceed the earnings. The eventual prevalence of consumption could be temporarily solved by external loans; on the other side, a higher level of the income is encouraging the imports, so that the long range equilibrium is always achieved. The succession of the weights does matter, but its importance diminishes as the number of the construction steps in GDSC increases. For 4096 samples we choose the following correspondence between forces and GDSC weights:

Table 1.

The correspondence market forces-GDSC weights

Category	**Economic**	**Political**	**Psychological**	**Closure condition**
Weight	p_1	p_2	p_3	p_4

We briefly agree to make the assumption the weight is positive when the net effect of the corresponding factor is to raise the demand for USD i.e. to increase the number of ROLs payed for one USD; consequently one has:

i) The weight p_1 for economic factors is positive if the external trade balance is negative i.e. the imports exceed exports thus exerting devaluating pressure on ROL and shortage on USD. Assuming the impossibility of yearly unbalances greater than 20% of gross domestic product (GDP), we choose a linear sizing scale as follows: for any year p_1=0 if the exports perfectly compensate the imports, and p_1=±1 if the trade unbalance accounts for 20% GDP (the positive sign stands for the prevalence of the imports). The value for p_1 is the averaged value over all the years in the interval subjected to the analysis. Yearly data for GDP and trade balance deficit are provided by the National Institute of Statistics [12]. As a general remark, it should be noted that in Romanian case we deal only with *deficit,* so that p_1 is positive every year.

ii) The weight p_2 for political conditions is negative (like a regulatory feedback) if the aggregated IEF (AIEF) indicates a significant degree of freedom i.e. the role of the political authorities (government included) is mostly a regulatory one. The government normally takes the appropriate measures to increase the confidence in the national currency by open market instruments. A positive value is assigned when its role distorts the repartition of the resources on economic bases. Again, for each year belonging to the analyzed interval, the etalon condition would assign p_2=0 for the median value of the AIEF=0.5. The additional fixed points are p_2=−1 for AIEF=1, and consequently p_2=1 for AIEF=0. Analogously, the value of p_2 is taken as the value averaged over all the years belonging to the analyzed interval.

iii) The weight p_3 for market psychology is positive if the “bearish” behaviour prevail against the “bullish” (e.g. the economic agents buy USD on the assumption their price will further increase; this is the typical case during the unstable and inflationary periods). This is the most delicate point since there are not confident studies focused on the Romanian market psychology. We decide to relate the value of p_3 to the linear trend of the exchange rate along each currently analyzing time interval. The etalon condition has to be robust against very different periods: on one side the interval 1990-1997 exhibited a depreciation of more than 20000% (current values), and on the other side the interval 2003-2007 when a certain stability began to manifest its benefiting effects. We fix the extreme limits p_3=0 for the case when the slope of the linear least square errors approximating polynomial of the log values is also zero, and p_3=1 when the slope is 0.001day^{-1}; correspondingly, the negative values are symmetrical.

Obviously, all sizing we have chosen above contain a certain degree of arbitrary. Besides the assumptions related to the impossibility of occurring events like unbalances of the external trade greater than 20% of GDP (in 2007 the deficit was of around 21.7 billion Euros i.e. 17.8% of GDP), or daily depreciations/appreciations larger than 10% (in the case of the huge depreciation during the passive transition époque 1990-1997 this is equivalently to the overall value of 22000% that corresponds to the slope of 10^{-3}day^{-1} in decimal log values), the weight for political conditions has already a built-in limit since IEF is referring to the *existing* state of the art in economic analysis.

2.3 The parameters that characterize the non linear dynamics

In order to compare the real and the simulated series we select the following parameters:

i) The correlation dimension (*CorrD*), for the complexity of the attractor, if any [13];

ii) The Hurst exponent, for checking the type of long time correlations i.e. the persistence or the anti-persistence of the series, or, more unlikely, the lack of correlations if the value of the exponent is close to 0.5;

iii) The largest Lyapunov exponent [14], for allowing the chaotic behaviours to be distinguished from the regular ones: if the exponent is positive, there is chaos and the longer the time, the weaker the prediction;

iv) The difference δ between the generalized Hurst exponents $h(0)$ and $h(2)$ as a measure of deviation from the monofractal structure [15]

$$\delta = |h(0) - h(2)|. \quad (4)$$

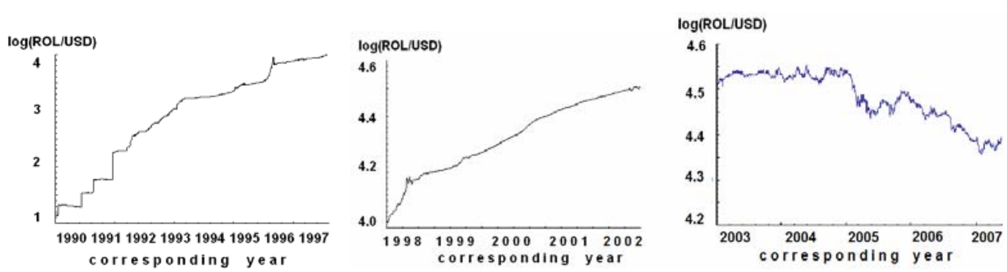
While denoting $x(n)$ the samples of the series, the generalized Hurst exponent $h(q)$ is related to the q-order fluctuations when removing the local linear trend $x_{\mathrm{TREND},n}$ in the current box of size n [16]:

$$F(q,n)=\left\langle \left\{ \frac{1}{n}\sum_{k=1}^{n}\left[x(n_0+k)-x_{\mathrm{TREND},n}(n_0+k)\right]^q \right\}^{1/q} \right\rangle_{n_0} \sim n^{h(q)}. \quad (5)$$

In Eq. (5) the symbol $\langle\ \rangle_{n_0}$ means the average over all starting points n_0. One should note that for a monofractal series $h(0)=h(2)$.

3. Computational results

The lengths of the real series corresponding to the significant periods of the historical evolution 1990-1997, 1998-2002, and 2003-2007 are of 2956, 1828, and 1827 points, respectively (see Fig.2). In order to avoid the spurious broadening and translation of the multifractal spectrum, an additional lengthening procedure was used [17].



a. **b.** **c.**

Fig.2 The significant intervals of the ROL-USD exchange rate along its historical evolution

For the synthesized series we considered equal lengths of 4096 points that are lowering the dependence on the permutations $p_1,\dots.p_4$ down to the error margins of the processing algorythms [17].

A MatLab program provides the simulated values given by Eq.(1); the series is integrated, after that both simulated and real series are undergoing the same steps: take the log values, detrend with a linear polynomial, and finally the residuals are processed with the appropriate software packages. The results are shown in Table 2.

Table 2

Simulated series versus real series

	Corr.D	Hurst exponent	Deviation coefficient δ	Lyapunov exponent
1990-1997				
Real series	1.5	0.597	0.4108	0.037
Simulated series $p_1=0.19$, $p_2=0.16$, $p_3=0.96$, $p_4=-0.31$	2.1	0.608	0.3112	0.116
1998-2002				

Real series	2.9	0.548	0.2586	0.056
Simulated series p_1=0.26, p_2=0.02, p_3=0.39, p_4=0.23	3.2	0.643	0.2561	0.133
2003-2007				
Real series	4.5	0.502	0.2109	0.102
Simulated series p_1=0.45, p_2=−0.24, p_3=−0.01, p_4=0.80	3.9	0.534	0.2577	0.121

As general remarks, we can point out the following:

i) The simulation agrees with the increasing trend of *CorrD* in real series, as the effect of the transition from a simple economy of command to a functional market one, endowed with sensitive instruments of prediction and correction inside the system;

ii) The real series evolves toward a monofractal profile while the GDSC simulation slightly deviates from the general tendency;

iii) The real series evolves toward a Brownian walk with the Hurst exponent slightly below 0.5. This is not exactly the case with the GDSC simulation. However, the series remain persistent for both real and simulated data.

iv) The largest Lyapunov exponents are all positive, that support the assumption the GDSC theoretical self similar model is statistically consistent with the real evolution of the exchange rate ROL-USD.

4. Conclusions

We presented a GDSC simulation of the ROL-USD exchange rate, daily series, covering a large time interval with very different dynamic features. We adopted here the GDSC because it generates a fractal structure and exhibits an apparent simple morphism between the number of variables and the number of market forces, including the closure condition. The main groups of market forces acting upon the exchange rate were quantitatively quantized by taking into account the sense of the pressure exerted on the market price of USD. The closure condition is inserted in a natural way into the model in order to preserve the long run equilibrium i.e. to have no long run trend in the genuine series.

Although there is no perfect fit of the values of the selected parameters, there are promising results related to the acceptable progress of the correlation dimension and the evolution toward a dominant monofractal behaviour as well. The presence of positive largest Lyapunov exponents validates the presence of chaos in all cases. The Hurst exponent is not monotonously decaying as in the real case, but it seems to preserve the coarse feature of the series namely its persistence. However, supplemental studies are necessary for a better allocation of the numerical weights in various real situations.

The overall simulation is limited and is strongly depending on the processing procedures, so cautious conclusions could be drawn. Every procedural filtering like integrating, logarithm taking, or trend removal induces spurious coherence that influence the final results. More profound analysis should be undertaken to adequately assess the overall variations of the simulated parameters with respect to the GDSC variables.

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