SINGULAR PERTURBATION DETECTION USING WAVELET FUNCTION REPRESENTATION

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Calitatea distribuţiei energiei electrice este o problemă foarte importantă în contextul actual. Detecţia perturbaţiilor singulare rămâne dificil de rezolvat pentru cele mai multe metode de analiză. Acest tip de perturbaţie este caracteristică pentru anumite evenimente anormale ce pot apare în reţea. Identificarea lor depinde de eficienţa instrumentelor de detecţie şi analiză. Lucrarea de faţă prezintă un nou mod de lucru bazat pe descompunerea în subspaţii de funcţii Wavelet. Această procedură are o eficienţă ridicată chiar când semnalul conţine o perturbaţie aleatoare permanentă (zgomot). Conţinutul lucrării vizează prezentarea modului de lucru şi interpretarea rezultatelor obţinute. Caracteristicile procedurilor dovedesc avantajele şi aplicabilitatea metodei.

The distribution power quality is a very important issue in present context. The singular perturbation detection remains a serious challenge for the most analysis methods. This type of perturbation is characteristic for certain abnormal events which may occur. Their identification depends on the efficiency of the detection and analysis tools. The presented paper gives a new methodology based on the Wavelet function decomposition. This procedure has a good efficiency even when the signal is affected by a permanent noise perturbation. The paper content is focused on presenting the working mode and the results' interpretation. The features of the procedures prove the advantages and the applicability of the methods.

Keywords: Wavelet functions, functional space, singular perturbation.

1. Introduction

The electric equipment may be monitored using the analysis of the electric signal that is measured on the power supply connections. Certain anomalies produce singular perturbations on the current or voltage signals. These perturbations consist in rapid variations with irregular occurrence on very short time durations. Frequency domain analysis may give good results in certain situations. This is mainly based on using high pass filter. The band-pass filter is not recommended because the spectrum of the perturbation signal cannot be well

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specified. However, in certain cases efficiency becomes very low. This is explained by two characteristics specific to singular perturbation. The first refers to the very short duration of evolution. The action of a high pass filter is related in a certain way to the signal duration. Even if this is conformable to the band-pass of the filter, the output signal depends on the signal duration, because of the power dissipated by the filter. The second problem is related to the presence of noise. This represents a permanent perturbation but with variable characteristics with random features. Usually a part of the noise signal spectrum conforms to the filters transfer function. Therefore, it may happen that the singular perturbation that needs to be detected is covered by the noise passing to the filter.

2. Characteristics of Wavelet analysis

In principle, the analysis of a signal may be done in the time domain, in the frequency domain or using a combination of the both domains. The frequency domain analysis, for example by Fourier transform, gives a useful global image, but it doesn't offer relevant results in the case of singular perturbations. This can be explain by the constant value of the Fourier transform for a Dirac "delta" pulse, which can be used as mathematical model for a singular perturbation.

Better results may be obtained using the "Short Time Fourier Transform" (STFT) or the Wavelet transform. Their characteristics may be observed in Fig.1, 2 and 3. In all the cases, using a 2D window, for each time moment (in abscissa), and for each frequency (in ordinate, with relative units) the corresponding transform value is given by a gray scale representation. Here, a particular test signal is taken under consideration. Its variation is based on two frequency sinusoidal components perturbed by two "delta" pulses singular perturbations. Although the Wavelet transform would offer promising results, as it can be seen in Fig.3, it must be observed that, when an additive random "noise" is present, the singular perturbation detection is poor for the both transforms (Fig.1 and 2).

It must be mentioned that the presented numerical results are based on a graphical representation, where the levels of gray in the upper window correspond to the Wavelet transform values for time and frequency coordinates. The shaded content of the image is due to the distribution of the computed values. For the Short time Fourier transform, the discontinuous distribution is due to the local frequency domain window, which is translated, step by step, along the whole interval. In order to give a better solution, the paper presents a special type of four levels Wavelet decomposition, matched for this kind of singular perturbation. This procedure carries out a functional projection of the time dependent signal, on certain subspaces, defined by corresponding Wavelet functions bases.

It is very interesting to point out one of the essential difference of the Wavelet representation, compared to frequency domain filtering. The latter is

defined by the "pass-band", that is the frequency interval where the attenuation is reduced, so that, if for certain of the spectrum attenuation is null, applying the filtering operation repeatedly this not change the output signal. However, in Wavelet function representation the projection process is based on the selection of certain variation speeds of original signal, which is, in a certain way, similar to the Taylor series development, where terms correspond to derivatives of different order. For a signal with finite variation speed, the maximum value of the derivative decreases as the derivation order increases. Thus projections corresponding to higher order Wavelet functions result in lower values. Therefore increasing the level of the decomposition is not similar to successively applying frequency domain filtering. Unlike filtering in the frequency domain, projections on spaces generated by the Wavelet functions do not lead to disjoint frequency domains. At each stage (level) of the projection two "components" of the signal are obtained: one corresponding to low frequencies and one corresponding to high frequencies. By choosing a certain base of Wavelet functions, we can obtain a "hierarchy" or a "tree" of several levels decomposition for a certain original signal, each level having a partition in a low and a high range of frequencies.

An interesting propriety of Wavelet decomposition is that elements composing the signal do not conserve their characteristics when we use different levels of decomposition, as in the case of frequency components in classic filtering. For example, the sign of certain components may change depending on the level of decomposition, as we will see in Fig.6. However, one of important characteristic of Wavelet decomposition is the fact that the signal can be precisely reconstructed after the decomposition. In the case of frequency filtering, this is not possible due to the additive structure of the decomposition and to the impossibility of obtaining an ideal transfer function (that ensures disjoint frequency bands).

Very important is to point that the Wavelet decomposition is a powerful analysis tool, giving more relevant information comparative with the Wavelet discrete transform (WDT).
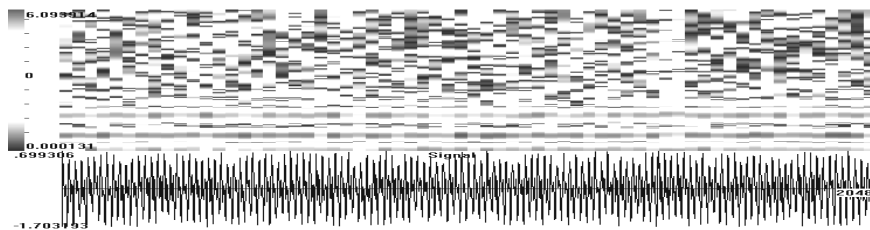

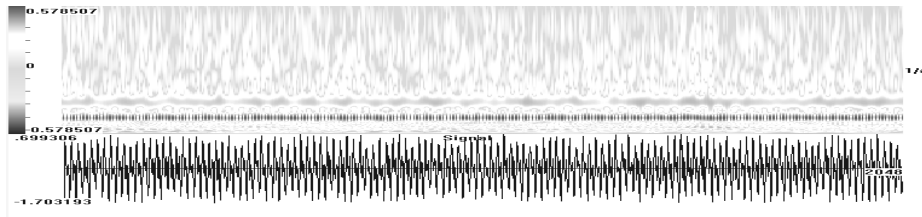Fig.1. "Short time Fourier transform" (for 32 samples window).

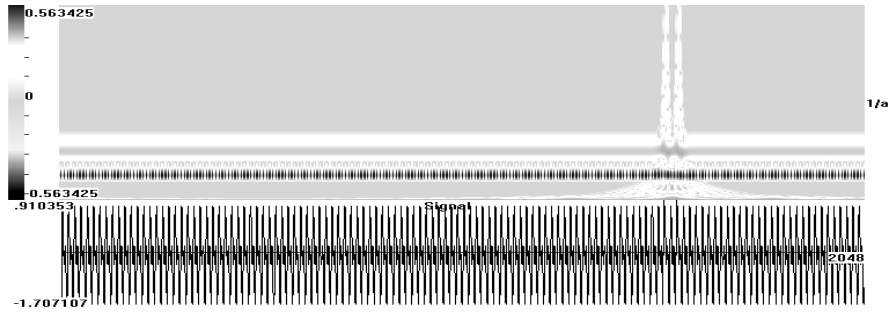Fig.2. Wavelet discrete transform ("noised" signal).

Fig.3 Wavelet discrete transform ("noiseless" signal).

3. Mathematical formalism

As it was showed above, the Wavelet analysis is based on a projection procedure using the spaces defined by the Wavelet functions. In the following, it is presented the working principle, considering a more simple Wavelet function class, known as Haar functions. These are simple shaped functions, with step variation, very different compared to the Daubechies functions, whose evolution, having an asymptotical variation, is described in Fig.4 (for the 10th order).
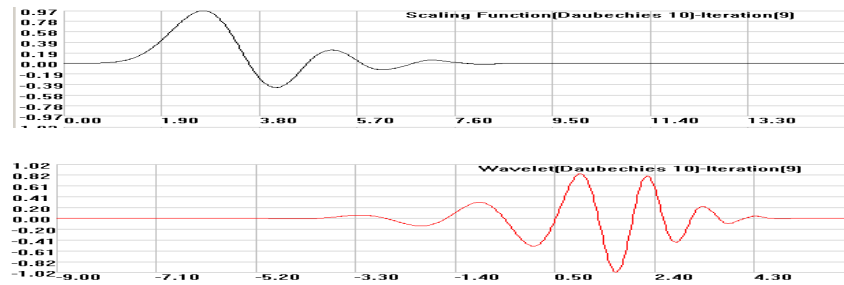

Fig.4. The Daubechies function shape (scaling function and Wavelet function)

The Wavelet projection process implies two categories of functions, named as "scaling functions" and "Wavelet functions" (in proper sense). Generally, the first have a semi-interval defined support and the second a well localized support. The "scaling functions", denoted as $\Phi(t)$ are given by the following representation (for the Haar functions case):

$$\phi(t) = \begin{cases} 1 & t \in [0,1) \\ 0 & t \notin [0,1) \end{cases} \tag{1}$$

and the "Wavelet" class functions, denoted as $\Psi(t)$, are represented by:

$$\psi(t) = \begin{cases} 1 & t \in [0,1/2) \\ -1 & t \in [1/2,1) \\ 0 & t \notin [0,1) \end{cases} \tag{2}$$

The elements of the function base, corresponding to $\phi(t)$ type function are defined by:

$$\phi_i^j(t) = \sqrt{2^j}\,\phi(2^j \cdot t - i) \quad \text{for} \quad j = 0,1,\ldots; \quad i = 0,1,\ldots,2^j\text{-}1 \tag{3}$$

where the j index correspond to an expansion of the function shape and the i index corresponds to a translation. These functions generate the spaces V^j:

$$V^j = sp\left\{\phi_i^j\right\}_{i=0,\ldots,2^j-1} \quad \text{having the propriety:} \quad V^j \subseteq V^{j+1} \tag{4}$$

The elements of the function base, corresponding to $\Psi(t)$ type function are defined by:

$$\psi_i^j(t) = \sqrt{2^j}\,\psi(2^j \cdot t - i) \quad \text{for} \quad j = 0,1,\ldots; \quad i = 0,1,\ldots,2^j\text{-}1 \tag{5}$$

These functions generate the spaces W^j:

$$W^j = sp\left\{\psi_i^j\right\}_{i=0,\ldots,2^j-1} \quad \text{having the propriety:} \quad W^j \subseteq V^{j+1} \tag{6}$$

It must be mentioned that these definitions may be also formulated with other relations, which may have other rules for intervals and for signs. In our case the $\sqrt{2^j}$ constant is chosen in order to satisfy the norm condition:

$$\left\langle \psi_i^j, \psi_i^j \right\rangle = \int_0^1 \left(\psi_i^j(t)\right)^2 dt = 1 \tag{7}$$

where we denote by $< . , . >$ the scalar product.

The spaces generated by the function above defined satisfy the following direct sum relation:

$$V^{j+1} = V^j \oplus W^j \tag{8}$$

Using these functions, the projection process is equivalent to direct sum decomposition. This process is well known as a multiresolution analysis, in the composed space:

$$V^0 \oplus W^0 \oplus W^1 \oplus W^2 \qquad\qquad (9)$$

corresponding to the following representation:

$$
\begin{aligned}
f =\ & \left\langle f, \phi_0^0 \right\rangle \phi_0^0 + \left\langle f, \psi_0^0 \right\rangle \psi_0^0 + \\
& \left\langle f, \psi_0^1 \right\rangle \psi_0^1 + \left\langle f, \psi_1^1 \right\rangle \psi_1^1 + \\
& \left\langle f, \psi_0^2 \right\rangle \psi_0^2 + \left\langle f, \psi_1^2 \right\rangle \psi_1^2 + \left\langle f, \psi_2^2 \right\rangle \psi_2^2 + \left\langle f, \psi_3^2 \right\rangle \psi_3^2
\end{aligned}
\qquad (10)
$$

4. Case study, numerical simulations

In order to emphasize the characteristics of the presented analysis procedure, a composed signal is taken under consideration, containing two "sinus" components superposed by two "delta" distribution pulses and a random "noise" signal function. The "delta" pulses correspond to singular perturbations of the sinusoidal signal (as we can see in an expanded form in Fig.5). The composed signal function may be described by an analytical relation:

$$f(t) = \sin(1000\pi t) + \sin(2000\pi t) + \alpha\left[\delta(t - t_1) + \delta(t - t_2)\right] + Rnd(1) \qquad (11)$$
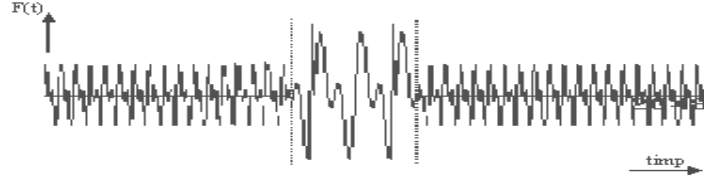

Fig.5. The composed signal (magnification containing two pulses of "δ" singular perturbation).

The main objective of the Wavelet analysis is the singular perturbation detection (represented by the delta pulses), when a high amplitude "noise" is present. The data were processed by a program package referred in [1]. The discrete form of the original signal, containing the sinusoidal functions and the singular perturbations is added by a unitary amplitude pseudo-random signal. For the numerical simulation 2048 samples were processed. The Wavelet decomposition was done for four levels, using the 5th order, biorthogonal Daubechies functions. The four level results, represented in Fig.6 are obtained from the decomposition resulting data, applying a 50% threshold. Thus, the noise contribution is completely eliminated. The presented numerical results are an optimal choice, derived from several numerical experiments, where there are used different functional bases as: Spline or non-orthogonal Daubechies wavelet functions of inferior or superior rank.

Noiseless composed signal

Composed signal (A_0)

"1A- high" level

"2B- high" level selection with a 50% threshold

"3D- high" level selection with a 50% threshold

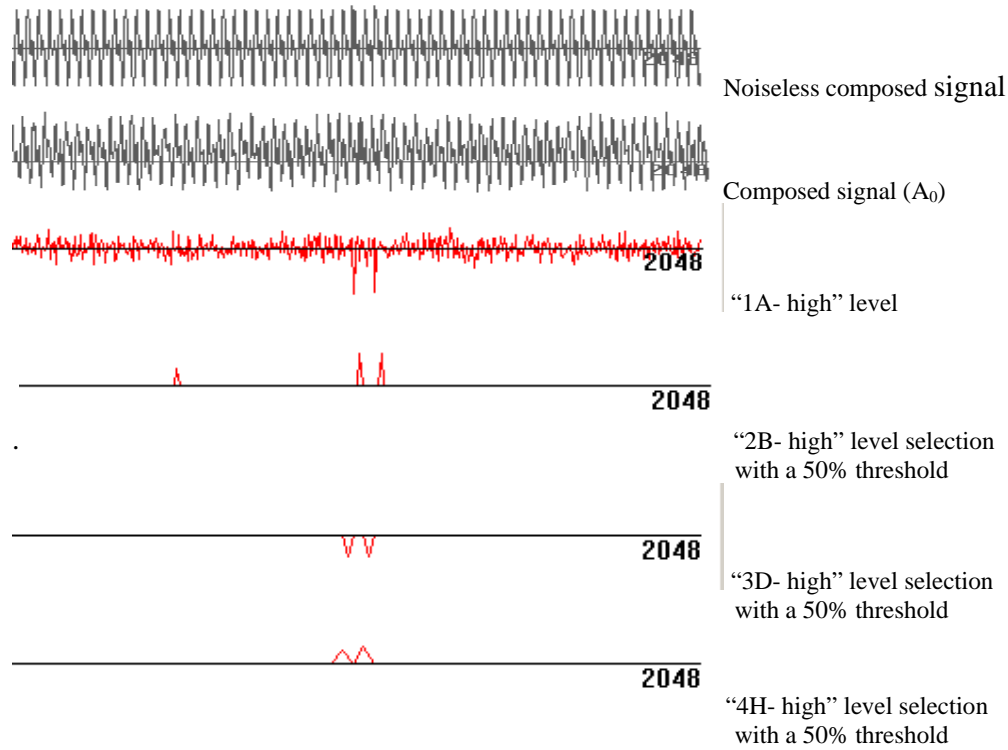"4H- high" level selection with a 50% threshold

Fig.6. Four decomposition levels, corresponding to the Wavelet spaces for 2048 samples.

The presented results are very interesting for a comparative analysis between the Wavelet type analysis and the classical detection possibilities. It must be emphasized that the Wavelet analysis, used for the singular perturbation detection is not equivalent with a high-pass filtering process, combined with controlled level peak detection. This approach is affected by some inherent restrictions:

- the singular perturbation signal must exceed (in amplitude) the level of the high frequency component contained by the random noise.
- the detection threshold level must be chosen so that to discriminate the singular perturbation from the "noise" signal.

The Wavelet analysis can eliminate these problems. Thus, as it can be see in Fig.7 that the "noise" high frequency component is higher or comparable with the singular perturbation level. This signal results from a value selection, using a 75% threshold, applied to the original composed signal (A_0).

The efficiency of the Wavelet analysis, used as a singular perturbation detection tool, is based especially on the functional projecting procedure, and the Wavelet functions characteristics. This class contains functions with short time evolution (limited support), comparable with the singular perturbation duration.

Thus, by the projection process (similar with a "correlation" function) are selected those signal variations that are comparable, from the shape and duration point of view, with the singular perturbation. When we gradually increase the decomposition level, the short time variations are emphasized, having a certain intensity level relative to the other components of the signal. In Fig.8, which shows a magnification of the Fig.7 representation, it can be observed that, when the noise is present, the signal peaks are more frequent when the singular perturbation occurs. So, in that zone, the intensity (the energy) of the short time signal variations is more important.

DB1[0A]<+0.000/ 2.699>

Fig.7. The original signal remainder, after applying a 75% threshold processing.
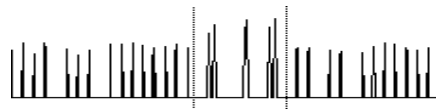
Fig.8. The magnification of the perturbation zone, for the Fig.7 signal.

5. Conclusions

The presented results yield some unique features of Wavelet multiresolution analysis. This enables the detection of some signal perturbations with unpredictable characteristics, with minimum computing effort and maximum efficiency. This gives better results compared to "Wavelet discrete transform" and the other classical or algorithmic methods. In our case, the imposed threshold is used only for clarity reasons. The perturbation may be detected as well by amplitude peak detection, after the Wavelet decomposition. In contrast, no good results can be obtained if the amplitude peak detection actions on the original signal (with high frequency components shown in Fig.6).

R E F E R E N C E S

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