KINEMATICS OF THE 3-PUU TRANSLATIONAL PARALLEL MANIPULATOR

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Articolul stabileşte relaţii matriceale pentru analiza cinematică a unei maşini cinematice de translaţie cu structură paralelă (PKM), adică manipulatorul prismatic-universal-universal (3-PUU). Cunoscând mişcarea de translaţie a platformei, problema de cinematică inversă este rezolvată printr-un procedeu bazat pe relaţii de conectivitate. În final, se obţin câteva grafice pentru deplasările, vitezele şi acceleraţiile de la intrarea în sistemul mecanic.

Recursive matrix relations for kinematics analysis of a translational parallel kinematical machine (PKM), namely the prismatic-universal-universal (3-PUU) manipulator are established in this paper. Knowing the translational motion of the platform, the inverse kinematics problem is solved based on the connectivity relations. Finally, some simulation graphs for the input displacements, velocities and accelerations are obtained.

Keywords: Connectivity relations; Kinematics; Parallel manipulator

List of symbols

$a_{k,k-1}$, $b_{k,k-1}$, $c_{k,k-1}$: orthogonal transformation matrices;
θ_1, θ_2 : two constant orthogonal matrices
$\vec{u}_1, \vec{u}_2, \vec{u}_3$: three right-handed orthogonal unit vectors
r : radius of the circular moving platform
l_2 : length of the limb of each leg
θ : angle of inclination of three sliders
ν : twist angle of the circular moving platform
λ_{10}^{A}, λ_{10}^{B}, λ_{10}^{C} : displacements of three prismatic actuators
$\varphi_{k,k-1}$: relative rotation angle of T_k rigid body
$\vec{\omega}_{k,k-1}$: relative angular velocity of T_k
$\vec{\omega}_{k0}$: absolute angular velocity of T_k
$\tilde{\omega}_{k,k-1}$: skew-symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$

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$\vec{\varepsilon}_{k,k-1}$: relative angular acceleration of T_k

$\vec{\varepsilon}_{k0}$: absolute angular acceleration of T_k

$\tilde{\varepsilon}_{k,k-1}$: skew-symmetric matrix associated to the angular acceleration $\vec{\varepsilon}_{k,k-1}$

$\vec{r}^{\,A}_{k,k-1}$: relative position vector of the centre A_k of joint

$\vec{v}^{\,A}_{k,k-1}$: relative velocity of the centre A_k

$\vec{\gamma}^{\,A}_{k,k-1}$: relative acceleration of the centre A_k

1. Introduction

Parallel kinematical machines (PKM) are closed-loop structures presenting very good potential in terms of accuracy, stiffness and ability to manipulate large loads. In general, these mechanisms consist of two main bodies coupled via numerous legs acting in parallel. One of the main bodies is fixed and is called *the base*, while the other is regarded as movable and hence is called *the moving platform* of the manipulator. The number of actuators is typically equal to the number of degrees of freedom. Each leg is controlled at or near the fixed base [1].

Compared with traditional serial manipulators, the following are the potential advantages of parallel architectures: higher kinematical accuracy, lighter weight and better structural stiffness, stable capacity and suitable position of actuator's arrangement, low manufacturing cost and better payload carrying ability. But, from the application point of view, the limited workspace and complicated singularities are two major drawbacks of the parallel manipulators. Thus, PKM are more suitable for situations where high precision, stiffness, velocity and heavy load-carrying capability are required within a restricted workspace [2].

Over the past two decades, parallel manipulators have received much attention from research and industry. Important companies such as Giddings & Lewis, Ingersoll, Hexel and others have developed them as high precision machine tools. Accuracy and precision in the direction of the tasks are essential since the positioning errors of the tool could end in costly damage.

Considerable efforts have been devoted to the kinematics and dynamic investigations of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [3]; Di Gregorio and Parenti Castelli [4]). They are used in flight simulators and more recently for PKMs. The prototype of the Delta parallel robot (Clavel [5]; Tsai and Stamper [6]; Staicu [7]) developed by Clavel at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper at the University of Maryland, as well as the Star parallel manipulator (Hervé and Sparacino [8]), are equipped with three motors, which train on the mobile platform in a three-degree-of-freedom general, translational motion. Angeles [9], Wang and Gosselin [10], Staicu [11] analysed the kinematics, dynamics and singularity loci of Agile Wrist spherical robot with three actuators.

In the previous works of Li and Xu [12], [13], [14] the 3-PRS parallel robot, the 3-PRC and 3-PUU parallel kinematical machines with relatively simple structure were presented with their kinematics solved in details. The potential application as a positioning device of the tool in a new parallel kinematics machine for high precision blasting attracted a scientific and practical interest to this manipulator type.

In the present paper, a recursive matrix method, already implemented in the inverse kinematics of parallel robots, is applied to the direct analysis of a spatial 3-DOF mechanism. It has been proved that the number of equations and computational operations reduces significantly by using a set of matrices for kinematics modelling.

2. Kinematics analysis

The 3-*PUU* architecture parallel manipulators are already well known in the mechanism community and several TPMs have been designed and analysed separately [15], [16]. Although these manipulators use different methods for the actuator's arrangement, they still can be considered as the same type of mechanism since they can be resolved using the same kinematics technique. The manipulator consists of a fixed triangular base $A_0B_0C_0$, a circular mobile platform and three limbs with an identical kinematical structure. Each limb connects the fixed base to the moving platform by a prismatic (*P*) joint followed by two universal (*U*) joints in sequence, where the *P* joint is driven by a lead screw linear actuator (Fig. 1).

Since each *U* joint consists of two intersecting revolute (*R*) joints, each limb is equivalent to a *PRRRR* kinematical chain. This mechanism can be arranged to achieve only translational motions with certain conditions satisfied, i.e., in each kinematical chain the axis of the first revolute joint is parallel to that of the last one and the two intermediate joint axes are parallel to one another. Since the geometric conditions stated above do not require the *U* joint axes to intersect in a point, any 3-*PRRRR* parallel manipulator whose revolute joint axes satisfy the above conditions will result in a manipulator with a pure translational motion. There are three active prismatic joints, and six passive universal joints. The lines of action of the three prismatic joints may be inclined from the fixed base by a constant angle θ as an architectural parameter. The first leg A is typically contained within the Ox_0z_0 vertical plane, whereas the remaining legs B, C make the angles $\alpha_B = 120^0, \alpha_C = -120^0$ respectively, with the first leg (Fig. 2).

For the purpose of analysis, we assign a fixed Cartesian coordinate system $Ox_0y_0z_0(T_0)$ at the centred point O of the fixed base platform and a mobile frame $Gx_Gy_Gz_G$ on the mobile platform at its centre G. The angle ν between Ox_0 and Gx_G axes is defined as the *twist* angle of the manipulator.

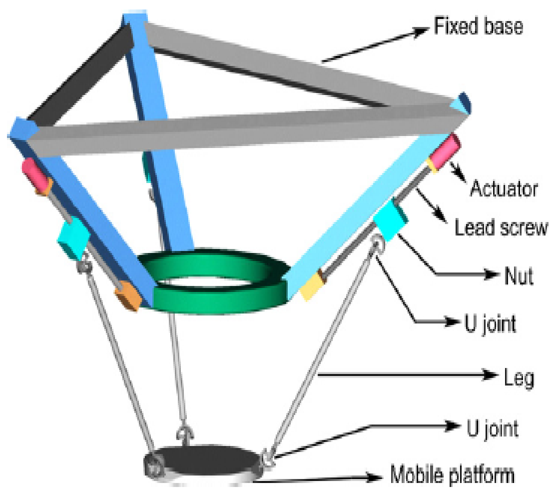

Fig. 1 Virtual prototype for the 3-*PUU* PKM

The moving platform is initially located at a *central configuration*, where the platform is not translated with respect to the fixed base and the origin O of the fixed frame is located at an elevation $OG = h$ above the mass centre G. The first active leg A, for example, consists of a prismatic joint with the nut linked at the $A_1x_1^Ay_1^Az_1^A$ moving frame, having a translation with the displacement λ_{10}^A, the velocity $v_{10}^A = \dot{\lambda}_{10}^A$ and the acceleration $\gamma_{10}^A = \ddot{\lambda}_{10}^A$, an universal joint $A_2x_2^Ay_2^Az_2^A$ characterised by the angular velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$ and the angular acceleration $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$ and a moving link of length $A_3A_4 = l_2$ having a relative rotation about $A_3z_3^A$ axis with the angular velocity $\omega_{32}^A = \dot{\varphi}_{32}^A$ and the angular acceleration $\varepsilon_{32}^A = \ddot{\varphi}_{32}^A$. Finally, a universal joint A_4 is introduced at the edge of a planar moving platform, which can be schematised as a circle of radius r.

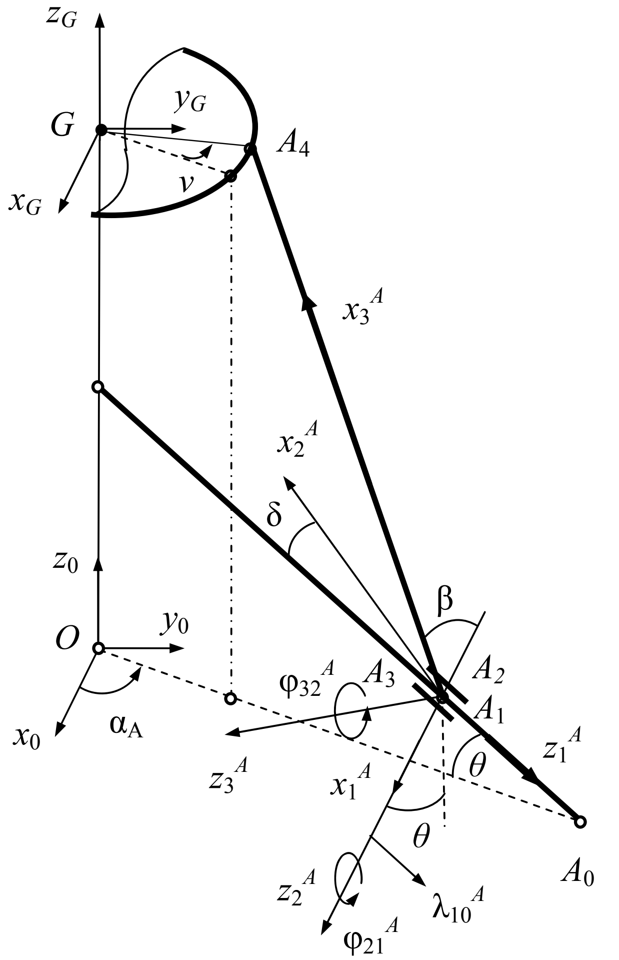

Fig. 2 Kinematical scheme of first leg *A* of upside-down mechanism

At the central configuration, we also consider that the three sliders are initially starting from the same position $A_0A_1 = l_1 = h\sin\theta + (l_0 - r\cos\nu)\cos\theta - l_2\sin\beta\cos\delta$ and that the angles of orientation of the legs are given by

$$\alpha_A = 0, \alpha_B = \frac{2\pi}{3}, \alpha_C = -\frac{2\pi}{3},\ \theta = \frac{\pi}{6},\ \nu = \frac{\pi}{6} \quad (1)$$

$$l_2\cos\beta = h\cos\theta - (l_0 - r\cos\nu)\sin\theta,\ l_2\sin\beta\sin\delta = r\sin\nu,$$

where δ and β are two constant angles of rotation around the axis z_2^A and z_3^A, respectively.

Starting from the reference origin O and pursuing along the independent legs $OA_0A_1A_2A_3A_4$, $OB_0B_1B_2B_3B_4$, $OC_0C_1C_2C_3C_4$, we obtain the transformation matrices

$$p_{10} = a_\theta \theta_1 a_\alpha^i,\; p_{21} = p_{21}^\varphi a_\delta \theta_1, \qquad p_{32} = p_{32}^\varphi a_\beta \theta_1 \theta_2^T \tag{2}$$

$$p_{20} = p_{21} p_{10},\; p_{30} = p_{32} p_{20} \;\; (p = a, b, c), \;\; (i = A, B, C),$$

where we denote the matrices [17], [18]

$$a_\alpha^i = \begin{bmatrix} \cos\alpha_i & \sin\alpha_i & 0 \\ -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix}, a_\beta = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}, a_\delta = \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, \theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \theta_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_{k,k-1}^\varphi = \begin{bmatrix} \cos\varphi_{k,k-1}^i & \sin\varphi_{k,k-1}^i & 0 \\ -\sin\varphi_{k,k-1}^i & \cos\varphi_{k,k-1}^i & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

The angles $\varphi_{21}^A, \varphi_{32}^A$, for example, characterise the sequence of rotations for the first universal joint A_2.

In the inverse geometric problem, the position of the mechanism is completely given through the coordinate x_0^G, y_0^G, z_0^G of the mass centre G. Consider, for example, that during three seconds the moving platform remains in the same orientation and the motion of the centre G along a *rectilinear trajectory* is expressed in the fixed frame $Ox_0y_0z_0$ through the following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{h - z_0^G}{z_0^{G*}} = 1 - \cos\frac{\pi}{3}t, \tag{4}$$

where the values $2x_0^{G*}, 2y_0^{G*}, 2z_0^{G*}$ denote the final position of the moving platform.

Nine independent variables $\lambda_{10}^A, \varphi_{21}^A, \varphi_{32}^A, \lambda_{10}^B, \varphi_{21}^B, \varphi_{32}^B, \lambda_{10}^C, \varphi_{21}^C, \varphi_{32}^C$ will be determined by vector-loop equations

$$\vec{r}_{10}^{\,A} + \sum_{k=1}^{3} a_{k0}^T \vec{r}_{k+1,k}^{\,A} - \vec{r}_G^{\,A_4} = \vec{r}_{10}^{\,B} + \sum_{k=1}^{3} b_{k0}^T \vec{r}_{k+1,k}^{\,B} - \vec{r}_G^{\,B_4} = \vec{r}_{10}^{\,C} + \sum_{k=1}^{3} c_{k0}^T \vec{r}_{k+1,k}^{\,C} - \vec{r}_{\mathrm{G}}^{\,C_4} = \vec{r}_0^{\,G}, \tag{5}$$

where

$$\vec{r}_{10}^{\,i} = l_0 a_\alpha^{iT} \vec{u}_1 + (\lambda_{10}^i - l_1) p_{10}^T \vec{u}_3,\ \vec{r}_{21}^{\,i} = \vec{0},\ \vec{r}_{32}^{\,i} = \vec{0}, \vec{r}_{43}^{\,i} = l_2 \vec{u}_1$$
$$\vec{r}_G^{\,i_5} = [r\cos(\alpha_i + \nu) \quad r\sin(\alpha_i + \nu) \quad 0]^T, \quad (i = A, B, C). \tag{6}$$

From the vector equations (5) we obtain the inverse geometric solution for the spatial manipulator:

$$l_2 \cos(\varphi_{32}^i + \beta) = (r\cos\nu + x_0^G \cos\alpha_i + y_0^G \sin\alpha_i)\sin\theta + z_0^G \cos\theta - l_0 \sin\theta$$
$$l_2 \sin(\varphi_{21}^i + \delta)\sin(\varphi_{32}^i + \beta) = r\sin\nu - x_0^G \sin\alpha_i + y_0^G \cos\alpha_i \tag{7}$$
$$\lambda_{10}^i = l_2 \cos(\varphi_{21}^i + \delta)\sin(\varphi_{32}^i + \beta) + (r\cos\nu + x_0^G \cos\alpha_i + y_0^G \sin\alpha_i)\cos\theta -$$
$$- z_0^G \sin\theta + l_1 - l_0 \cos\theta.$$

The motion of the component elements of the leg A, for example, are characterized by the relative velocities of the joints

$$\vec{v}_{10}^{\,A} = \dot{\lambda}_{10}^A \vec{u}_3,\ \vec{v}_{21}^{\,A} = \vec{0},\ \vec{v}_{32}^{\,A} = \vec{0} \tag{8}$$

and by the following relative angular velocities

$$\vec{\omega}_{10}^A = \vec{0},\ \vec{\omega}_{21}^A = \dot{\varphi}_{21}^A \vec{u}_3,\ \vec{\omega}_{32}^A = \dot{\varphi}_{32}^A \vec{u}_3, \tag{9}$$

which are *associated* to skew-symmetric matrices

$$\tilde{\omega}_{10}^A = \tilde{0},\ \tilde{\omega}_{21}^A = \dot{\varphi}_{21}^A \tilde{u}_3,\ \tilde{\omega}_{32}^A = \dot{\varphi}_{32}^A \tilde{u}_3. \tag{10}$$

From the geometrical constraints (5), we obtain the *matrix conditions of connectivity* and the relative velocities $v_{10}^A, \omega_{21}^A, \omega_{32}^A$ of the first leg A [19], [20]

$$v_{10}^A\, \vec{u}_j^{\,T} a_{10}^T \vec{u}_3 + \omega_{21}^A \vec{u}_j^{\,T} a_{20}^T \tilde{u}_3 a_{32}^T \vec{r}_{43}^{\,A} + \omega_{32}^A \vec{u}_j^{\,T} a_{30}^T \tilde{u}_3 \vec{r}_{43}^{\,A} = \vec{u}_j^{\,T} \dot{\vec{r}}_0^{\,G}, \quad (j = 1, 2, 3). \tag{11}$$

If the other two kinematical chains of the robot are pursued, analogous relations can be easily obtained.

From these equations, we obtain the *complete* Jacobian matrix of the manipulator. This matrix is a fundamental element for the analysis of the robot workspace and the particular configurations of singularities where the spatial manipulator becomes uncontrollable [21].

To describe the kinematical state of each link with respect to the fixed frame, we compute the angular velocity $\vec{\omega}_{k0}^A$ and the linear velocity $\vec{v}_{k0}^{\,A}$ in terms of the vectors of the preceding body, using a recursive manner:

$$\vec{\omega}_{k0}^A = a_{k,k-1} \vec{\omega}_{k-1,0}^A + \dot{\varphi}_{k,k-1}^A \vec{u}_3,\quad \vec{v}_{k0}^{\,A} = a_{k,k-1} \vec{v}_{k-1,0}^{\,A} + a_{k,k-1} \tilde{\omega}_{k-1,0}^A \vec{r}_{k,k-1}^{\,A} + \dot{\lambda}_{k,k-1}^A \vec{u}_3. \tag{12}$$

Rearranging, the above nine constraint equations (7) can be written as three independent relations

$$
\begin{aligned}
&[(r\cos\nu + x_0^G \cos\alpha_i + y_0^G \sin\alpha_i)\sin\theta + z_0^G \cos\theta - l_0 \sin\theta]^2 + \\
&\quad + (r\sin\nu - x_0^G \cos\alpha_i + y_0^G \sin\alpha_i)^2 + \\
&\quad + [\lambda_{10}^i - l_1 - (r\cos\nu + x_0^G \cos\alpha_i + y_0^G \sin\alpha_i)\cos\theta + z_0^G \sin\theta + l_0 \cos\theta]^2 = \\
&\quad = l_2^2 . (i = A, B, C)
\end{aligned}
\tag{13}
$$

concerning the coordinates x_0^G, y_0^G, z_0^G and the displacements λ_{10}^A, λ_{10}^A, λ_{10}^A only.

The derivative with respect to the time of conditions (13) leads to the matrix equation

$$J_1 \dot{\vec{\lambda}}_{10} = J_2 [\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{z}_0^G]^T , \tag{14}$$

where the matrices J_1 and J_2 are, respectively,

$$J_1 = diag\,\{\delta_A \quad \delta_B \quad \delta_C\}\,,\; J_2 = \begin{bmatrix} \beta_1^A & \beta_2^A & \beta_3^A \\ \beta_1^B & \beta_2^B & \beta_3^B \\ \beta_1^C & \beta_2^C & \beta_3^C \end{bmatrix}, \tag{15}$$

with the notations

$$
\begin{aligned}
&\delta_i = -l_2 \cos(\varphi_{21}^i + \delta)\cos(\varphi_{32}^i + \beta), \quad (i = A, B, C) \\
&\beta_1^i = x_0^G + (l_1 - \lambda_{10}^i)\cos\alpha_i \cos\theta + r\cos(\alpha_i + \nu) - l_0 \cos\alpha_i \\
&\beta_2^i = y_0^G + (l_1 - \lambda_{10}^i)\sin\alpha_i \cos\theta + r\sin(\alpha_i + \nu) - l_0 \sin\alpha_i \,, \beta_3^i = z_0^G - (l_1 - \lambda_{10}^i)\sin\theta \,.
\end{aligned}
\tag{16}
$$

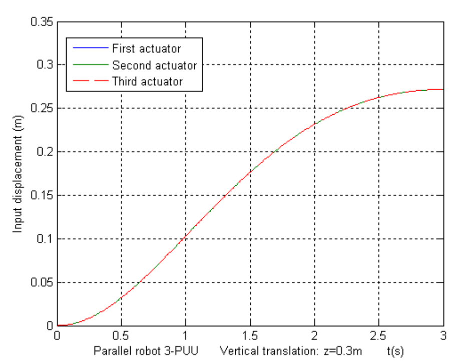

Fig. 3 Displacements λ_{10}^i of the three sliders

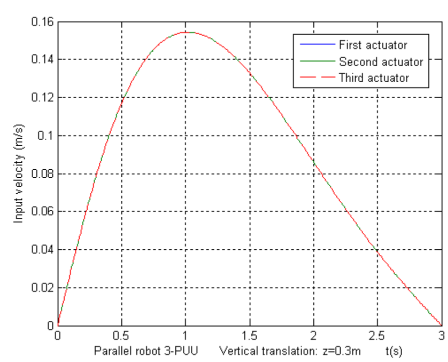

Fig. 4 Velocities v_{10}^i of the three sliders

The three kinds of singularities of the three closed-loop kinematical chains can be determined through the analysis of two Jacobian matrices J_1 and J_2 [22], [23], [24].

The acceleration γ_{10}^{A} and the angular accelerations ε_{21}^{A}, ε_{32}^{A} of leg A are expressed by the conditions of connectivity:

$$\gamma_{10}^{A}\,\vec{u}_j^{\,T} a_{10}^{T}\vec{u}_3 + \varepsilon_{21}^{A}\vec{u}_j^{\,T} a_{20}^{T}\tilde{u}_3 a_{32}^{T}\vec{r}_{43}^{\,A} + \varepsilon_{32}^{A}\vec{u}_j^{\,T} a_{30}^{T}\tilde{u}_3\vec{r}_{43}^{\,A} = \vec{u}_j^{\,T}\dot{\vec{r}}_0^{\,G} - \omega_{21}^{A}\omega_{21}^{A}\vec{u}_j^{\,T} a_{20}^{T}\tilde{u}_3\tilde{u}_3 a_{32}^{T}\vec{r}_{43}^{\,A} -$$

$$- \omega_{32}^{A}\omega_{32}^{A}\vec{u}_j^{\,T} a_{30}^{T}\tilde{u}_3\tilde{u}_3\vec{r}_{43}^{\,A} - 2\omega_{21}^{A}\omega_{32}^{A}\vec{u}_j^{\,T} a_{20}^{T}\tilde{u}_3 a_{32}^{T}\tilde{u}_3\vec{r}_{43}^{\,A}, \quad (j = 1, 2, 3). \qquad (17)$$

Computing the derivatives with respect to the time of equations (12), we obtain a recursive form of accelerations $\vec{\varepsilon}_{k0}^{\,A}$ and $\vec{\gamma}_{k0}^{\,A}$:

$$\vec{\varepsilon}_{k0}^{\,A} = a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{\,A} + \ddot{\varphi}_{k,k-1}^{A}\vec{u}_3 + \dot{\varphi}_{k,k-1}^{A} a_{k,k-1}\tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T}\vec{u}_3, \qquad (18)$$

$$\vec{\gamma}_{k0}^{\,A} = a_{k,k-1}\vec{\gamma}_{k-1,0}^{\,A} + a_{k,k-1}\{\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A}\}\vec{r}_{k,k-1}^{\,A} + 2\dot{\lambda}_{k,k-1}^{A} a_{k,k-1}\tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T}\vec{u}_3 + \ddot{\lambda}_{k,k-1}^{A}\vec{u}_3$$

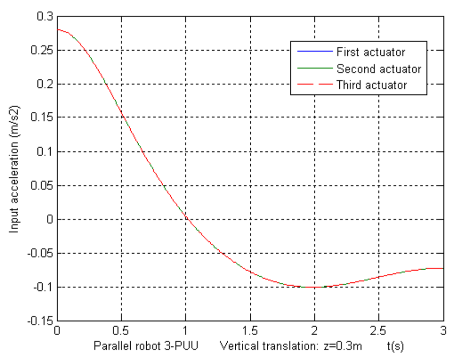

Fig. 5 Accelerations γ_{10}^{i} of the three sliders

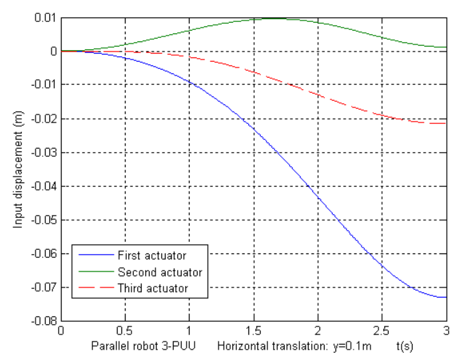

Fig. 6 Displacements λ_{10}^{i} of the three sliders

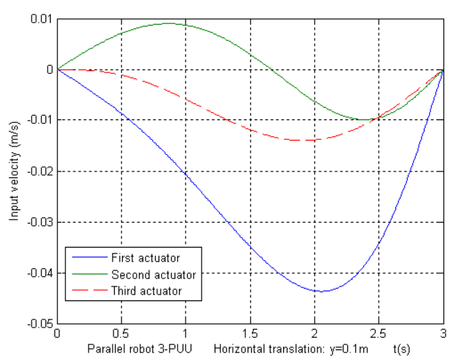

Fig. 7 Velocities v_{10}^{i} of the three sliders

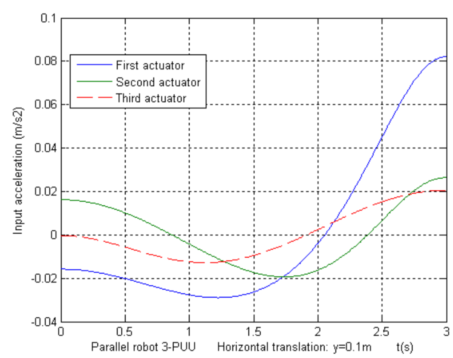

Fig. 8 Accelerations γ_{10}^{i} of the three sliders

As an application let us consider a 3-*PUU* PKM which has the following characteristics

$$x_0^{G*} = 0.05\,m\,,\; y_0^{G*} = 0.05\,m\,,\; z_0^{G*} = 0.15\,m$$

$$r = 0.1m\,,\ OA_0 = l_0 = 0.3\,m,\ l_2 = 0.3m,\ h = 0.4\,m,\ \Delta t = 3\,s\,, \qquad (19)$$

$$A_0A_1 = l_1 = h\sin\theta + (l_0 - r\cos\nu)\cos\theta - l_2\sin\beta\cos\delta\,.$$

Using MATLAB software, a computer program was developed to solve the kinematics of the 3-*PUU* parallel robot. To develop the algorithm, it is assumed that the platform starts at rest from a central configuration and moves pursuing successively rectilinear and general translations.

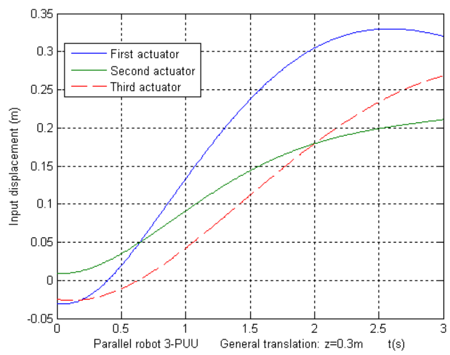

Fig. 9 Displacements λ_{10}^i of the three sliders

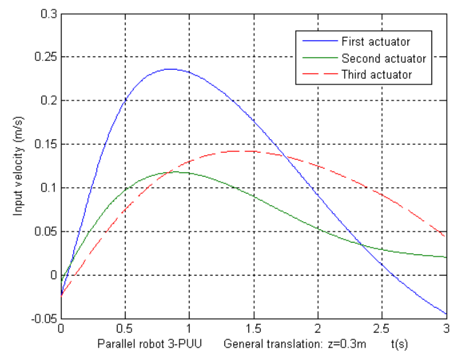

Fig. 10 Velocities v_{10}^i of the three sliders

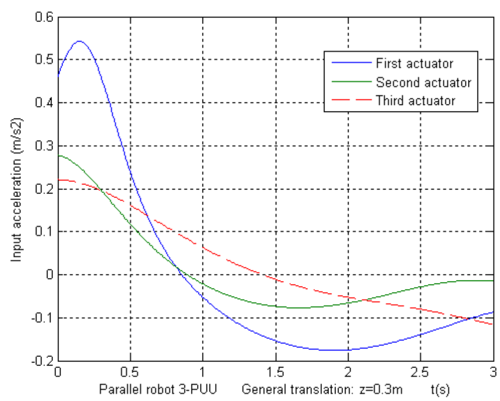

Fig. 11 Accelerations γ_{10}^i of the three sliders

Some examples are solved to illustrate the algorithm. For the first example, the platform moves along the *vertical direction* z_0 with variable acceleration while all the other positional parameters are held equal to zero. The time-histories for the input displacements λ_{10}^i (Fig. 3), velocities v_{10}^i (Fig. 4) and accelerations γ_{10}^i

(Fig. 5) are carried out for a period of $\Delta t = 3$ seconds in terms of analytical equations (4).

For the case when the platform's centre G moves along a *rectilinear horizontal trajectory* without any rotation of the platform, the graphs are illustrated in Fig. 6, Fig. 7 and Fig. 8.

Further on, we shall consider a spatial evolution of the platform, combining a curvilinear horizontal translation and a vertical translational ascension. The centre G of the platform starts from a point of coordinates (x_0^{G*}, x_0^{G*}, h) and may describe uniformly a half-circle of radius x_0^{G*}, in agreement with equations

$$x_0^G = x_0^{G*}(1+\sin\frac{\pi}{3}t),\; y_0^G = x_0^{G*}\cos\frac{\pi}{3}t,\;\; t\in[0,3]. \tag{20}$$

But, in the same time interval, the platform performs a non-uniform vertical ascension in accord to the law of motion

$$z_0^G = h - z_0^{G*}(1-\cos\frac{\pi}{3}t). \tag{21}$$

The input displacements, velocities and accelerations in the case of the general translation are graphically sketched in Fig. 9, Fig. 10 and Fig. 11.

3. Conclusions

By the kinematics analysis some exact relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in the present paper. The simulation through the program certifies that one of the major advantages of the current matrix recursive formulation is the accuracy and a reduced number of additions or multiplications and consequently a smaller processing time for the numerical computation.

Choosing the appropriate serial kinematical circuits connecting many moving platforms, the present method can be easily applied in forward and inverse mechanics of various types of parallel mechanisms, complex manipulators of higher degrees of freedom and particularly *hybrid structures*, with increased number of components of the mechanisms.

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