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FRACTALITY INFLUENCES ON A FREE GAUSSIAN "PERTURBATION" IN THE HYDRODINAMIC VERSION OF THE SCALE RELATIVITY THEORY. POSSIBLE IMPLICATIONS IN THE BIOSTRUCTURES DYNAMICS

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Various differentiable models are frequently used to describe the dynamics of complex systems (kinetic models, fluid models). Given the complexity of all the phenomena involved in the dynamics of such systems, it is required to introduce the dynamic variables dependences both on the space-time coordinates and on the resolution scales. Therefore, in this case an adequate theoretical approach may be the use of non-linear physical models. In such framework, using a simplified version of the fractal hydrodynamic model, the dynamics of a free Gaussian "perturbation" is analyzed. Possible implications of the model in dynamics of biological structures are also studied.

Keywords: complex systems, non-differentiability, fractal hydrodynamic model.

1. Introduction

The standard models [1, 2] used to study the complex system dynamics are based on the hypothesis of the differentiability of the physical variables that describes it. The success of the differentiable models must be understood sequentially, i.e. there are domains large enough for the differentiability to be valid.

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But differential methods fail when facing the physical reality, such as instabilities of the complex system that can generate chaos or patterns through self-structuring, and, therefore, we are “required” to work with non-differentiable (fractal) method [3].

In order to describe some of the dynamics observed in a complex system by means of the non-differentiable method, and still remaining treatable as differential method, it is necessary to introduce, the scale resolution, both in the expressions of the physical variables and the dynamics equations. This means that any dynamic variable dependent, in a classical meaning, on the spatial coordinates and time, become in a non-differential meaning dependent also on the scale resolution. In other words, instead of working on a dynamic variable, described by means of a mathematical function strictly non-differentiable, we will work just with different approximations of the function, derived though their averaging at different scale resolutions. Consequently, any dynamic variable acts as the limit of a family of functions, the functions being non-differentiable for a non-zero resolution scale and differentiable for a null scale resolution.

This approach, well adapted to the complex system dynamics, where any real determination is conducted at a finite scale resolution, clearly implies the development both of a new geometric structure and of a physical theory applied to complex system dynamics, for which the motion laws, invariant to spatial and temporal coordinate transformations, completed with scales laws, are invariant to the scale transformations. Such a physical theory that includes the geometric structure based on the above presented assumptions was developed in the Scale Relativity Theory with fractal dimension 2 [4] and in Scale Relativity Theory with an arbitrary constant fractal dimension [5]. In the field of complex system, if we assume that the complexity of the interactions in the system is replaced by non-differentiability (fractality), the constrained motion on continuous, but differentiable curves in a Euclidian space of the complex system structural units are replaced with the free motions, without any constrains, on continuous but non-differentiable curves in a fractal space of the same complex system structural units [6-9]. This is the reasoning for which at the time resolution scales that prove to be large when compared with the inverse of the highest Lyapunov exponent, the deterministic trajectories are replaced by a collection of potential states, so that the concept of “definite position” is substituted by that of an ensemble of positions having a definite probability density [10-13].

As a consequence, the determinism and the potentiality (non-determinism) become distinct parts of the same “evolution” of a complex system, through reciprocal interactions and conditioning, in such way that the complex system structural units are substituted with the geodesics themselves [14-18].

Considering the above affirmations, in this paper, we study the influence of the fractality degree on a free Gaussian perturbation, using the hydrodynamic version of Scale Relativity with constant arbitrary fractal dimension.

2. The basis of a fractal model in the hydrodynamic representation

Let us now reconsider the fractal hydrodynamic equations with an arbitrary fractal dimension (kept constant), i.e. the specific momentum and state density conservation laws [5]:

$$\partial_t v^i + v^l \partial_l v^i = -\partial^i (Q+U) \tag{1}$$

$$\partial_t \rho + \partial_l (\rho v^l) = 0 \tag{2}$$

with Q the specific fractal potential:

$$Q = -2\lambda^2 (dt)^{\left(\frac{4}{D_F}\right)-2} \frac{\partial_i \partial^i \rho^{-\frac{1}{2}}}{\rho^{\frac{1}{2}}} = \frac{u_l u^l}{2} + \lambda (dt)^{\left(\frac{2}{D_F}\right)-1} \partial_l u^l \tag{3}$$

$v^l = 2\lambda (dt)^{\left(\frac{2}{D_F}\right)-1} \partial^l s$ the standard classical velocity which is differentiable and independent of the scale resolution dt, $u^l = \lambda (dt)^{\left(\frac{2}{D_F}\right)-1} \partial^l \ln \rho$ is the non-standard non-differentiable (fractal) velocity, dependent on the resolution scale, ρ is the state density, s is the phase, U is the external scalar potential, λ is the fractal-non-fractal transition coefficient and D_F is the fractal dimension of the motion curves. We note that the fractal dimension D_F is the ratio describing a statistical index of system complexity comparing the variation of a fractal pattern with changes in the measuring scale [3, 19]. For D_F one can choose different definition for the fractal dimensions, i.e. the fractal dimension in a Kolmogorov sense, in a Hausdorff-Besikovici sense, etc. (for details, see [3, 19]). But, once chosen the definition, it has to remain constant during whole analysis of the complex system dynamics. In our case, each specific process is characterized by specific geodesics (fractal trajectories) which correspond to a specific fractal dimension D_F.

The velocities $v^l$and u^l define, in the fractal space, the complex velocity [5]:

$$\hat{V}^l = -2i\lambda (dt)^{\left(\frac{2}{D_F}\right)-1} \partial^l \ln \Psi = v^l - iu^l \tag{4}$$

with $\Psi = \sqrt{\rho} e^{is}$ the equivalent of the wave function, $\sqrt{\rho}$ the amplitude and s the phase.

Equations (1) and (2) will further be used to study the dynamics of a spatial Gaussian "perturbation" free of any external constrains.

The influence of white Gaussian noise on the fluctuations of different physical systems is a problem intensively studied, especially interpreted as positive influence of noise in the increasing of the predictability of apparently chaotic dynamics [20, 21]. Fractal or multifractal detrended fluctuation analysis are usually used to study, how the multifractality strength is increased by the noisy perturbation amplitude [22].

Given the non-linear nature of the equations it is difficult to obtain an analytical solution for a general case. However, there are some particular circumstances for which analytical solution can be obtained. Let us consider the one-dimensional case for the equations (1) and (2) in the absence of any external constraint ($U = 0$), i.e.:

$$\partial_t v + v\partial_x v = -2\lambda^2 (dt)^{\left(\frac{4}{D_F}\right)-2} \rho^{-\frac{1}{2}} \partial_{xx} \rho^{-\frac{1}{2}} \tag{5}$$

$$\partial_t v + \partial_x \rho v = 0 \tag{6}$$

We consider the initial conditions:

$$v(x, t = 0) = c \tag{7}$$

$$\rho(x, t = 0) = \rho_0 e^{(-x/\alpha)^2} \tag{8}$$

and the boundary ones:

$$v(x = ct, t) = c \tag{9}$$

$$\rho(x = -\infty, t) = \rho(x = \infty, t) = 0 \tag{10}$$

In this case, we assume that at $t = 0$ the center of the spatial Gaussian "perturbation" $\rho(a, x)$ is at $\langle x(t=0) \rangle = 0$ and has the velocity $\langle v(t=0) \rangle = c$ The boundary conditions (9) and (10) mean that at any $t \rangle 0$ or $t \langle 0$ one obtains $\langle \partial_x Q \rangle = 0$ (for details, see [5]). A similar outcome is obtained for $\langle x \rangle = ct$.

Using the method presented in [5], the analytical solution for the equations (5) and (6), with the initial condition (7), (8) and the boundary ones (9), (10) becomes:

$$v(x,t) = \frac{c\alpha^2 + \left[\frac{2\lambda(dt)^{(2/D_F)-1}}{\alpha}\right]^2 xt}{\alpha^2 + \left[\frac{2\lambda(dt)^{(2/D_F)-1}}{\alpha}\right]^2 t^2} \tag{11}$$

for the velocity and

$$\rho(x,t) = \frac{\pi^{-1/2}}{\left\{\alpha^2 + \left[\frac{2\lambda(dt)^{(2/D_F)-1}}{\alpha}\right]^2 t^2\right\}^{1/2}} \exp\left\{-\frac{(x-ct)^2}{\alpha^2 + \left[\frac{2\lambda(dt)^{(2/D_F)-1}}{\alpha}\right]^2 t^2}\right\} \tag{12}$$

for the states density. One can further reconstruct the current density in the form:

$$j(x,t)=v(x,t)\rho(x,t)=\pi^{-1/2}\frac{c\alpha^2+\left[\frac{2\lambda(dt)^{(2/D_F)-1}}{\alpha}\right]^2 xt}{\left\{\alpha^2+\left[\frac{2\lambda(dt)^{(2/D_F)-1}}{\alpha}\right]^2 t^2\right\}^{3/2}}\cdot\exp\left\{-\frac{(x-ct)^2}{\alpha^2+\left[\frac{2\lambda(dt)^{(2/D_F)-1}}{\alpha}\right]^2 t^2}\right\} \tag{13}$$

For $t\neq 0$ it results that the dynamics variables defined through (11) - (13) are non-homogeneous in x and t, while for $x=ct$ these become non-homogeneous either in x or t.

Given the multiple functional dependences of the dynamic variables (v,ρ,j) on the external parameters (α, c, λ, $(dt)^{(2/D_F)-1}$, etc.) we will choose an adequate normalization that will allow us to obtain a more compact and simplified dependences. Basically, we want to reduce the explicit dependences on the external parameters. Thus, we will choose the normalization:

$$\xi\to\frac{x}{\alpha},\ \tau\to\frac{tc}{\alpha},\ 2\lambda\to\alpha c,\ (dt)^{(4/D_F)-2}=\mu \tag{14}$$

This leads to dependences of the complex system dynamics variables on the external parameters, which can be rewritten as:

i) normalized velocity:

$$V(\xi,\tau)=\frac{1+\mu\xi\tau}{1+\mu\tau^2} \tag{15}$$

ii) normalized state density:

$$N(\xi,\tau)=\left[1+\mu\,\tau^2\right]^{-1/2}\exp\left[-\left(\frac{(\xi-c\tau)^2}{1+\mu\tau^2}\right)\right] \tag{16}$$

iii) normalized current density:

$$J(\xi,\tau)=\frac{1+\mu\xi\tau}{(1+\mu\tau^2)^{3/2}}\exp\left[-\left(\frac{(\xi-c\tau)^2}{1+\mu\tau^2}\right)\right] \tag{17}$$

3. Numerical simulation

Using the mathematical approach given above, we performed a series of graphical representations to obtain information regarding the spatial and temporal evolution of the velocity, states density and current density. Also, we studied the effect of the value attributed to the parameter μ, which now implicitly contains the contribution of the external parameters. We remind that the parameter μ (fractal degree) is directly connected to the fractal contribution on the dynamic variables, which implies fractalization by means of different statistics (from Levy type movements to Brownian ones, either by means of non-Markovian processes or Markovian ones [3, 4, 19]). From a physical point of view, these statistics are dictated by the fundamental processes involved in the evolution of the complex system (formation and expansion).

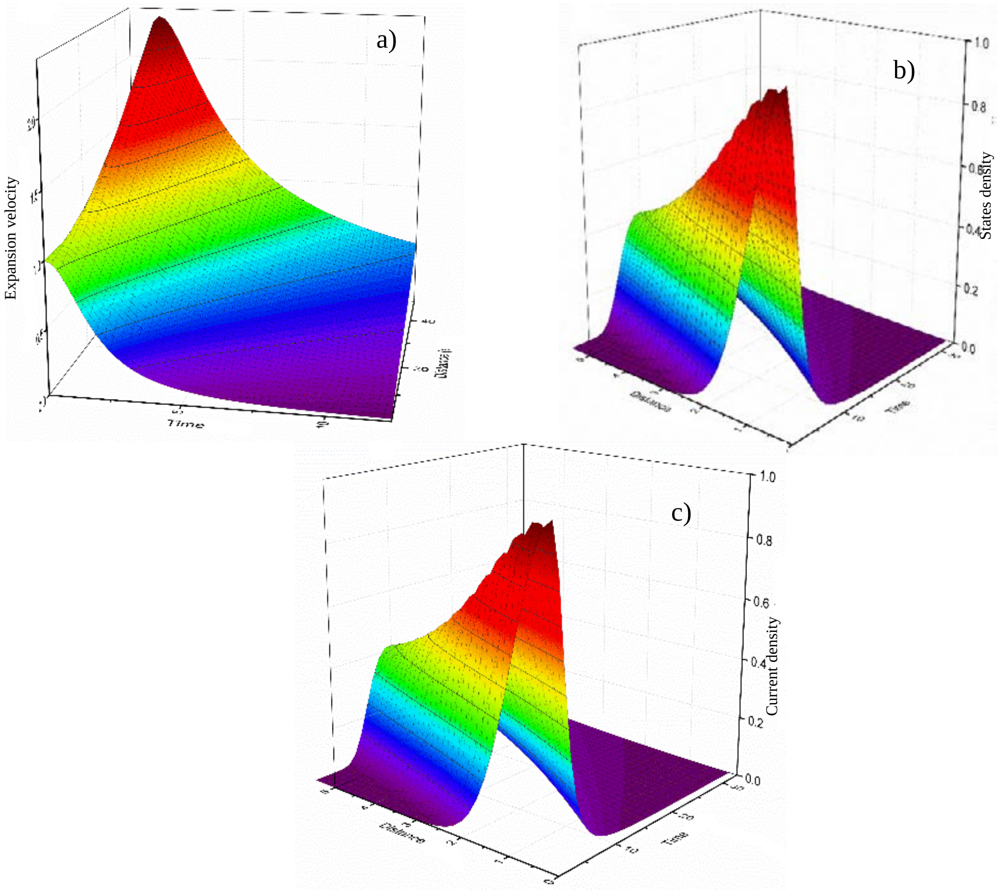
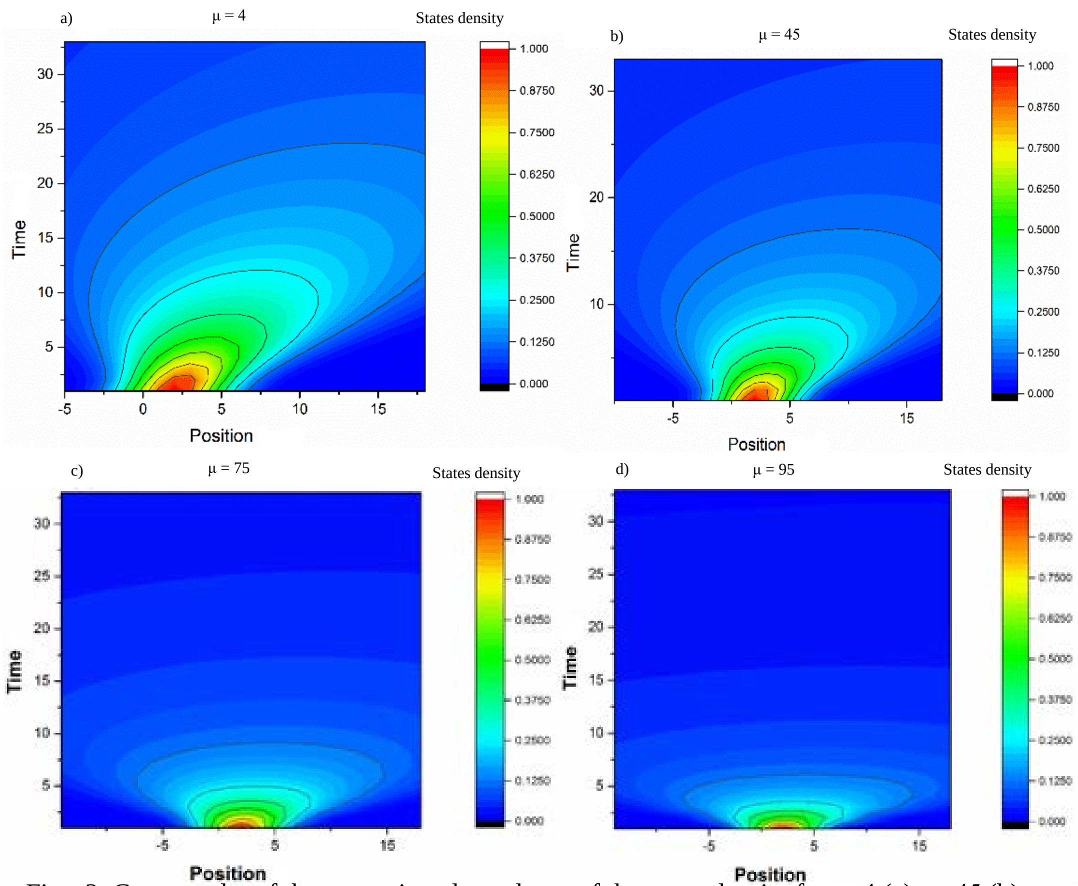


Fig. 1: Space-time evolution of the expansion velocity (a), states density (b), current density (c) for $\mu \equiv 6$

In Fig. 1 is represented the space-time evolution of the previously mentioned dynamic variables determined by the equations (15)–(17). All the graphical depictions (Fig. 1) were made for a constant value of $\mu \rangle 1$ (at a scale resolution $dt = 5$ and motions on fractal curves with the fractal dimension $D_f = 1.28$). The dependences reveal a space-time decrease of all the complex system dynamic variables during evolution. Both states density and current density present a quasi-exponential decrease in both time and space.

The changes to the states density, with variations of the external constraints, will also be seen in the shape of the complex system “perturbation”. This mathematical approach allows us to investigate these changes for different values of the external parameters described through μ. In Fig. 2 we present the contour plot of the states density for different values of μ. We observe that for smaller values ($\mu = 4$) the “perturbation” has a longer life time, presents a higher states density and has an elongated shape. With the increase of the external parameter’s value, a decrease both of the states density and of the life time of the “perturbation” is observed. The decrease is most probable an effect given by the properties of the expanding “medium” and it is attributed to strong interaction processes. The “perturbation” becomes confined for values higher than 45.

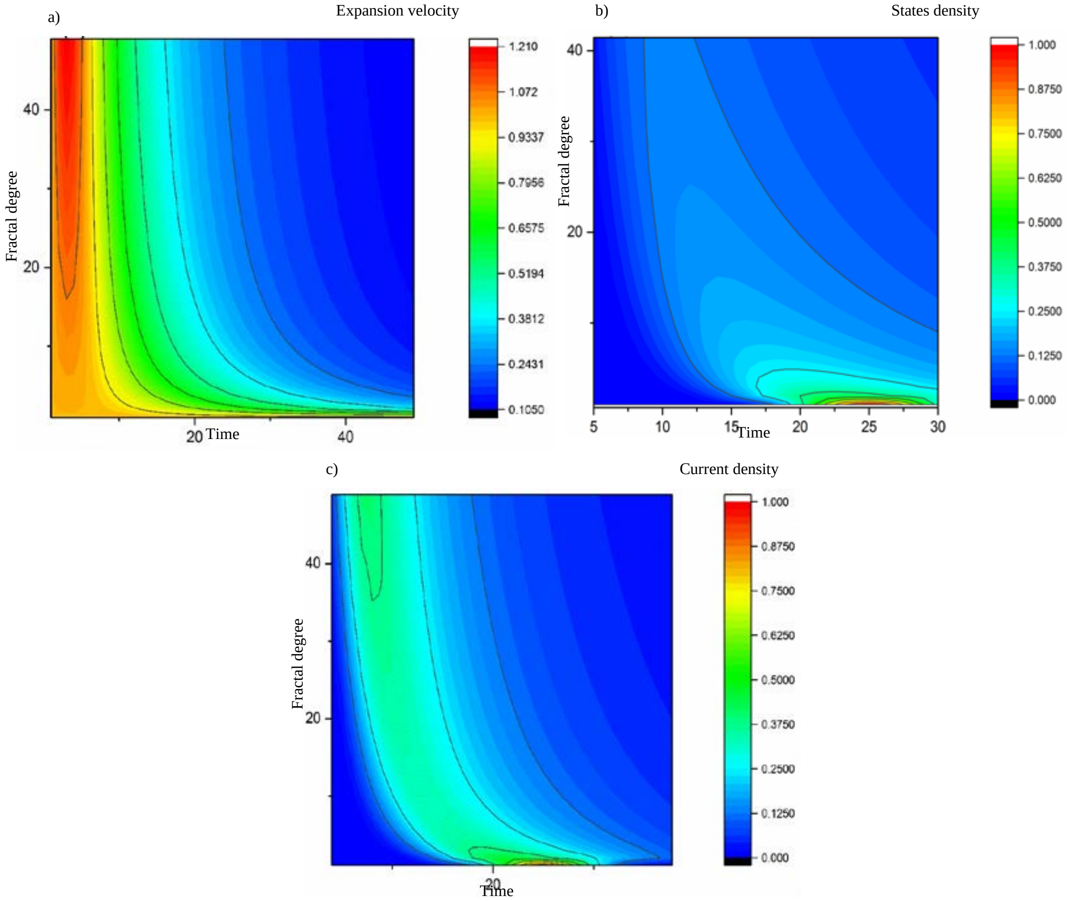
The values of the fractal degree for our system (μ) affects not only the values and evolution of the states density, but also all the other investigated complex system dynamic variables.



Figs. 2: Contour plot of the space-time dependence of the states density for μ=4 (a), μ=45 (b), μ=75 (c) and μ=95 (d)

We now focus on the influence of μ on the temporal evolution of the complex system. In Fig. 3 are represented the contour plot of the functions depicted in (15) – (17) for a constant distance. We observed that the current density presents two maxima as the value of μ is increased. The first one is observed for values of $\mu\langle 1$ and short evolution times, while the second maximum is observed for $\mu\rangle 1$ and longer evolution times. We attribute the presence of these two maxima to the temporal evolution given by changes in the "perturbation" dynamics and to the structure induced by the external parameters (through μ).

There are several parameters like background pressure, external potential, etc., which influences the structure and shape of the perturbation. Given the manner in which we defined μ, we cannot differentiate between various external parameters and it is difficult to attribute the presence of these two maxima in current density strictly to only one of them.



Figs. 3: Contour plot of the following normalized complex system dynamic variables: expansion velocity (a), states density (b), current density (c) for a constant distance $\xi = 6$

From these representations, we find that the current density has a decrease in its amplitude and presents a shift towards higher space-time coordinates.

4. Blood as a complex fluid

Previous results can be applied to the blood dynamics, considered a complex system, i.e. complex fluid. Therefore, both the “changing” of “perturbation” shape (Fig. 2), and its “breaking” (Fig. 3) can be assimilated to the initiation of the self-organization process having as consequence the blockage of blood flow through “stopper effect” (for example, the acute arterial occlusion). Let us note that without contradicting the above stated theory, which is sustained by some morphopathological evidence, we prove through the mathematical modeling that the blocking of the lumen of an absolutely healthy artery can happen as a result of the “stopping effect” (even in the absence of the at least disputable cracked and non-protrusive atheroma plaque), in the conditions of a normal sanguine circulation [23].

This happens due to the fact that blood is a complex non-Newtonian fluid made of: plasma and formed cells, cholesterol vesicles and other suspended elements [24]; thus, the laws of fractal physics are completely applicable to sanguine circulation. For conformity, the perfect Newtonian fluid is a fluid in which viscosity is independent of the shear stress, thus having no relation to the sanguine fluid. However, not only the complex structure of blood justifies the using of fractality, but also the complex structure of the arterial system, with its multiple ramifications, which generate turbulence areas and interruptions of the linear flow that make classical physics not applicable in this context. We really discuss about multi fractality: a morphological one due to complex structure of the arterial tree as well as a functional one due to blood flow “regimes” [25-27].

To explain the above statements, let us observe first that blood as non-Newtonian complex fluid has a behavior of Bingham type [28]. This results from the conservation law of the specific momentum in the one dimensional case with axial symmetry, i.e. for the velocity field $(v_\theta = 0, v_r = 0, v_z \neq 0)$ under the form [5]:

$$\tau = \tau_0 + \nu \frac{dv_z}{dt} \tag{18}$$

where τ is the tangential unitary type effort, τ_0 is the deformation unitary type effort that is direct correspondence with the specific fractal potential (3), $\nu = \lambda (dt)^{(2/D_F)-1}$ is the viscosity type coefficient and dv_z/dr is the radial gradient of the velocity field. From (18), for $\tau_0 \to 0$, results the Newtonian type behavior of the complex fluid. Bingham type behavior of the complex fluid through a circular pipe of radius R and length l implies the simultaneous existence of two flow sub-domains: a) a sub-domain for $r \in [0, r_0]$, where the tangential unitary type effect is lower than the flow limit τ_0. As a consequence, the complex fluid moves as an apparently undistorted rigid system (in the form of a stopper with quasi-parallel walls to those of the pipe). The solid stopper flows with constant velocity, without changing its structure; b) a sub-domain for $r \in [r_0, R]$ where the tangential unitary type effort excludes the value τ_0. As a consequence, the complex fluid flows so that a layer with finer structural units and lower concentration appears. In these conditions, following the procedure from [29], the stopper radius is obtained in the form:

$$r_0 = \frac{2\tau_0 l}{\Delta p} \tag{19}$$

where Δp is the pressure drop on the pipe. This result can be applied in the analysis of blood dynamics, both in arteries and capillaries, by calculating the diameter of the thrombus assimilated with a cylinder of radius r_0 and length l subjected to a pressure drop Δp in the form $D = 4\tau_0 l / \Delta p$. This result is validated experimentally, as it results from [30, 31].

Regarding the recovery of such biological diseases, there are a vast number of techniques. We remind that the external electrical stimulation can cause changes in the blood vessels. Although atherosclerosis cause vasodilation in the affected area and blood flow remains unchanged for an extended period of time, the vascular wall stiffness will increase the pulse pressure. James E. Tracy et al. developed, in 1950, a study whose purpose was to measure the effects of electrical stimulation (ES) on blood flow and blood pressure. All subjects received electrical stimulation of intensity sufficient to produce torque equal to 15% of the predetermined maximal voluntary contraction of their right quadriceps femoris muscle. The conclusions were that the increase in blood flow occurred within 5 minutes after the onset of ES and dropped to resting levels within 1 minute after a 10-minute period of ES [32].

From the physiotherapeutic point of view, treatment is directed towards improving blood flow and towards decreasing the disparity between the demand for blood and its supply [33].

5. Conclusions

We formulated a simplified version of the fractal hydrodynamic model and used it to describe the dynamic of a fractal fluid which is assimilated to a complex system. The space and time evolution of expansion velocity, current density and state density were investigated. The model manages to simulate the formation and expansion of complex system perturbation taking into account the scale differences between different mechanisms.

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