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DAMAGE DETECTION BY UPDATING USING CORRELATION FUNCTIONS

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A finite element model updating method is proposed for damage detection in mechanical structures using frequency measurements. The cost function is made up of a frequency residual formulated by the gradient of a correlation function in the frequency domain. An optimization algorithm is proposed for the resolution of the numerical problem. The suggested technique is applied to simulated structures considering the effect of noisy measurements. The simulation tests results show the effectiveness of this new damage identification technique. Mathematical and algorithmic analyses highlight very interesting characteristics of the proposed optimization algorithm. The updating method thus obtained has application in structural damage detection and finite elements models validation. It allows also a structural health monitoring of large mechanical structures.

Keywords: damage detection, model updating, frequency measurements, correlation

1. Introduction

Damage detection in structures drew a great attention in civil, mechanical and aerospace engineering. In this context, many vibration measurements based methods were developed.

Genetic algorithms took a significant part in this field. These algorithms are able to find global minima or maxima. They can thus be usable for the minimization of cost functions. Larson and Zimmerman [1] developed an effective program using genetic algorithms. Applied for a finite element model updating of a six elements bar with 25 degrees of freedom, the results show a considerable improvement even when the modes are disturbed by experimental noise. Carlin and Garcia [2] used the algorithm for the detection of defects in mechanical structures. After comparison with other damage detection methods, they conclude that genetic algorithms are more powerful and avoid announcing defects where they do not exist. Li et al. [3] proposed another type of methods based on Perturbed Boundary Condition (PBC) in experimental test. The pmain utility of the PBC is to overcome the principal problem of the updating methods which is the insufficiency of information, and to improve conditioning of the

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systems. One of the problems of this type of methods is the difficulty of associating measurements of the various tests in only one updating procedure, because if one simply increases the number of equations, the resolution of the system becomes more complex. Instead of using several sets of tests as for the PBC, Ibrahim [4] used an analytical model with multiple disturbances. Even if information on the structural behaviour comes from only one experimental test, the method increases the quantity of information obtained, and consequently helps to improve conditioning of equations.

Some researchers used antiresonance frequencies with the natural frequencies, mode shapes, and Frequency Response Functions (FRFs) in the updating method [5 – 7]. Even if there is a multitude of finite element model updating methods, the problem remains always posed since none of them can correctly update the FE model of all industrial structures. There are cases considered to be satisfactory but their success is limited. A good outline of existing methods is given by Sohn et al. [8]. These methods are based on the fact that the defects cause usually the reduction in the rigidity of the structure which results in the change of the vibratory characteristics (like damping, eigen frequencies and eigen modes). The defects cause also the change of the geometrical and mechanical parameters of the structure which one finds in the mass, damping, stiffness and flexibility matrices. The finite element method can be employed for damage detection by inverse techniques or models updating.

This work presents a new finite elements model updating method for the detection and the quantification of defects using a correlation function in frequency domain.

2. Parametrisation of the updating method

The dynamic behaviour of a linear mechanical structure is governed by the following equation

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = f(t) \tag{1}$$

where M, C and K are mass, damping and stiffness real symmetric matrices which can be discretized as follow

$$M = \sum_{i=1}^{N} M_i^{(e)}, \; C = \sum_{i=1}^{N} C_i^{(e)} \text{ and } K = \sum_{i=1}^{N} K_i^{(e)} \tag{2}$$

For a harmonic excitation of pulsation ω, the particular solution of Eq. 1 is

$$\left(-M\omega^2 + j\omega C + K\right)y = f \tag{3}$$

which we can deduce the expression of the displacement vector

$$y(\omega) = \left(-M\omega^2 + j\omega C + K\right)^{-1} f \tag{4}$$

with $\omega \in \{\omega_1 \; \omega_2 \; .., \omega_S\}$

Damping is considered proportional to the mass and the stiffness

$C = \alpha M + \beta K$

By introducing the parameterization of the structure, the global mass and stiffness matrices from Eq. 2 can be expressed in the following form

$$M^{(s)} = \sum_{i=1}^{N} m_i M_i^{(e)}\ , \tag{5}$$

$$K^{(s)} = \sum_{i=1}^{N} k_i K_i^{(e)} \tag{6}$$

where $M^{(e)}$ et $K^{(e)}$ are mass and stiffness elementary matrices, and m_i and k_i are the updating parameters. Case where $m_i = k_i = 1$ imply that the ith element is well modelized and thus doesn't comprise any defect or errors. The updating process consists then in quantifying parameters m_i and k_i to localize and quantify defects in the considered structure. For this purpose we use a measurement set of frequency response functions FRFs.

Let $y^{(s)}(\omega_k)$ be an incomplete measurement set of « s » frequency response functions. In practice FRFs are not completely measured. This problem of incomplete data can be solved by the extension of the experimental data or the reduction of the analytical model. In the method presented here, the unmeasured degrees of freedom are approximated by their analytical equivalents.

3. Cost function and minimization procedure

Before updating a finite elements model it is useful to have an idea on its precision. For that, correlation functions exist; they make a comparison between experimental measurements and values predicted by the analytical model. One of the first correlation methods is based on modes correlation to evaluate the errors modelling. Modal Assurance criterion (MAC) is the most used correlation function; this is defined by Allemang and Brown [9]

$$MAC\left(\{\phi_A\}_i, \{\phi_X\}_j\right) = \frac{\left(\sum_{k=1}^{N} \phi_{Aki} \phi_{Xkj}^{*}\right)^2}{\sum_{k=1}^{N} \phi_{Aki} \phi_{Aki}^{*} \sum_{k=1}^{N} \phi_{Xkj} \phi_{Xkj}^{*}} \tag{7}$$

where ϕ_A is the analytical eigenvector and ϕ_X is the experimental eigenvector.

When measurements noise is important other quantities like frequency response functions FRF are used. The use of FRF instead of modes is more effective since experimental FRFs are directly obtained in experiments whereas the modes require calculations based on these measures. This passage of FRFs to the modes utilizes thus additional error.

One of the principal advantages of the use of FRF is that the damping parameters can be corrected whereas the modal parameters are not sensitive to damping. The use of FRF by selecting a sufficient number of measurement points can attenuate the problem of incomplete measurements. This is due to the fact that FRFs contain the influence of all the modes.

The objective of methods with cost function is to make a correlation between the measured data and those of the analytical model. For the choice of the cost function let us consider the various correlation functions used in frequency domain. They are summarized what follows in

The Frequency Domain Assurance Criterion (FDAC) is given by Pascual et al. [10]

$$FDAC(\omega_A, \omega_X)_k = \frac{\left(\sum_{j=1}^{N} H_{Ajk} H^*_{Xjk}\right)^2}{\left(\sum_{j=1}^{N} H_{Ajk} H^*_{Ajk}\right)\left(\sum_{j=1}^{N} H_{Xjk} H^*_{Xjk}\right)} \tag{8}$$

where H_A is the analytical FRF for an analytical frequency ω_A, and H_X is a measured FRF for a corresponding working frequency ω_X. k is the excitation degree of freedom.

The Frequency Response Scale Factor (FRSF) is given by Pascual et al. [11]

$$FRSF(\omega_A, \omega_X) = \frac{H_A^T(\omega_A)[S]H_X(\omega_X)}{H_A^T(\omega_A)[S]H_A(\omega_A)} \tag{9}$$

where [S] is the a weighting matrix.

The FRSF gives values between - 1 and 1. The FDAC gives a quantitative comparison of FRFs; on the other hand the FRSF gives a qualitative comparison. Consequently the FRSF is not sufficient to evaluate the degree of correlation.

The Frequency Response Assurance Criterion (FRAC) is given by Nefske and Sung [12]

$$FRAC_{jk} = \frac{\left(\sum_{\omega_1}^{\omega_m} H_{Ajk} H^*_{Xjk}\right)^2}{\left(\sum_{\omega_1}^{\omega_m} H_{Ajk} H^*_{Ajk}\right)\left(\sum_{\omega_1}^{\omega_m} H_{Xjk} H^*_{Xjk}\right)} \tag{10}$$

j being the measurement dof and k the excitation dof.

Fotsch and Ewins [13] proposed the Modal FRF Assurance Criterion (MFAC) defined by

$$MFAC(\omega_A,\omega_X)_k = \frac{\left(\sum_{j=1}^{N}\phi_{Ajk}H^*_{Xjk}\right)^2}{\left(\sum_{j=1}^{N}\phi_{Ajk}\phi_{Ajk}\right)\left(\sum_{j=1}^{N}H_{Xjk}H^*_{Xjk}\right)} \tag{11}$$

where ϕ_A is the analytical eigenvector for an analytical frequency ω_A.

Put aside the FRSF, all correlation functions vary between 0 and 1, value 1 indicates a perfect correlation and 0 indicates a bad correlation.

FRSF does not give a good quantitative correlation and thus cannot quantify possible defects objectively. FDAC and FRAC are limited to the use of frequency response functions only.

Some correlation functions are used in literature, Gao and Spencer [14] used the total modal assurance criterion (TMAC) to determine the analytical model to correlate for a damage localization method. Zang et al. [15] used global shape criterion (GSC) and global amplitude criterion (GAC) to update finite element model. In this present work, the method is based on the MFAC (11) which uses the analytical modes in addition to FRFs. The cost function which results is

$$J = \sum_{k=1}^{s}\left(1 - MFAC(\omega_A,\omega_X)_k\right)^2 \tag{12}$$

The minimization of this cost function amounts minimizing the square of the residue

$$R_k = \left(\sum_{j=1}^{N}\phi_{Ajk}\phi_{Ajk}\right)\left(\sum_{j=1}^{N}H_{Xjk}H^*_{Xjk}\right) - \left(\sum_{j=1}^{N}\phi_{Ajk}H^*_{Xjk}\right)^2 \tag{13}$$

what is equivalent to

$$R_k = \left(\phi^T_{Ak}\phi_{Ak}\right)\left(H^T_X H^*_X\right) - \left(\phi_{Ak}H^*_{Xk}\right)^2 \tag{14}$$

The minimization process is written then

$$\min_{m_i,k_i}\left(\delta = \sum_{k=1}^{s}R_k^2\right) \tag{15}$$

4. Gradient of the cost function

The calculation of the gradient of the cost function requires derivations of the analytical eigenvectors according to the updating parameters m_i and k_i. This type of calculation is developed in [16] by the following form:

The mode vector derivative may be expressed as a linear combination of all eigenvectors

$$\frac{\partial \phi_{Ak}}{\partial m_i} = \sum_{q=1}^{N} \mu_{kq} \phi_{Aq} \ , \tag{16}$$

where the coefficients μ_{kq} are determined using the generalized eigenvalue and orthogonalisation properties of eigenvectors

$$\mu_{kq} = \begin{cases} \phi_{Aq}^{T}\left[\left(\dfrac{\partial K}{\partial m_i} - \lambda_k \dfrac{\partial M}{\partial m_i}\right) \Big/ \left(\lambda_k - \lambda_q\right)\right], & \mathrm{q} \neq \mathrm{k} \\ -\dfrac{1}{2}\phi_{Ak}^{T} \dfrac{\partial M}{\partial m_i} \phi_{Ak}, & \mathrm{q} = \mathrm{k} \end{cases} \tag{17}$$

knowing that ϕ_A are the analytical eigenvectors and λ are the analytical eigen values.

In the same way, we have

$$\frac{\partial \phi_{Ak}}{\partial k_i} = \sum_{q=1}^{N} \gamma_{kq} \phi_{Aq} \ , \tag{18}$$

with

$$\gamma_{kq} = \begin{cases} \phi_{Aq}^{T}\left[\left(\dfrac{\partial K}{\partial k_i} - \lambda_k \dfrac{\partial M}{\partial k_i}\right) \Big/ \left(\lambda_k - \lambda_q\right)\right], & \mathrm{q} \neq \mathrm{k} \\ -\dfrac{1}{2}\phi_{Ak}^{T} \dfrac{\partial M}{\partial k_i} \phi_{Ak}, & \mathrm{q} = \mathrm{k} \end{cases} \tag{19}$$

On the other hand, for the calculation of derivations of the mass and stiffness matrices we use the computation formulae clarified by Asma and Bouazzouni [17]

The derivation of the global mass and stiffness are

$$\frac{\partial M}{\partial m_i} = M_i^{(e)}, \frac{\partial M}{\partial k_i} = 0, \tag{20,21}$$

$$\frac{\partial K}{\partial m_i} = 0, \ \frac{\partial K}{\partial k_i} = K_i^{(e)} \tag{22, 23}$$

The FRF is expressed as

$$H(\omega) = Z^{-1}(\omega) = \left(-M\omega^2 + j\omega C + K\right)^{-1} \tag{24}$$

The derivation of this matrix is calculated by

$$\frac{\partial H(\omega)}{\partial m_i}=\frac{\partial Z^{-1}(\omega)}{\partial m_i}=-H(\omega).\frac{\partial Z(\omega)}{\partial m_i}.H(\omega) \quad (25)$$

$$\frac{\partial H(\omega)}{\partial k_i}=\frac{\partial Z^{-1}(\omega)}{\partial k_i}=-H(\omega).\frac{\partial Z(\omega)}{\partial k_i}.H(\omega) \quad (26)$$

Finally, the derivation of the damping stiffness matrix led to

$$\frac{\partial Z(\omega)}{\partial m_i}=M_i^{(e)}\left(-\omega^2+j\alpha\omega\right) \quad (27)$$

$$\frac{\partial Z(\omega)}{\partial k_i}=K_i^{(e)}\left(1+\beta\omega\right) \quad (28)$$

5. Optimization Algorithm

A good choice of the objective function influences the quality and the effectiveness of the method. The cost function must be monotonous and continuous compared to the updating parameters. When the cost function is well chosen, the optimization method plays an important part. The least squares method is one of the most used methods. Some methods based on genetic algorithms improve the updating process.

Usually in damage detection and finite element model updating the optimization process is ill-conditioned and causes local entrapments. To avoid this, Duan et al. [18] propose a float-encoding genetic algorithm. The optimization algorithm used for the minimization of the cost function δ is the Gauss – Newton algorithm, which is an iterative algorithm with Jacobian matrix. The nonlinear system of N unknown and N equations is obtained by

$$\frac{\partial \delta}{\partial m_i}=0\,, \quad (29)$$

$$\frac{\partial \delta}{\partial k_i}=0 \quad (30)$$

The resolution by Gauss – Newton method led to an iterative system of the form

$$\begin{pmatrix} m_1 \\ \dots \\ m_N \\ k_1 \\ \dots \\ k_N \end{pmatrix}^{(v+1)} = \begin{pmatrix} m_1 \\ \dots \\ m_N \\ k_1 \\ \dots \\ k_N \end{pmatrix}^{(v)} - \left(J^{(v)}\right)^{-1} \begin{pmatrix} \frac{\partial\delta}{\partial m_1} \\ \dots \\ \frac{\partial\delta}{\partial m_N} \\ \frac{\partial\delta}{\partial k_1} \\ \dots \\ \frac{\partial\delta}{\partial k_N} \end{pmatrix}^{(v)} \tag{31}$$

where J is the Jacobian matrix

The system is completely constructed using equations (16) to (28). The convergence criterion is based on the relative difference between two successive *m*- and *k*-values during the iteration process; a relative value of 0.1% is acceptable to have a good accuracy with 5% measurement noise. To assure a numerical stability a relaxation coefficient is used to limit the variation of the *m*- and *k*-values at each iteration.

6. Simulated cases studied

First case:

To test the suggested method, we consider the structure of Fig. 1. Meshez with into 70 finite elements and 120 degrees of freedom, with $E=2.1\cdot10^{11}N/m^2$ and $\rho=7800kg/m^3$. The simulating model of the structure is built by introducing simultaneous defects of +40% and -30% of the stiffness respectively in elements 24 and 59 with -20% and +15% of the mass, respectively in elements 14 and 69, by adding 5% of random noise. Measurements are taken according to degrees of freedom 94, 95, 97, 98, 100, 101, 103, 104, 106, 107, 109, 110, 112, 113, 115, 116, 118 and 119 of the structure (which are the displacement of nodes 35 to 43 in Fig. 1.).

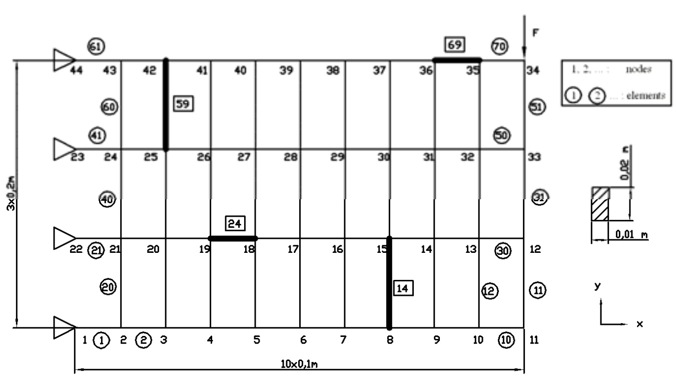


Fig. 1. Simulated test structure

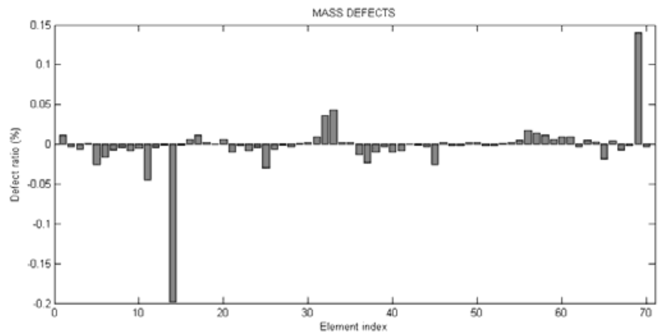


Fig. 2. Mass updating results obtained

Solid vertical line denotes detected mass modelling error. We can see on the defect ratio in Fig. 2. that the introduced errors in the 14th and 69th elements have been detected and quantified close to -0.2 and 0.15 respectively. Other not perturbed elements like 5, 11, 25, 32, 33, 37, 45, and 56 are also updated.

We can see in the defect ratio in Fig. 3. that the introduced stiffness errors in elements 24 and 59 are detected and well quantified respectively 0.4 and -0.3. Some other elements are also considered to be corrected like 10, 17, 37, 45 and 61.

The obtained results represented in Figs. 2. and 3. show that the simulated defects are localized and quantified. Some other element are lightly updated, this is because these elements are geometrically close to the damaged elements, and probably because of the measurement noise.

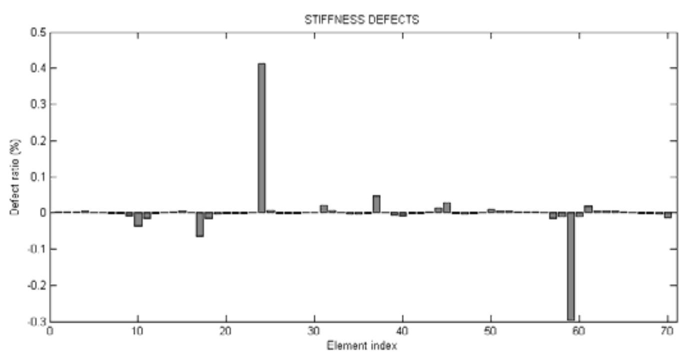


Fig. 3. Stiffness updating results obtained

Second case:

In this second example the suggested method is used to detect damages. We consider the structure represented in Fig. 4.

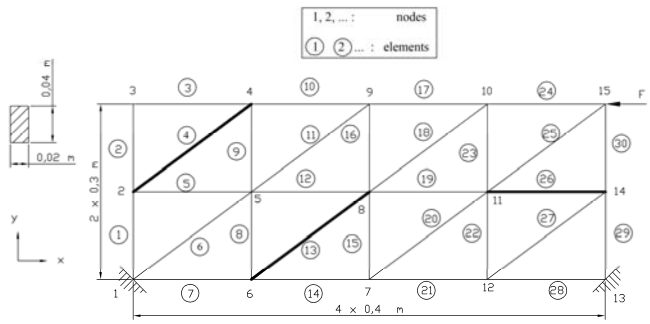


Fig. 4. Simulated damaged structure

This is discretized into 30 finite elements and 39 degrees of freedom. The simulating model of the structure is built by introducing damages 40%, 25%, and 30% of the stiffness respectively in elements 4, 13 and 26, adding 5% of random noise. Measurements are taken according to degrees of freedom 7, 8, 13, 14, 16, 17, 22, 23, 25, 26, 31 and 32 (which are the displacement of nodes 4, 6, 7, 9, 10 and 12 in Fig. 4.).

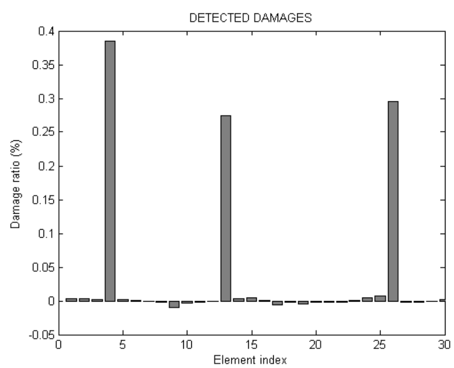
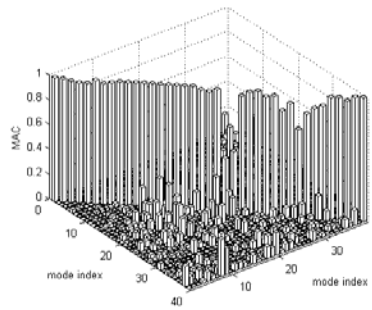


Fig. 5. Detected damages

Fig. 5. shows that introduced damages in elements 4, 13 and 26 are localized and well quantified.

In this work, a correlation is made using the Modal Assurance Criterion (MAC). Fig. 6. and Fig. 7. represent respectively the correlation before and after using the damage detection algorithm. This shows a bad correlation of the initial finite elements model. In fact from the 19^{th} mode to the 35^{th}, MAC diagonal values are less than 1 in Fig. 6.



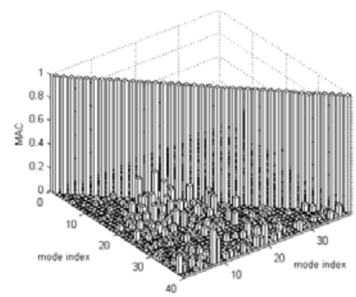


Fig. 6. MAC correlation before using the damage detection algorithm

Fig. 7. MAC correlation after using MAC correlation after using the damage detection algorithm

After application of the proposed algorithm, the diagonal values of the MAC (Fig. 7.) are all close to 1. This shows that the corrected finite elements model is well correlated.

Finally this second example highlights the performance of the method to detect damages in structures.

7. Conclusion

A damage detection method in mechanical structures based on the correlation function MFAC (Modal FRF Assurance Criterion) is proposed. This uses the inverse technique by updating the finite element model. This method is expensive from the point of view of calculation but a numerical stability is acquired since it considers in the derivative only the eigen modes.

The tests carried out on a simulated truss structure shows very interesting qualities of detection and correction in term of quantification and localization. The correlation function and the optimization algorithm used show good characteristics of stability and convergence. The updating method thus obtained will find its application in detection of damage and modelling errors, as for the validation of finite elements models. It allows also the structural health monitoring of large mechanical structures.

R E F E R E N C E S

[1]. *C. B. Larson, D. C. Zimmerman*, Structural model refinement using a genetic algorithm approach, Proceedings of the 11th International Modal Analysis Conference, 1993, pp. 1095- 1101.

[2]. *R. A. Carlin, E. Garcia*, Parameter optimisation of a genetic algorithm for structural damage detection, Proceedings of the 14th International Modal Analysis Conference, 1996, pp. 1292 – 1298.

[3]. *S. Li, S. Shelley, D. Brown*, Perturbed boundary condition testing concepts. Proceedings of the 13th International Modal Analysis Conference, 1995, pp. 902 – 911.

[4]. *S. R. Ibrahim*, Natural Frequency Based Finite Element Model Updating Using Multiple Analytical Models, Proceedings of the 18th International Modal Analysis Conference, 2000, pp. 1018 – 1027.

[5]. *W. D'Ambrogio, A. Fregolent*, Promises and pitfalls of antiresonance based dynamic model updating, Proceedings of the second international conference of Identification in Engineering Systems, 1999, pp. 112-121.

[6]. *J. E. Mottershead*, On the zeros of structural frequency response functions and their sensitivities, Mechanical Systems and Signal Processing, **vol. 12**, no.5, 1998, pp. 591-597.

[7]. *D. A. Rade, G. Lallement*, A strategy for the enrichment of experimental data as applied to an inverse eigen-sensitivity-based FE model updating method, Mechanical Systems and Signal Processing, **vol. 12**, no.2, 1998, pp. 293-307.

[8]. *H. Sohn, C. R. Farrar, F. M. Hemez, D. D. Shunk, D. W. Stinemates, B. R. Nadler*, A review of structural health monitoring literature: 1996–2001, Research report n° LA-13976-MS, Los Alamos National Laboratory, USA. 2003

[9]. *R. J. Allemang, D. L. Brown*, A Correlation Coefficient for Modal Vector Analysis, Proceedings of the 1st International Modal Analysis Conference, 1982, pp. 110-119.

[10]. *R. Pascual, J. C. Golinval, M. Razeto*, Testing of FRF Based Model Updating Methods Using a General Finite Elements Program, 21st International Seminar on Modal Analysis, Louvain, Belgique. 1996 [18]. *Z. Duan, B. F. Spencer, G. Yan, J. Ou*, An improved optimal elemental method for updating finite element models, Earthquake Engineering and Engineering Vibration, **vol.3**, no.1, 2004, pp. 67-74

[11]. *R. Pascual, J. C. Golinval, M. Razeto*, A Frequency Domain Correlation Technique for Model Correlation and Updating, Proceedings of the 15th International Modal Analysis Conference, 1997, pp. 587-592.

[12]. *D. J. Nefske, S. H. Sung*, Correlation of a Coarse-Mesh Finite Element Model Using Structural System Identification and a Frequency Response Assurance Criterion, Proceedings of the 14th International Modal Analysis Conference, 1996, pp. 597-602.

[13]. *D. W. Fotsch, D. J. Ewins*, Application of the MAC in the Frequency Domain, Proceedings of the 18th International Modal Analysis Conference, 2000, pp. 1225-1231.

[14]. *Y. Gao, B. F. Spencer*, Damage localization under ambient vibration using changes in flexibility, Earthquake Engineering and Engineering Vibration, **vol. 1**, no.1, 2002, pp. 136-144.

[15]. *C. Zang, H. Grafe, M. Imregun*, Frequency Domain Criteria for Correlating and Updating Finite Element Models, Mechanical Systems and Signal Processing, **vol. 15**, no.1, 2001, pp. 139- 155.

[16]. *B. Jaishi, W. X. Ren*, Damage detection by finite element model updating using modal flexibility residual", Journal of Sound and Vibration, **vol. 290**, 2006, pp. 369-387.

[17]. *F. Asma, A. Bouazzouni*, Finite element model updating using FRF measurements, Shock and Vibration, **vol. 12**, no. 5, 2005, pp. 377-388.

[18]. *Z. Duan, B. F. Spencer, G. Yan, J. Ou*, An improved optimal elemental method for updating finite element models, Earthquake Engineering and Engineering Vibration, **vol.3**, no.1, 2004, pp. 67-74