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FEEDBACK DESIGN METHOD FOR DESIRED LONGITUDINAL STABILITY CHARACTERISTICS

Bogdan C. TEODORESCU[1]

În lucrarea prezentă se propune o metodă originală de proiectare a unei structuri de reacție având drept scop îmbunătățirea caracteristicilor de stabilitate longitudinală ale aeronavelor.

Adoptând o structură de reacție proporțională, restricțiile algebrice de tip inegalitate care definesc un nivel prescris de calități de zbor sunt transpuse analitic în domenii admisibile ale constantelor de reacție.

Această reprezentare analitică directă a cerințelor de calități de zbor în planul constantelor de reacție constituie o simplificare importantă a metodei clasice a "spațiului K" a lui Ackermann.

Se formulează o condiție de compatibilitate pentru rezolvarea problemei de proiectare considerate la un regim de zbor specificat, precum şi o condiție privind existența soluțiilor cu amplificare fixă pentru un domeniu dat de viteze de zbor.

Abordarea teoretică este validată printr-un exemplu numeric relevant.

In the present paper an original feedback design method is proposed for improving aircraft longitudinal stability characteristics.

Assuming a proportional feedback structure, the algebraic inequality-type constraints that define a prescribed level of flying qualities are transposed analytically into admissible feedback-gain domains.

This direct analytical representation of flying qualities requirements into the feedback-gain plane is an important simplification of Ackerman's classic "K-space" technique.

A compatibility condition for solving the considered synthesis problem at a specified flight regime and a condition concerning the existence of fixed-gain solutions over a given flight speed range are formulated.

The theoretical approach is validated by a relevant numerical example.

Keywords: flight stability, flying qualities, feedback control

Nomenclature

a	= speed of sound;
c, S	= wing mean chord, wing surface;
C_D, C_L, C_m, C_T, C_W	= drag, lift, pitching-moment, thrust and weight coefficients, respectively;

[1] Lecturer, Dept. of Aerospace Engineering, University POLITEHNICA of Bucharest, ROMANIA, e-mail: b_teodor@yahoo.com

$C_{D_\alpha}, C_{L_\alpha}, C_{m_\alpha}$ = derivatives of drag, lift and pitching-moment coefficient with respect to the aircraft angle-of-attack, $C_{D_\alpha} = \partial C_D / \partial\alpha$, $C_{L_\alpha} = \partial C_L / \partial\alpha$, $C_{m_\alpha} = \partial C_m / \partial\alpha$;

$C_{L_{\dot\alpha}}$, $C_{m_{\dot\alpha}}$ = derivatives of lift and pitching-moment coefficient with respect to the nondimensional angle-of-attack rate of change, $C_{L_{\dot\alpha}} = \partial C_L / \partial(0.5\,\dot\alpha c / V_0)$, $C_{m_{\dot\alpha}} = \partial C_m / \partial(0.5\,\dot\alpha c / V_0)$;

$C_{D_q}, C_{L_q}, C_{m_q}$ = derivatives of drag, lift and pitching-moment coefficient with respect to the nondimensional pitch rate, $C_{D_q} = \partial C_D / \partial(0.5qc / V_0)$, $C_{L_q} = \partial C_L / \partial(0.5qc / V_0)$, $C_{m_q} = \partial C_m / \partial(0.5qc / V_0)$;

$C_{D_V}, C_{L_V}, C_{m_V}, C_{T_V}$ = derivatives of drag, lift, pitching-moment and thrust coefficient with respect to the nondimensional flight speed, $C_{D_V} = \partial C_D / \partial(V / V_0)$, $C_{L_V} = \partial C_L / \partial(V / V_0)$, $C_{m_V} = \partial C_m / \partial(V / V_0)$, $C_{T_V} = \partial C_T / \partial(V / V_0)$;

$C_{L_{\delta_e}}$, $C_{m_{\delta_e}}$ = derivatives of lift and pitching-moment coefficient with respect to the elevator deflection angle, $C_{L_{\delta_e}} = \partial C_L / \partial\delta_e$, $C_{m_{\delta_e}} = \partial C_m / \partial\delta_e$;

$C_{T_{\delta_t}}$ = derivative of thrust coefficient with respect to the thrust setting input, $C_{T_{\delta_t}} = \partial C_T / \partial\delta_t$;

M = Mach number, $M = V / a$;

V = flight speed.

Note: Used as a subscript, “0” denotes the considered reference flight condition.

1. Introduction

Stability characteristics are critically important in estimating the so-called *flying (handling) qualities,* i.e. those aircraft characteristics that determine the ease, precision and safety with which a human pilot is able to accomplish the flight tasks required by a specific mission.

Since, due to his physiological limits, a human pilot cannot efficiently influence the short-term aircraft dynamics, flying qualities depend essentially on the characteristics the *rapid modes of motion*; typically, these modes of motion are the *short-period* longitudinal mode and the aperiodic *roll* and oscillatory *Dutch-roll* lateral-directional modes. Being relatively slow, the other two typical modes of motion, namely the longitudinal *phugoid* oscillation and the lateral-directional *spiral* mode, manifest themselves, usually, as minor trimming problems.

Flying qualities are rated by pilots using the well-known *Cooper-Harper* scale, [1], [2]. Mathematically, different levels of flying qualities are defined in terms of algebraic *inequality-type constraints* applied to significant stability parameters, [3].

In the present work, *a feedback design problem* is formulated and solved in order *to improve longitudinal stability characteristics* of high-performance airplanes and obtain *desired levels of flying qualities.*

Appropriate algebraic constraints defining a prescribed level of flying qualities are imposed to the *short-period modal characteristics.* Additionally, an algebraic constraint is considered for *avoiding speed divergence* (occurrence of an aperiodic unstable longitudinal mode affecting, dominantly, the airplane's speed).

A proportional feedback law is designed based on the considered algebraic constraints. These constraints are *analytically represented* in the feedback gain space (also-called "*K*-space"), in which *admissible gain domains* are determined.

Using the typical geometric characteristics of the determined admissible gain domains, *a compatibility condition* for solving the considered design problem at specified flight regimes is formulated. On this basis, *an existence condition for fixed-gain solutions* within given flight-speed and altitude intervals is obtained.

The proposed feedback-law design method represents a more direct and simple design method than Ackerman's classic technique, [4], [5].

2. Mathematical model of open-loop dynamics

The following linear differential model describing the longitudinal motion of an airplane is considered, [6],

$$\frac{d}{dt}\Delta V = a_{11}\,\Delta V + a_{12}\,\Delta\alpha + a_{13}\,\Delta q + a_{14}\,\Delta\theta + b_{11}\,\Delta\delta_e + b_{12}\,\Delta\delta_t, \qquad (1)$$

$$\frac{d}{dt}\Delta\alpha = a_{21}\,\Delta V + a_{22}\,\Delta\alpha + a_{23}\,\Delta q + a_{24}\,\Delta\theta + b_{21}\,\Delta\delta_e + b_{22}\,\Delta\delta_t, \quad (2)$$

$$\frac{d}{dt}\Delta q = a_{31}\,\Delta V + a_{32}\,\Delta\alpha + a_{33}\,\Delta q + a_{34}\,\Delta\theta + b_{31}\,\Delta\delta_e + b_{32}\,\Delta\delta_t, \quad (3)$$

$$\frac{d}{dt}\Delta\theta = \Delta q, \quad (4)$$

where ΔV, $\Delta\alpha$, Δq, $\Delta\theta$ are the components of the state disturbance vector Δx, while $\Delta\delta_e$ and $\Delta\delta_t$ represent the components of the control disturbance vector Δu, i.e.

$$\Delta x = [\Delta V \quad \Delta\alpha \quad \Delta q \quad \Delta\theta]^T, \quad (5)$$

$$\Delta u = [\Delta\delta_e \quad \Delta\delta_t]^T. \quad (6)$$

Specifically, the longitudinal state variables are the vehicle's flight speed (V), angle-of-attack (α), pitch rate (q) and pitch angle (θ), whilst the control variables are the thrust setting input (δ_t) and the elevator deflection angle (δ_e).

The considered reference flight condition, about which the equations of motion are linearized, is a steady, straight, symmetric flight at constant altitude. In this case, the stability coefficients $a_{11}, \ldots, a_{34}$ are

$$a_{11} = \frac{V_0}{\mu c}\left[C_{T_V}\cos(\alpha_0+\tau) - C_{D_V}\right], \qquad a_{12} = \frac{V_0^2}{\mu c}\,(C_{L_0} - C_{D_\alpha}),$$

$$a_{13} = -\frac{V_0}{2\mu}C_{D_q} \cong 0, \qquad a_{14} = -g,$$

$$a_{21} = -\frac{2}{c}\cdot\frac{C_{T_V}\sin(\alpha_0+\tau) + C_{L_V} + 2C_{W_0}}{2\mu + C_{L_{\dot\alpha}}} \equiv \frac{2}{c}\hat a_{21},$$

$$a_{22} = -\frac{2V_0}{c}\cdot\frac{C_{L_\alpha} + C_{D_0}}{2\mu + C_{L_{\dot\alpha}}} \equiv \frac{2V_0}{c}\hat a_{22}, \qquad a_{23} \cong 1, \qquad a_{24} = 0, \quad (7)$$

$$a_{31} = \frac{4V_0}{c^2}\cdot\frac{1}{\hat I_y}\,(C_{m_V} + \hat a_{21}C_{m_{\dot\alpha}}), \quad a_{32} = \left(\frac{2V_0}{c}\right)^2\cdot\frac{1}{\hat I_y}\,(C_{m_\alpha} + \hat a_{22}C_{m_{\dot\alpha}}),$$

$$a_{33} = \frac{2V_0}{c}\cdot\frac{1}{\hat I_y}\,(C_{m_q} + C_{m_{\dot\alpha}}), \qquad a_{34} = 0$$

and the control coefficients $b_{11}, \ldots, b_{32}$ are expressed as follows

$$b_{11} \cong 0, \qquad b_{12} = \frac{V_0^2}{\mu c}C_{T_{\delta_t}}\cos(\alpha_0+\tau),$$

$$b_{21} = -\frac{2V_0}{c}\cdot\frac{C_{L\delta_e}}{2\mu + C_{L\dot{\alpha}}} \equiv \frac{2V_0}{c}\hat{b}_{21},$$

$$b_{22} = -\frac{2V_0}{c}\cdot\frac{C_{T\delta_t}\,\sin(\alpha_0+\tau)}{2\mu + C_{L\dot{\alpha}}} \equiv \frac{2V_0}{c}\hat{b}_{22}\ , \tag{8}$$

$$b_{31} = \left(\frac{2V_0}{c}\right)^2\cdot\frac{1}{\hat{I}_y}\cdot(C_{m\delta_e} + \hat{b}_{21}\,C_{m\dot{\alpha}})\,,$$

$$b_{32} = \left(\frac{2V_0}{c}\right)^2\cdot\frac{1}{\hat{I}_y}\cdot(C_{T\delta_t}\cdot t/c + \hat{b}_{22}\,C_{m\dot{\alpha}})\,,$$

where μ and $\hat{I}_y$ denote dimensionless mass and inertia parameters,

$$\mu = \frac{2m}{\rho S c}\ , \qquad \hat{I}_y = \frac{8I_y}{\rho S c^3}\ . \tag{9}$$

The characteristic equation of the fourth-order differential system (1)-(4) is

$$\lambda^4 + c_3\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 = 0\,, \tag{10}$$

where

$$c_3 = -\left(a_{11} + a_{22} + a_{33}\right), \tag{11}$$

$$c_2 = a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32}\ , \tag{12}$$

$$c_1 = a_{11}\left(a_{23}a_{32} - a_{22}a_{33}\right) + a_{12}\left(a_{21}a_{33} - a_{23}a_{31}\right) + a_{13}\left(a_{22}a_{31} - a_{21}a_{32}\right) - a_{14}a_{31}, \tag{13}$$

$$c_0 = a_{14}\left(a_{22}a_{31} - a_{21}a_{32}\right). \tag{14}$$

Typically for conventional configuration airplanes, Eq. (10) has two pairs of complex-conjugate roots corresponding to two oscillatory modes of motion: the rapid, *short-period* (*sp*) mode, involving, primarily, the airplane's angle-of-attack and pitch rate, and the slow, long-period *phugoid* (*p*) mode, in which the dominant variables are the flight speed and the longitudinal attitude angle.

Under certain conditions (associated, for example, with the transition from subsonic to supersonic flight regimes), the normal oscillatory phugoid is replaced by two aperiodic modes; usually, one of these aperiodic modes is a subsidence and the other one a divergence ("speed divergence").

3. Longitudinal flying qualities requirements

As previously mentioned, the longitudinal flying qualities of an airplane are mainly determined by the short-period modal characteristics.

In the present paper, the most significant short-period parameters, namely the *control anticipation parameter (CAP)* and the *short-period damping ratio* (ζ_{sp}), are considered. These parameters are algebraically constrained in order to obtain a specified level of flying qualities.

Proposed by Bihrle Jr., [7], the control anticipation parameter is defined as the ratio of the initial pitch angular acceleration ($\dot{q}(t=0)$) to the steady-state change in normal load factor (Δn) following a step longitudinal control input,

$$CAP = \frac{\dot{q}(t=0)}{\Delta n}. \tag{15}$$

If the control anticipation parameter is too small, the pilot will appreciate the pitch response as sluggish and overcontrol the airplane, thus exceeding the desired response by generating extremely large Δn values. On the contrary, if the control anticipation parameter is too large, the pilot will appreciate the pitch response as too fast (sensitive) and will reduce or even reverse the control input, thus generating too small Δn values and, consequently, not reaching the desired response.

Using the classical short-period approximation

$$\frac{d}{dt}\Delta\alpha = a_{22}\,\Delta\alpha + a_{23}\,\Delta q, \tag{16}$$

$$\frac{d}{dt}\Delta q = a_{32}\,\Delta\alpha + a_{33}\,\Delta q, \tag{17}$$

and accounting for the usually negligible values of derivatives C_{L_q} and $C_{L_{\delta_e}}$, it can be shown that

$$CAP = \frac{\omega_{n_{sp}}^2}{\Delta n / \Delta\alpha}, \tag{18}$$

where $\omega_{n_{sp}}$ is the short-period undamped (angular) frequency and $\Delta n / \Delta\alpha$ is the so-called *normal acceleration sensitivity parameter,* which can be expressed as

$$\frac{\Delta n}{\Delta\alpha} = \frac{V}{g}\cdot\frac{a_{22}b_{31} - a_{32}b_{21}}{a_{33}b_{21} - a_{23}b_{31}} \tag{19}$$

or

$$\frac{\Delta n}{\Delta\alpha} \cong \frac{2V^2}{g\,c}\cdot\frac{(C_{L_\alpha} + C_D)C_{m_{\delta_e}} - C_{m_\alpha}C_{L_{\delta_e}}}{(2\mu - C_{L_q})C_{m_{\delta_e}} + C_{m_q}C_{L_{\delta_e}}}. \tag{20}$$

To obtain a prescribed level of flying qualities according to the military flying qualities standard MIL-STD 1797A, [3], the control anticipation parameter and the short-period damping ratio are algebraically constrained in the form

$$CAP_{\min} \leq \frac{\omega_{n_{sp}}^2}{\Delta n / \Delta\alpha} \leq CAP_{\max}\,, \tag{21}$$

$$\zeta_{\min} \leq \zeta_{sp} \leq \zeta_{\max}\,, \tag{22}$$

the limit values depending on the airplane *class,* flight phase *category* and flying qualities *level.*

An additional constraint is considered for *avoiding speed divergence.*

Specifically, the constant term in the longitudinal characteristic equation is constrained to be positive, i.e.

$$c_0 > 0\ . \tag{23}$$

It should be noticed that the preceding constraint represents one of the critical Routh-Hurwitz stability conditions. Indeed, as pointed out by Duncan, if c_0 changes its sign from positive to negative, then an aperiodic divergent mode appears in the solution of the linear equations of motion, [8].

Generally, relying exclusively on airplane's natural stability and control characteristics, inequalities (21)-(23) cannot be satisfied at all operational flight conditions. Hence, it is necessary to design and implement appropriate feedback control laws such that

$$CAP_{\min} \leq \frac{\tilde{\omega}_{n_{sp}}^2}{\Delta n / \Delta\alpha} \leq CAP_{\max}\,, \tag{24}$$

$$\zeta_{\min} \leq \tilde{\zeta}_{sp} \leq \zeta_{\max}\,, \tag{25}$$

$$\tilde{c}_0 > 0\,, \tag{26}$$

where the upper symbol "~" designates the corresponding closed-loop parameters.

Note: It can be readily shown that the considered feedback structure does not influence the values of the normal acceleration sensitivity parameter.

4. Mathematical model of closed-loop dynamics

In the present paper, in order to satisfy the mentioned flying qualities constraints over the entire operational flight envelope, the following *feedback law* is considered

$$\Delta\delta_e = k_\alpha\, \Delta\alpha + k_q\, \Delta q\ , \tag{27}$$

i.e. the airplane's angle-of-attack and pitch rate variations $\Delta\alpha$ and Δq are fed back proportionally to the elevator deflection variation $\Delta\delta_e$, with the gain factors k_α and k_q as design parameters to be determined.

By implementing the chosen feedback law, the following *closed-loop model* is obtained (assuming $\Delta\delta_t \equiv 0$ and $b_{11} = 0$)

$$\frac{d}{dt}\Delta V = a_{11}\,\Delta V + a_{12}\,\Delta\alpha + a_{13}\,\Delta q + a_{14}\,\Delta\theta\ , \tag{28}$$

$$\frac{d}{dt}\Delta\alpha = a_{21}\,\Delta V + \tilde{a}_{22}\,\Delta\alpha + \tilde{a}_{23}\,\Delta q + a_{24}\,\Delta\theta\,, \tag{29}$$

$$\frac{d}{dt}\Delta q = a_{31}\,\Delta V + \tilde{a}_{32}\,\Delta\alpha + \tilde{a}_{33}\,\Delta q + a_{34}\,\Delta\theta\ , \tag{30}$$

$$\frac{d}{dt}\Delta\theta = \Delta q\,, \tag{31}$$

where the closed-loop ("augmented") coefficients $\tilde{a}_{22}$, $\tilde{a}_{23}$, $\tilde{a}_{32}$, $\tilde{a}_{33}$ are given by

$$\tilde{a}_{22} = a_{22} + k_\alpha b_{21}\ , \quad \tilde{a}_{23} = a_{23} + k_q b_{21}\ , \tag{32}$$

$$\tilde{a}_{32} = a_{32} + k_\alpha b_{31}\ , \quad \tilde{a}_{33} = a_{33} + k_q b_{31}\ . \tag{33}$$

Based on the *closed-loop short-period approximation*

$$\frac{d}{dt}\Delta\alpha = \tilde{a}_{22}\,\Delta\alpha + \tilde{a}_{23}\,\Delta q\,, \tag{34}$$

$$\frac{d}{dt}\Delta q = \tilde{a}_{32}\,\Delta\alpha + \tilde{a}_{33}\,\Delta q\,, \tag{35}$$

it follows that

$$\tilde{\omega}^2_{n_{sp}} = \tilde{a}_{22}\tilde{a}_{33} - \tilde{a}_{23}\tilde{a}_{32}\ , \tag{36}$$

$$2\tilde{\zeta}_{sp}\tilde{\omega}_{n_{sp}} = -(\tilde{a}_{22} + \tilde{a}_{33})\,. \tag{37}$$

Thus, the closed-loop short-period characteristics $\tilde{\omega}^2_{n_{sp}}$ and $2\tilde{\zeta}_{sp}\tilde{\omega}_{n_{sp}}$ can be expressed as

$$\tilde{\omega}^2_{n_{sp}} = \omega^2_{n_{sp}} + m_1 k_\alpha + m_2 k_q\ , \tag{38}$$

$$2\tilde{\zeta}_{sp}\,\tilde{\omega}_{n_{sp}} = 2\zeta_{sp}\omega_{n_{sp}} - b_{21}k_\alpha - b_{31}k_q\ , \tag{39}$$

where $\omega_{n_{sp}}^2$ and $2\zeta_{sp}\omega_{n_{sp}}$ are the corresponding open-loop characteristics,

$$\omega_{n_{sp}}^2 = a_{22}a_{33} - a_{23}a_{32} \ , \tag{40}$$

$$2\zeta_{sp}\omega_{n_{sp}} = -(a_{22} + a_{33}) \, , \tag{41}$$

and the coefficients m_1, m_2 are given by

$$m_1 = a_{33}b_{21} - a_{23}b_{31} \ , \tag{42}$$

$$m_2 = a_{22}b_{31} - a_{32}b_{21} \ . \tag{43}$$

The closed-loop short-period damping ratio ($\tilde{\zeta}_{sp}$) can be written as

$$\tilde{\zeta}_{sp} = \frac{2\zeta_{sp}\omega_{n_{sp}} - b_{21}k_\alpha - b_{31}k_q}{2 \cdot \sqrt{\omega_{n_{sp}}^2 + m_1 k_\alpha + m_2 k_q}} \ . \tag{44}$$

5. Feedback design methodology

The considered closed-loop constraints concerning the control anticipation parameter, short-period damping ratio and speed divergence can be rewritten in the form

$$CAP_{\min} \cdot (\Delta n / \Delta\alpha) \le \omega_{n_{sp}}^2 + m_1 k_\alpha + m_2 k_q \le CAP_{\max} \cdot (\Delta n / \Delta\alpha), \tag{45}$$

$$\zeta_{\min} \le \frac{2\zeta_{sp}\omega_{n_{sp}} - b_{21}k_\alpha - b_{31}k_q}{2 \cdot \sqrt{\omega_{n_{sp}}^2 + m_1 k_\alpha + m_2 k_q}} \le \zeta_{\max} \, , \tag{46}$$

$$c_0 + m_3 k_\alpha > 0 \, , \tag{47}$$

where

$$c_0 = a_{14}(a_{22}a_{31} - a_{21}a_{32}), \quad m_3 = a_{14}(a_{31}b_{21} - a_{21}b_{31}). \tag{48}$$

The previous flying qualities constraints define, at each flight condition, an *admissible domain in the* k_α-k_q *plane.* As illustrated in Fig.1, inequalities (45) and (46) define a domain limited by the parallel straight lines l_1, l_2,

$$(l_1): \quad m_1 k_\alpha + m_2 k_q = CAP_{\max} \cdot (\Delta n / \Delta\alpha) - \omega_{n_{sp}}^2 \, , \tag{49}$$

$$(l_2): \quad m_1 k_\alpha + m_2 k_q = CAP_{\min} \cdot (\Delta n / \Delta\alpha) - \omega_{n_{sp}}^2 \, , \tag{50}$$

and by the parabolic arcs p_1, p_2,

$$(p_1):\ 2\zeta_{sp}\omega_{n_{sp}} - b_{21}k_\alpha - b_{31}k_q = 2\zeta_{\max}\cdot\sqrt{\omega_{n_{sp}}^2 + m_1 k_\alpha + m_2 k_q}\,, \tag{51}$$

$$(p_2):\ 2\zeta_{sp}\omega_{n_{sp}} - b_{21}k_\alpha - b_{31}k_q = 2\zeta_{\min}\cdot\sqrt{\omega_{n_{sp}}^2 + m_1 k_\alpha + m_2 k_q}\,, \tag{52}$$

whilst inequality (47) is satisfied in the region situated on the right-hand side of the vertical straight line l_3,

$$(l_3):\quad k_\alpha \equiv (k_\alpha)_{\tilde{c}_0=0} = -c_0/m_3\,. \tag{53}$$

Note that the angular coefficient of the parallel straight lines l_1 and l_2 is negative for conventional configuration airplanes and usual flight conditions ($m_1 > 0$, $m_2 > 0$).

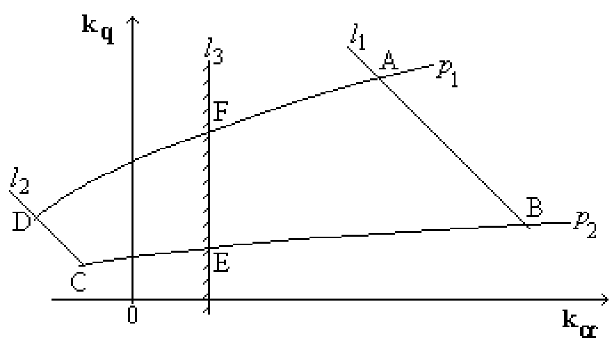

Fig. 1. Typical aspect of the admissible gain domain

Based on the typical aspect of the admissible gain domain represented in Fig.1, it follows that, *at each specified flight condition,* the considered feedback design problem can be solved if

$$(k_\alpha)_B > (k_\alpha)_{\tilde{c}_0=0}\ . \tag{54}$$

This *compatibility condition* can be written, in an extended form, as

$$\frac{2m_2\left(\zeta_{\min}\Omega_{\max} - \zeta_{sp}\omega_{n_{sp}}\right) + b_{31}\left(\Omega_{\max}^2 - \omega_{n_{sp}}^2\right)}{b_{31}m_1 - b_{21}m_2} > -\frac{c_0}{m_3}\,, \tag{55}$$

where

$$\Omega_{\max}^2 = CAP_{\max}\cdot(\Delta n/\Delta\alpha) \tag{56}$$

and m_1, m_2 and m_3 are functions of airplane's stability and control characteristics (according to expressions (42), (43) and (48)).

It should be noted the importance of the above-mentioned compatibility condition from a designer's point of view as it reveals the influence of different stability and control parameters in the context of a specified flying quality level.

An existence condition for fixed-gain solutions over given Mach number and altitude intervals (denoted I_M , respectively I_H) can be inferred as follows

$$\min_{\substack{M \in I_M \\ H \in I_H}} \left\{ (k_\alpha)_B \right\} > \max_{\substack{M \in I_M \\ H \in I_H}} \left\{ (k_\alpha)_{\tilde{c}_0 = 0} \right\} . \tag{57}$$

6. Numerical application

A typical light-weight supersonic fighter airplane has been considered for numerical studies. According to Ref.3, for this class of airplanes (*Class IV*) and *Category A* flight phases, the limit values defining the best level (*Level 1*) of flying qualities are

$$CAP_{\min} = 0.28 \ , \qquad CAP_{\max} = 3.6 \ , \tag{58}$$

$$\zeta_{\min} = 0.35 \ , \qquad \zeta_{\max} = 1.3 \ . \tag{59}$$

Admissible gain domains have been determined for specified values of flight speed and altitude. Figures 2 and 3 illustrate the obtained admissible gain domains corresponding to *two subsonic* (*M*=0.6, *M*=0.9) and *two supersonic* (*M*=1.2, *M*=1.6) *Mach numbers* and an altitude value of 9000 m. Note that, except for the low subsonic regime (*M*=0.6), *the constraint concerning speed divergence is an important limiting factor* for the obtained admissible gain domains.

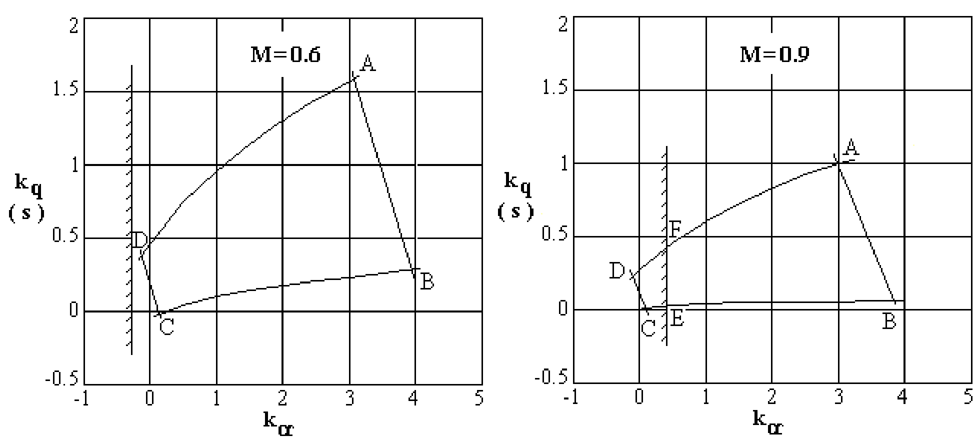

Fig. 2. Admissible gain domains for $M = 0.6$ and $M = 0.9$

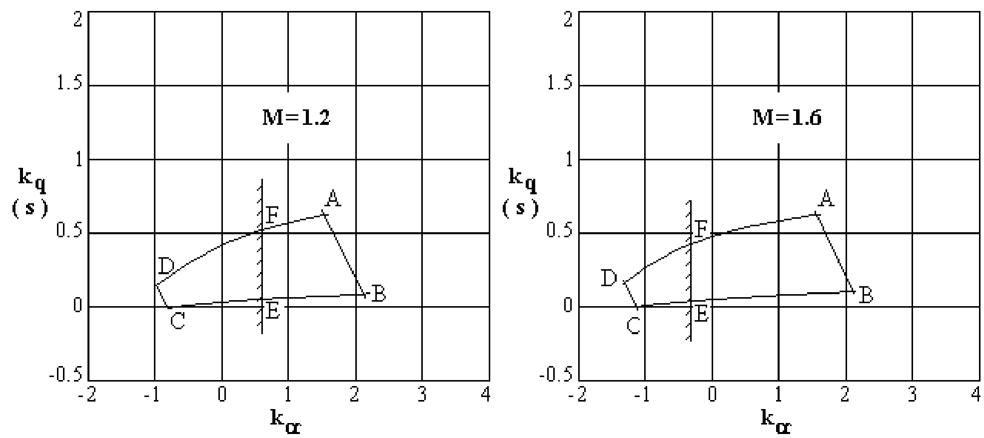

Fig. 3. Admissible gain domains for $M = 1.2$ and $M = 1.6$

As seen in Fig. 4, condition (57) is satisfied over the entire Mach number range. In this case, the considered feedback synthesis problem admits *fixed-gain solutions* with respect to the flight speed (at the specified altitude). These fixed-gain solutions are characterized by k_α -values belonging to the interval marked by dashed lines in Fig. 4, i.e. $1.48 < k_\alpha < 1.98$. Obviously, in case that inequality (54) is satisfied for different operational flight conditions and inequality (57) is not satisfied within the specified flight speed and altitude intervals, the considered design problem can be solved by using appropriate gain-scheduling laws, [9].

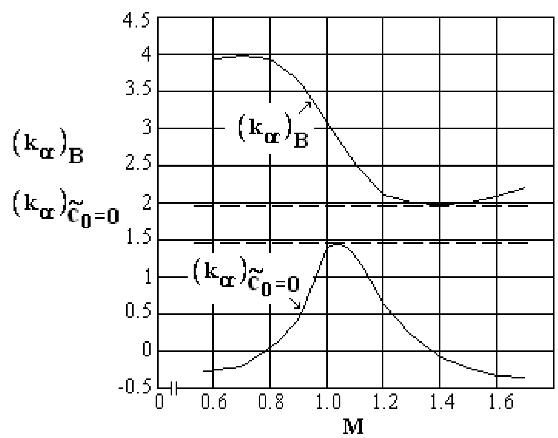

Fig. 4. Existence proof of fixed-gain solutions

The *open-loop* flying qualities parameters and the *corresponding closed-loop* parameters obtained for the fixed-gain solution ($k_\alpha = 1.75$, $k_q = 0.25$) are represented *as functions of flight speed (Mach number)* in Figs. 5-7.

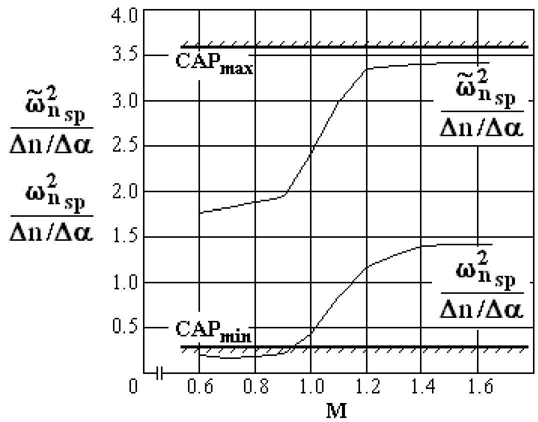

Fig. 5. Open-loop and the obtained closed-loop values of the control anticipation parameter

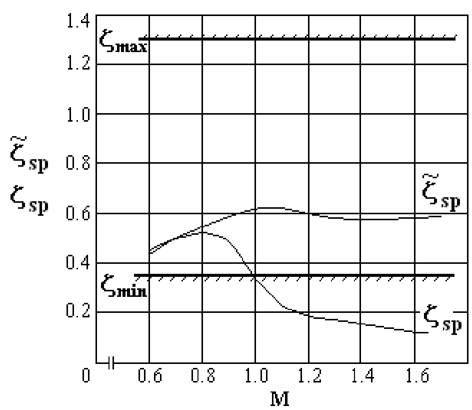

Fig. 6. Open-loop and the obtained closed-loop values of the damping ratio

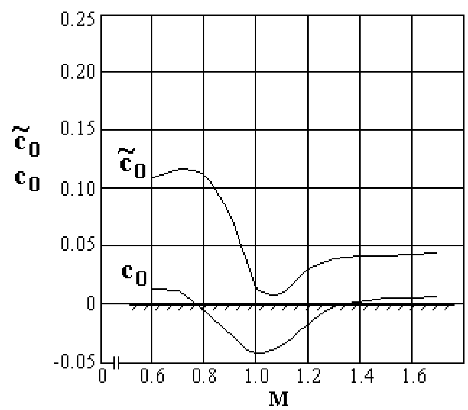

Fig. 7. Open-loop c_0 and the obtained closed-loop $\tilde{c}_0$

As it can be noticed, there are *three distinct flight-speed ranges* within which the considered parameters have unsatisfactory open-loop values. To be specific, the open-loop values of the control anticipation parameter are not satisfactory at subsonic Mach numbers, those of the short-period damping ratio – at supersonic Mach numbers, and the condition for avoiding speed divergence isn't satisfied within a speed range involving, primarily, the transonic flight regimes.

Remarkably, as illustrated in Figs. 5-7, the chosen fixed-gain solution ($k_\alpha = 1.75$, $k_q = 0.25$) provides *satisfactory closed-loop values* of the considered flying qualities parameters *over the entire operational flight speed range.*

7. Conclusions

An original feedback synthesis method has been proposed for improving longitudinal stability characteristics of high-performance airplanes and obtaining *desired levels of flying qualities.*

A proportional feedback structure relating the airplane's angle-of-attack and pith rate changes ($\Delta\alpha$ and, respectively, Δq) to the elevator control input ($\Delta\delta_e$) has been assumed and *admissible gain domains* have been *analytically determined,* at each specified flight condition, by imposing appropriate *inequality-type constraints* to the significant flying qualities parameters.

Taking into account the particular aspect of the determined admissible gain domains, *a compatibility condition* (at each specified flight regime) and *an existence condition for fixed-gain solutions* (over specified Mach number intervals) have been formulated.

The predicted existence of fixed-gain solutions in the studied numerical case has been verified by showing that the closed-loop flying qualities parameters meet the requirements corresponding to the desired level of flying qualities (i.e., level 1 according to Ref.3).

Since the proposed methodology provides entire admissible domains in the *K*-plane, robustness considerations can be easily included in the design process by appropriately choosing the operational gain values for both fixed-gain and gain scheduling solutions.

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