FINANCIAL DATA ANALYSIS USING NONLINEAR TIME SERIES METHODS. FLUCTUATIONS INTERPRETATION OF FOREIGN CURRENCY EXCHANGE RATES

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A financial data analysis using the time series method has been performed. At the same time a correct interpretation of daily exchange rates fluctuations, for both Swiss franc and Euro foreign currencies is presented. By means of powerful box-counting algorithm, the global fractal dimensions associated to the nonlinear evolutionary processes were calculated.

Keywords: time series, fractal analysis, correlation, attractors, embedding dimension

1. Introduction

For a long time, perhaps decades, the science of complex systems has been attempting to provide methods of understanding the dynamics of systems where conventional methods fail. These methods apply, with excellent results, across the various fields, e.g. chaotic dynamics that can be observed in physical and chemical systems, biological systems, economic systems, but also in many others [1, 2]. There are many systems in which the behaviour of the whole emerges from interactions between the parts, e.g. cells in bodies, cars on roads or traders in financial markets [3]. Being in similar situations, we try to use appropriate techniques and methods [4], in solving the proposed issue by means of this study.

As a natural continuation, we can affirm that in the real world most systems considered self consistent are, de facto, complex systems. By definition, the spatially and/or temporally extended nonlinear systems are entitled complex systems. As a distinctive look, they are characterized as having collective properties associated with the system as a whole, just that are different from the characteristic behaviors of the constituent parts [5].

In a consecrate language, a dynamical system consists of an abstract phase space or state space, whose coordinates describe the dynamical state at any

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instant. In addition, it is equipped with a dynamical rule, which specifies the immediate future trend of all state variables, given only the present values of those same state variables. Strictly mathematically speaking, a dynamical system is described by a classical “initial value problem”.

There are two general types of such systems. Firstly, dynamical systems can be "deterministic" if there is a unique consequent to every state. Secondly, dynamical systems can be "stochastic" or "random" if there is more than one consequent chosen from some probability distribution (see for example the "perfect" coin toss that has two consequents with equal probability for each initial state).

Another notion extensively used is phase space. In the natural continuation of the above statements, phase space is the collection of possible states of a dynamical system. A phase space can be finite (e.g. for the ideal coin toss, we have two states heads and tails), countably infinite (e.g. state variables are integers), or uncountably infinite (e.g. state variables are real numbers). Implicit in the notion is that a particular state in phase space specifies the system completely. It is all we need to know about the system to have complete knowledge of the immediate future. An attractor is simply a state into which a system settles (thus dissipation is needed). Thus in the long term, a dissipative dynamical system may settle into an attractor.

Interestingly enough, there is still some controversy on this statement, in the mathematics community. A lot of authors adopt a new definition. Thereby the attractor is a set in the phase space that has a neighborhood in which every point stays nearby and approaches the attractor as time goes to infinity.

In formal language, a map f is chaotic on a compact invariant set S if f is transitive on S (there is a point x whose orbit is dense in S) and f exhibits sensitive dependence on S. The same definition can be said in a more accessible manner. The Chaos is an effectively unpredictable long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. It must be emphasized that a deterministic dynamical system is perfectly predictable given perfect knowledge of the initial condition, and is in practice always predictable in the short term. In other words, the key to long-term unpredictability is a property known as sensitivity to initial conditions or sensitive dependence on the initial conditions. In general, for a dynamical system to be chaotic it must have a 'large' set of initial conditions, which are highly unstable. No matter how precisely one measures the initial condition in these systems, one’s prediction of its subsequent motion goes radically wrong after a short time.

2. Theoretical background and mathematical support

Fractal dimension

Let $N(X, \varepsilon)$ denote the minimum number of balls of radius ε required to cover X. Then the fractal dimension [6] is defined as

$$df(X) = \lim_{\varepsilon \to 0} \sup \frac{\log N(X, \varepsilon)}{\log(1/\varepsilon)} \tag{1}$$

The bound on the fractal dimension we prove here would essentially if the limit superior in (1) could be replaced by a straightforward limit. Not an exaggerated requirement, the "lim sup" is necessary, as there are simple sets for which the limit as $\varepsilon \to 0$ is inexistent.

Takens theorem ("embedding theorem")

Theorem (Takens 1981). Let M be a compact manifold of dimension d. For pairs (ϕ, h), where $\phi : M \to M$ is a smooth (at least C^2) diffeomorphism and $h : M \to R$ a smooth function, it is a generic property that the $(2d+1)$-fold observation map $H_k[\phi, h]:M \to R^{2d+1}$ defined by

$$x \to (h(x), h(\phi(x)),\dots,h(\phi^{2d+1}(x))) \tag{2}$$

is an immersion (i.e. H_k is one-to-one between M and its image with both H_k and H_k^{-1} differentiable).

The theorem can be applied to time series by taking ϕ to be the time T map of the underlying (continuous time) dynamical system, i.e. $\phi^j(x_0) = x(jT)$, where $x(\cdot)$ is the trajectory starting at x_0. The reconstruction that is then provided is 'accurate' in two ways, *i)* and *ii)*.

i). The first is topological: the map H_k is one-to-one between M and its image in R^{2d+1}, so that the time delay coordinates

$$[h(x(0)),h(x(T)),\dots,h(x(2dT))] \tag{3}$$

can be guaranteed to distinguish between points on M .

ii). The second is dynamical: the time T map on M is equivalent to a shift on the time series in 'delay coordinate space',

$$[h(x(0)),\ h(x(T)),\ \dots,\ h(x(2dT)),\ h(x((2d+1)T)),\ \ h(x((2d+2)T))\ \dots,]$$
$$\text{is } [h(x(0)),\ H_k(x(T)),\ \ h(x((2d+2)T))\ \dots,] \tag{4}$$
$$\text{and } [h(x(T)),\ \dots,\ h(x(2dT)),\ h(x((2d+1)T))] = H_k(x(T))$$

so we can hope to use these induced dynamics to obtain properties of the time T map on M. Since H_k is a diffeomorphism (its inverse is differentiable) this reconstruction preserves the dimension of any invariant set and the Lyapunov exponents of the flow [7, 8].

The state of a dynamical system at any time can be specified by a state-space vector where the coordinates of the vector are the freedom independent degrees of the system [9].

The “embedding theorem” establishes that, when there is only a single measured quantity from a dynamical system, it is possible to reconstruct a state space that is equivalent to the original (but unknown) state space composed of all the dynamical variables [10, 11].

The embedding theorem states that if the system produces orbits in the original state space that lie on a geometric object of dimension d_g (which need not be integer), then the object can be unambiguously seen without any spurious intersections of the orbit in another space of integer dimension $d_g > 2d$, or larger, comprised of coordinates that are (almost) arbitrary nonlinear transformations of original state-space coordinates [12, 13].

Regarding the nature of the orbits, the Lyapunov exponents measure the rate at which nearby orbits converge or diverge. Not that this is something wrong, but there are as many Lyapunov exponents as there are dimensions in the state space of the system. Essentially, the largest is usually the most important.

3. Results and discussion

After all, what is practically a time series analysis? Without being a rhetorical question we can say that this is the application of dynamical systems techniques to a data series, usually obtained by "measuring" the value of a single observable as a function of time. The major tool in a dynamicist's toolkit is "delay coordinate embedding" which creates a phase space portrait from a single data series. It seems remarkable at first, but one can reconstruct a picture topologically equivalent to the full attractor in three-dimensional space (x_1, x_2, x_3), by measuring only one of its coordinates. In other words, it can be said that we start from x(t), and plot the delay coordinates ($x_1 = x(t)$, $x_2 = x(t+\tau)$, $x_3 = x(t+2\tau)$) for a fixed τ.

The idea of using delay coordinates (derivatives) in time series modeling is nothing new in numerical simulation. It goes back at least to the paper of Yule, who in 1927 used an autoregressive model to make a predictive model for the sunspot cycle [14].

Currency exchange rate fluctuations

Returning to our approach started in this article, we mention that in mondial economy history, were such collective modes of fluctuation, market players would probably know about them. As a self-irony, the economic theory says that if many people recognized these patterns, the actions they would take to exploit them would quickly nullify the patterns. Market participants would probably not need to know chaos theory for this to happen [15].

{{"2015Jan", 1.094}, {"2014Dec", 1.2026}, {"2014Nov", 1.2027}, {"2014Oct", 1.2078},
{"2014Sep", 1.2076}, {"2014Aug", 1.2118}, {"2014Jul", 1.215}, {"2014Jun", 1.2181},
{"2014May", 1.2204}, {"2014Apr", 1.2189}, {"2014Mar", 1.2177}, {"2014Feb", 1.2212},
{"2014Jan", 1.2317}, {"2013Dec", 1.2245}, {"2013Nov", 1.2316}, {"2013Oct", 1.2316},
{"2013Sep", 1.2338}, {"2013Aug", 1.2338}, {"2013Jul", 1.2366}, {"2013Jun", 1.2322},
{"2013May", 1.2418}, {"2013Apr", 1.2199}, {"2013Mar", 1.2266}, {"2013Feb", 1.2298},
{"2013Jan", 1.2288}, {"2012Dec", 1.2091}, {"2012Nov", 1.2052}, {"2012Oct", 1.2098},
{"2012Sep", 1.2089}, {"2012Aug", 1.2011}, {"2012Jul", 1.2011}, {"2012Jun", 1.2011},
{"2012May", 1.2012}, {"2012Apr", 1.2023}, {"2012Mar", 1.2061}, {"2012Feb", 1.2071},
{"2012Jan", 1.2108}, {"2011Dec", 1.2276}, {"2011Nov", 1.2307}, {"2011Oct", 1.2295},
{"2011Sep", 1.2005}, {"2011Aug", 1.1203}, {"2011Jul", 1.1766}, {"2011Jun", 1.2092},
{"2011May", 1.2537}, {"2011Apr", 1.2977}, {"2011Mar", 1.2867}, {"2011Feb", 1.2974},
{"2011Jan", 1.2779}, {"2010Dec", 1.2811}, {"2010Nov", 1.3442}, {"2010Oct", 1.3452},
{"2010Sep", 1.3089}, {"2010Aug", 1.3413}, {"2010Jul", 1.346}, {"2010Jun", 1.3767},
{"2010May", 1.4181}, {"2010Apr", 1.4337}, {"2010Mar", 1.4482}, {"2010Feb", 1.4671},
{"2010Jan", 1.4765}, {"2009Dec", 1.5021}, {"2009Nov", 1.5105}, {"2009Oct", 1.5138},
{"2009Sep", 1.5148}, {"2009Aug", 1.5236}, {"2009Jul", 1.5202}, {"2009Jun", 1.5148},
{"2009May", 1.5118}, {"2009Apr", 1.5147}}

Table1. Matrix of correct values for the EUR / CHF exchange rate

The matrix of correct values for the EUR / CHF international exchange rate is presented in Table 1. On the basis of these values, we will construct the associated time series.

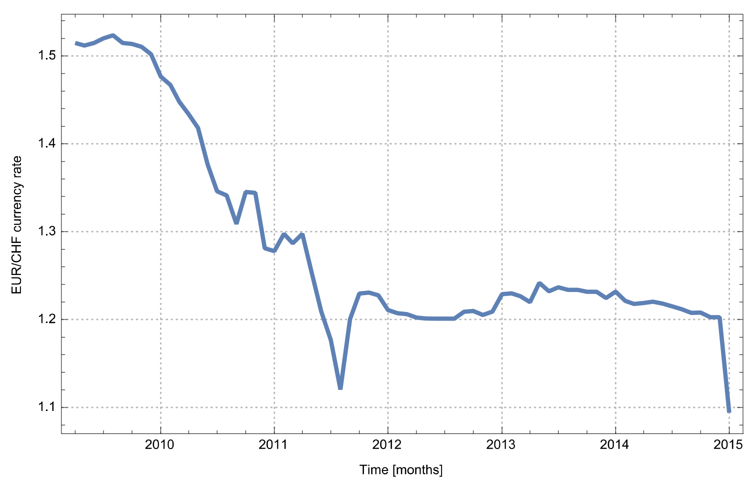

Figure 1. Time series of EUR/CHF currency rate, 2009-2015

In Fig. 1, the time series of EUR/CHF currency rate, for 2009-2015 years interval, is graphically represented. The same time series of EUR/CHF currency rate, according to the data in Table 1 and globally illustrated in Figure 1, is also presented in Fig. 2, for different samplings such as at 2 weeks, at 1 month, at quarter year, and at 1 year.

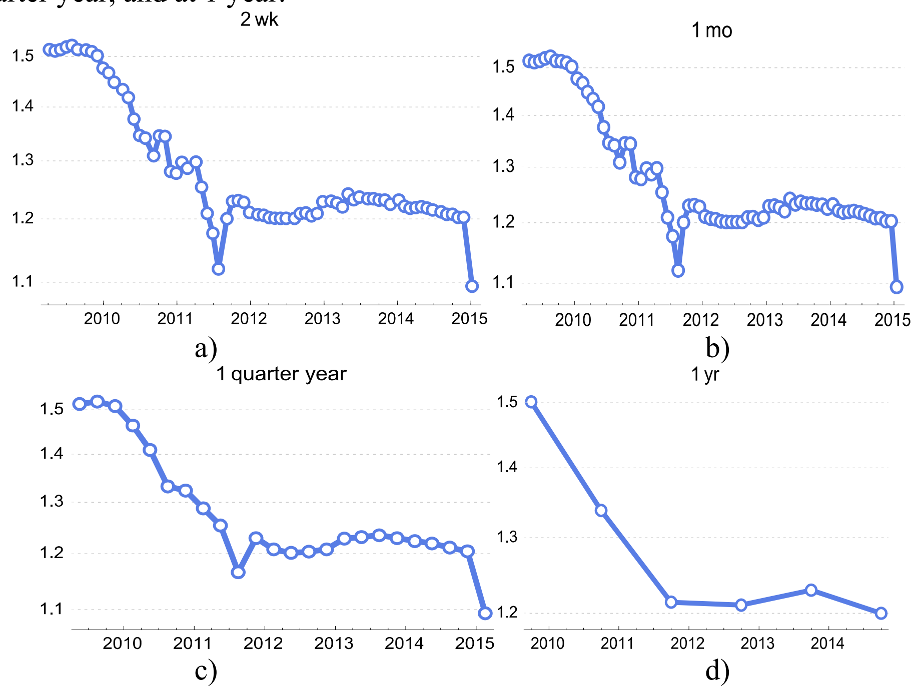

Figure 2. The same time series of EUR/CHF currency rate, for different sampling: a) at 2 weeks; b) at 1 month; c) at quarter year; d) at 1 year

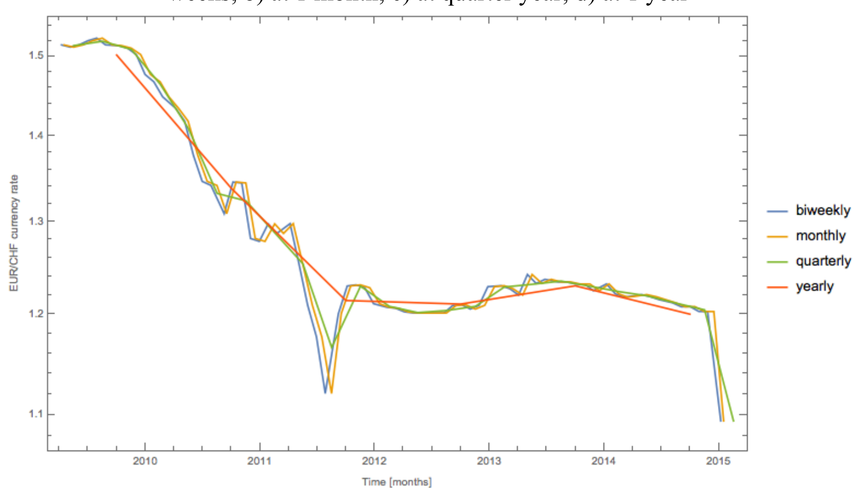

Figure 3. Time series of EUR/CHF currency rate for four different sampling

In Fig. 3 there are four overlapping time series, each for a different time period, (biweekly, monthly, quarterly and yearly), between years 2010 and 2015.

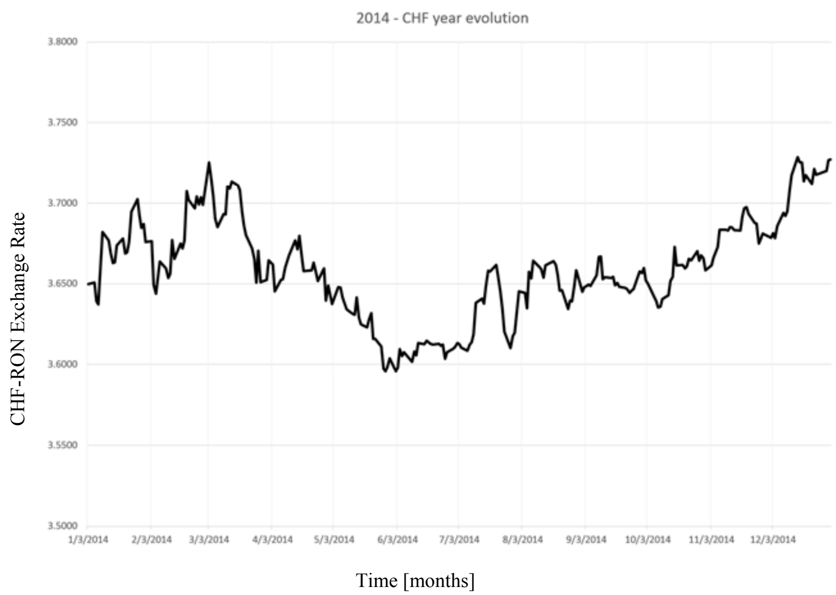

Figure 4. Time series of CHF-RON Exchange rate for year 2014

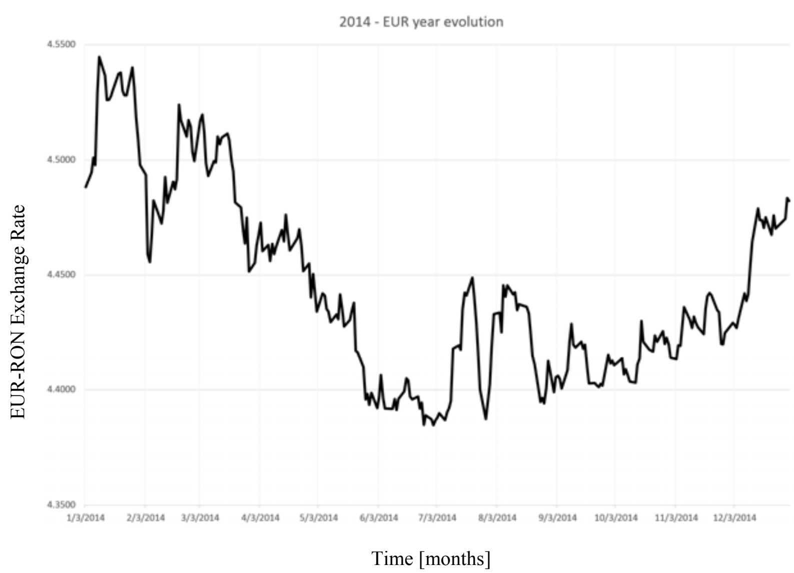

Figure 5. Time series of EUR-RON Exchange rate for year 2014

In Figs. 4 and 5, the evolutions for two major currencies, Euro and Swiss Franc respectively, relative to the Romanian RON, for the full year 2014, are presented. The Time series of CHF-RON and EUR-RON Exchange rate (for 12 months in 2014 year) will be used to obtain the relevant numerical indicators.

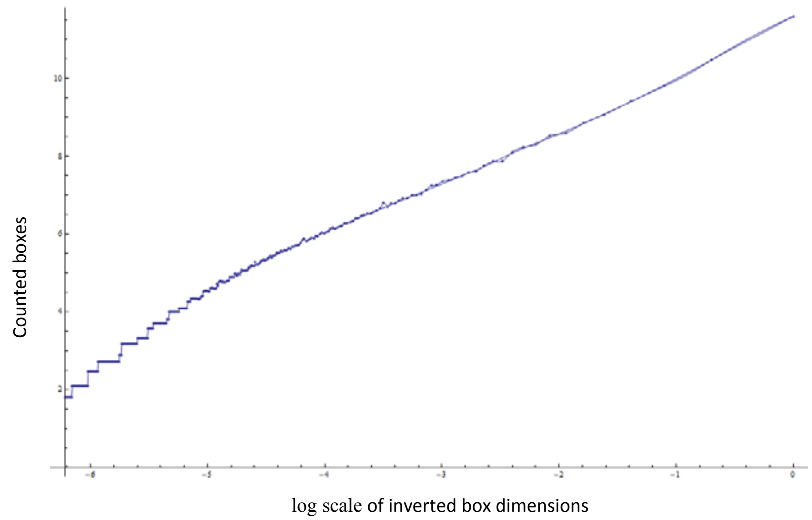

Figure 6. Counted boxes vs. log scale of inverted box dimensions, for CHF

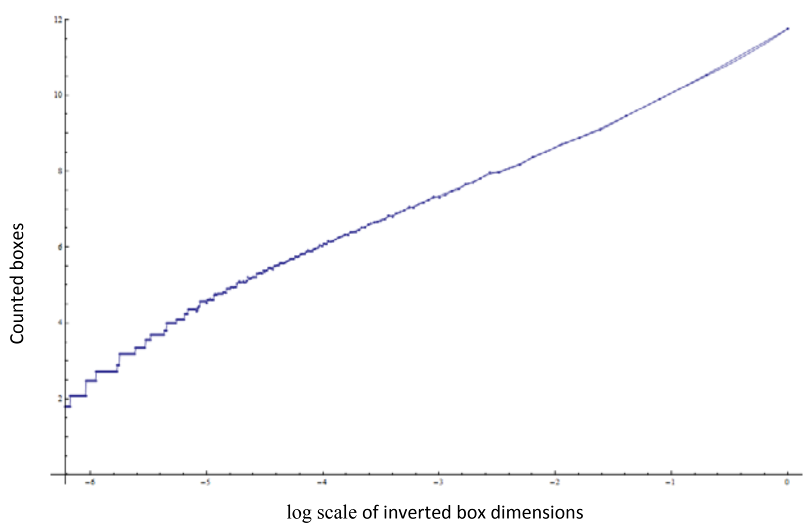

Figure 7. Counted boxes vs. log scale of inverted box dimensions, for EUR

In Figs. 6 and 7, the counted boxes vs. log scale of inverted box dimensions, for CHF and EUR, are plotted according to the algorithm used.

Both Swiss Franc (CHF) and Euro (EUR) currency evolutions for a fixed time period were analyzed. Financial Data was downloaded from the website NBR [http://bnr.ro/Exchange-Rates--3727.aspx] for the calendaristic range January 1st, 2013 - February 5th, 2015. Long-term developments (whole period) of daily value, for each currency nominated here (Fig. 8), have been graphically represented.

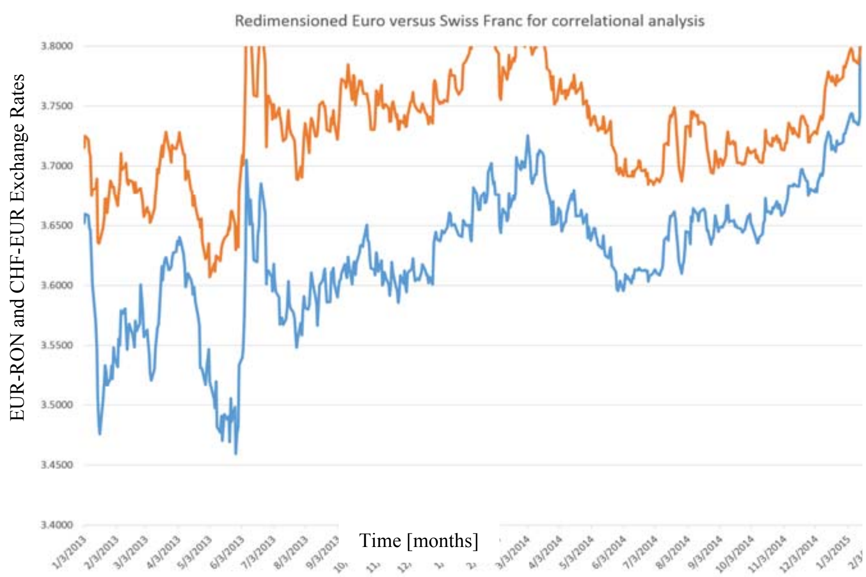

Figure 8. Time series of EUR and CHF currency rate

In Fig. 8 are simultaneously presented the both time series of EUR and CHF currency rate, but re-dimensioned, for a direct correlation analysis. To emphasize the parallelism in oscillations of the two currencies, Euro was scaled by subtracting the value of the currency difference in average daily rate for each currency over the period considered (approximately 0.7 RON). In this figure also we can see that the two currencies statistically fluctuate (quasi) in parallel manner.

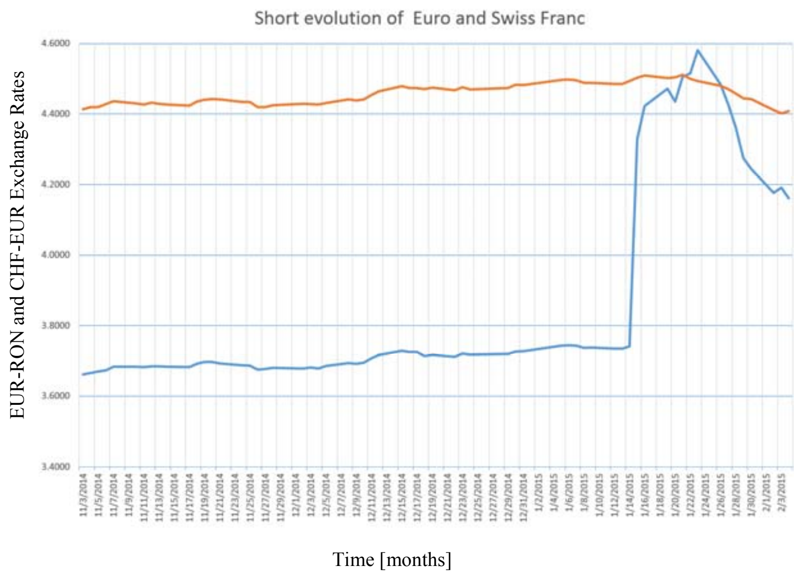

Figure 9. Time series of EUR and CHF currency rate (short evolution)

However in the last period (November 2014-February 2015) there has recently been a leap of Swiss Franc without any modification or previous visible oscillations (see Fig. 9, for short evolution and Fig. 10, for long evolution).

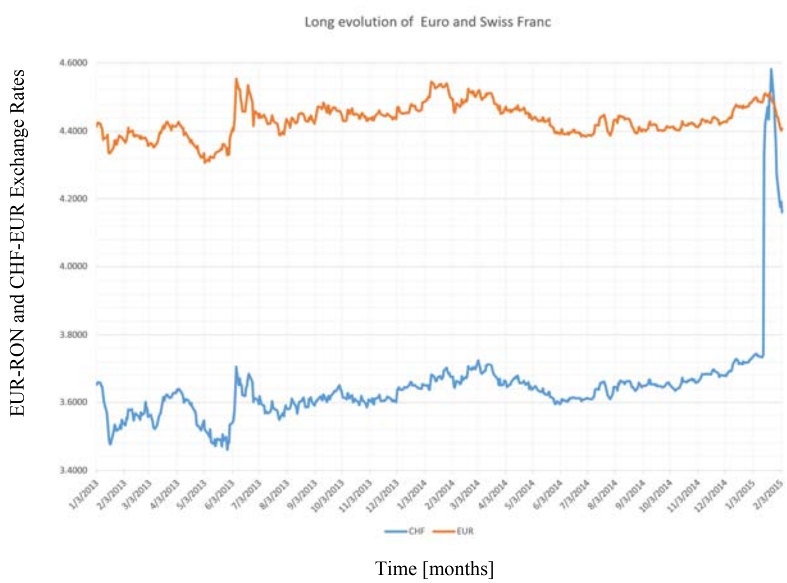

Figure 10. Time series of EUR and CHF currency rate (long evolution)

In order to analyze the preliminary variations of the two currencies chosen, in view to extract the information that can differentiate two developments, there have been used the fractal methods to reduce the complexity. Necessarily, this reduction was achieved by evaluating the fractal dimension.

The evaluation of fractal dimension

The fractal dimension evaluation has been performed through the use of the box-counting method, which allows for a rapid analysis of 2D images on two bits (white-black) [16, 17, 18]. A perfected version of this method has been employed to investigate the temporal evolution of a currency rate, which has oscillations predominantly on the vertical direction [19]. Please note that the same database was approached, several years ago for a similar topic, but on USD exchange rate fluctuation [20].

For this analysis, the currency exchange rates evolution for the year 2014 (please see the images Fig. 4-"2014-CHF evolution.bmp" and Fig. 5-"2014-EUR evolution.bmp") have been used. From these pictures, the 8-bit images (CHF8.bmp and EUR8.bmp) have been extracted for the intended analysis. After having imported these images in the developed dedicated software, they are subsequently transformed in 2-bit images. Further on, the global fractal dimensions are evaluated firstly for CHF (Fig. 6-"Counted boxes vs. log scale of inverted box dimensions CHF.bmp", Fig. 11-"Fractal dimension vs. box counting dimensions CHF.bmp"). In a similar manner, the EUR rate is analyzed (Fig. 7-"Counted boxes vs. log scale of inverted box dimensions EUR.bmp", Fig. 12-"Fractal dimension vs. box counting dimensions EUR.bmp").

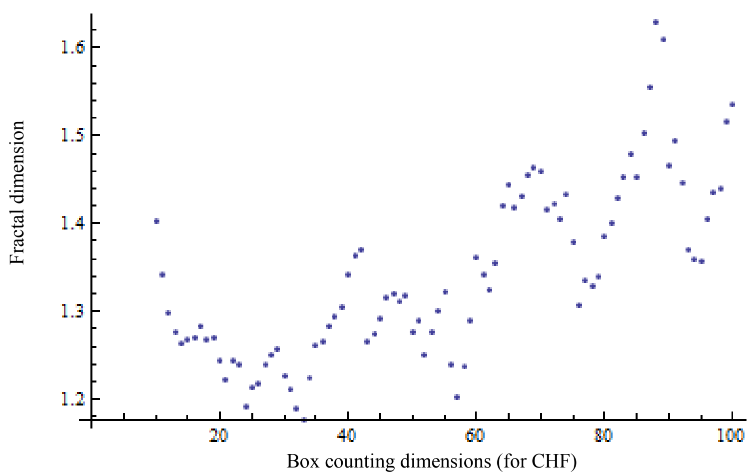

Figure 11. Fractal dimension vs. box counting dimensions, for CHF

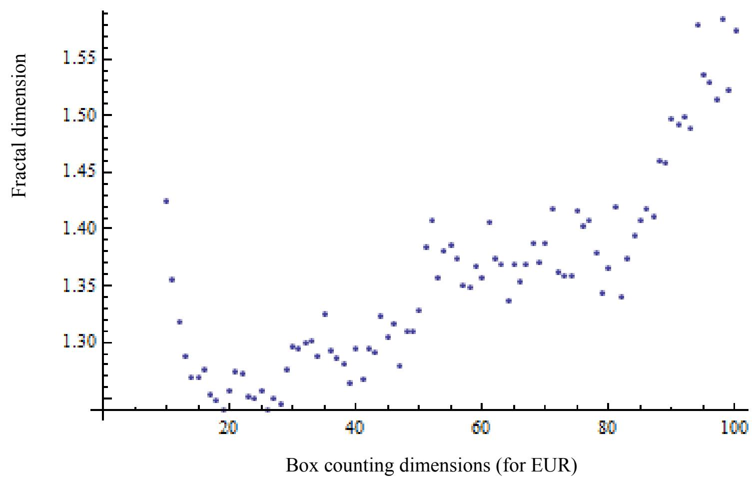

Figure 12. Fractal dimension vs. box counting dimensions, for EUR

Quantitatively, the differential fractal dimensions are represented in Figs. 13 and 14, after the two directions, Ox and Oy respectively, according to a new calibration method in fractal analysis [19]. Thus we have:

-with blue, the fractal dimension attached to the temporal evolution is represented (the abscissa on the temporal evolution graphic);

-with red, the fractal dimension attached to the oscillatory behavior of the exchange rate is represented (the ordinate on the temporal evolution graphic).

The final results for CHF and EUR are interpreted in the following figures:

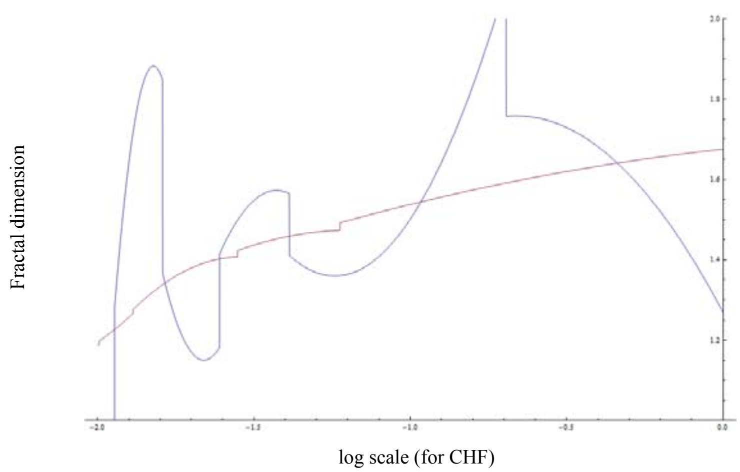

Figure 13. Fractal dimension vs. log scale, for CHF

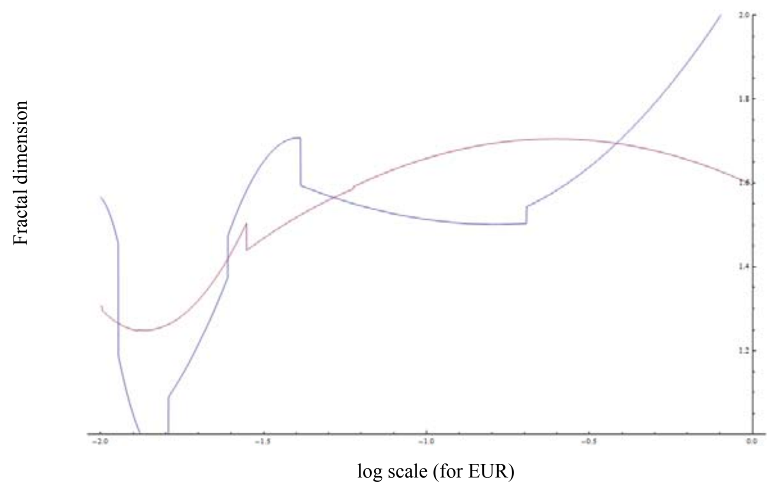

Figure 14. Fractal dimension vs. log scale, for EUR

Finally, some interesting observations can be made. The increasing slope (red color) for the Swiss Franc (Fig. 13) shows a continuous increase of fractal dimension with the covering cells (box) size decreasing, that hides a deeper unpredictable fluctuation of the franc against the euro where the fractal dimension, on this direction has a value that is much better determined by landing in (Figure 14). Although the fractal dimension, determined by this method, for the Euro $D_{EUR} \cong 1.7$ is slightly higher than for the Swiss Franc $D_{CHF} \cong 1.67 - 1.68$, obvious differences for further predictions are evident.

4. Conclusions

The time series method has been successfully applied for financial data analysis. The results often indicate that, at least, nonlinear structure is present. More specifically, the both Swiss Franc (CHF) and Euro (EUR) currency evolutions, for a fixed time period, were analyzed.

The fractal dimension evaluation has been performed through the use of the box-counting algorithm. The global fractal dimensions are evaluated firstly for CHF (‘Counted boxes vs. log scale of inverted box dimensions’ and ‘Fractal dimension vs. box counting dimensions’) and in a similar manner, the EUR exchange rate is also interpreted. Qualitatively speaking, the fractal dimension for the Euro is slightly higher than for the Swiss Franc, in accordance with Figs. 13 and 14. Quantitatively, the determined numerical values are $D_{EUR} \cong 1.7$ respectively $D_{CHF} \cong 1.67 - 1.68$.

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