DYNAMICS OF A 3-*PRR* PLANAR PARALLEL ROBOT

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Lucrarea prezentă stabileşte o modelare recurentă pentru cinematica şi dinamica unui robot paralel plan 3-PRR. Trei lanţuri cinematice plane ce conectează platforma mobilă sunt situate în plan vertical. Cunoscând mişcarea platformei, se dezvoltă mai întâi cinematica inversă şi se determină poziţiile, vitezele şi acceleraţiile robotului. În continuare, se utilizează principiul lucrului mecanic virtual în cadrul problemei de dinamică inversă. Unele ecuaţii matriceale oferă expresi iiterative şi grafice pentru o comparaţie a puterilor necesare celor trei sisteme de acţionare concepute în două scheme diferite: acţionori de translaţie şi acţionori de rotaţie.

Recursive modelling for the kinematics and dynamics of known 3-PRR planar parallel robot are established in this paper. Three identical planar legs connecting to the moving platform are located in a vertical plane. Knowing the motion of the platform, we develop first the inverse kinematics and determine the positions, velocities and accelerations of the robot. Further, the principle of virtual work is used in the inverse dynamics problem. Several matrix equations offer iterative expressions and graphs for the power requirement comparison of each of three actuators in two different actuation schemes: prismatic actuators and revolute actuators.

Keywords: dynamics, kinematics, planar parallel robot, virtual work

1. Introduction

Parallel manipulators are closed-loop mechanisms that consist of separate serial chains connecting the fixed base to the moving platform. Compared with serial manipulators, the followings are the potential advantages of parallel architectures: higher kinematical precision, lighter weight and better stiffness, greater load bearing, stabile capacity and suitable position of arrangement of actuators, but having limited workspace and complicated singularities.

Equipped with revolute or prismatic actuators, the parallel manipulators have a robust construction and can move bodies of large dimensions with high velocities and accelerations [1].

Over the past decades, parallel manipulators have received more and more attention from researches and industries. Accuracy and precision in the direction

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of the tasks are essential since the positioning errors of the tool could end in costly damage.

Considerable efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2]; Merlet [3]; Parenti-Castelli and Di Gregorio [4]). They are used in flight simulators and more recently for Parallel Kinematics Machines. The prototype of Delta parallel robot (Clavel [5]; Staicu and Carp-Ciocardia [6]; Tsai and Stamper [7]) developed by Clavel at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper at the University of Maryland as well as the Star parallel manipulator (Hervé and Sparacino [8]) are equipped with three motors, which train on the mobile platform in a three-degree-of-freedom general translation motion. Angeles, Gosselin, Gagné and Wang [9], [10], [11] analysed the kinematics, dynamics and singularity loci of Agile Wrist spherical robot with three actuators.
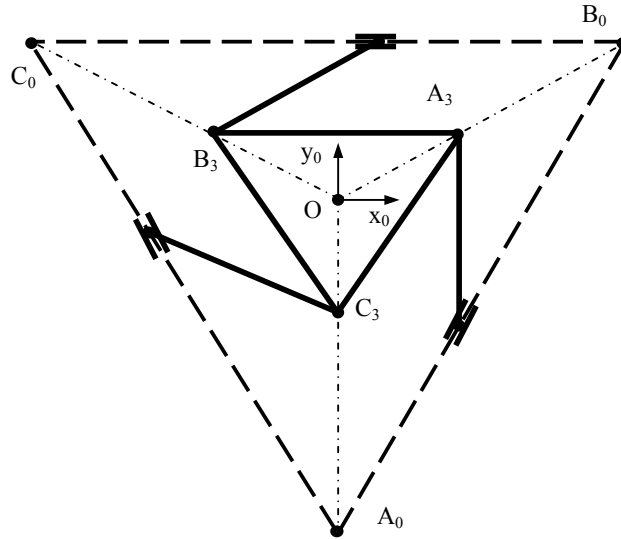

Fig. 1 The 3-*PRR* planar parallel robot

Planar parallel robots are useful for manipulating an object on a plane. A mechanism is said to be a *planar robot* if all the moving links in the mechanism perform the planar motions. For a planar mechanism, the loci of all points in all links can be drawn conveniently on a plane. In a planar linkage, the axes of all revolute joints must be normal to the plane of motion, while the direction of translation of a prismatic joint must be parallel to the plane of motion.

Aradyfio and Qiao [12] examined the inverse kinematics solution for the three different 3-DOF planar parallel robots. Gosselin and Angeles [13] and Pennock and Kassner [14] each present a kinematical study of a planar parallel robot, where a moving platform is connected to a fixed base by three links, each leg consisting of two binary links and three parallel revolute joints. Sefrioui and Gosselin [15] give an interesting numerical solution in the inverse and direct kinematics of this kind of planar robot.
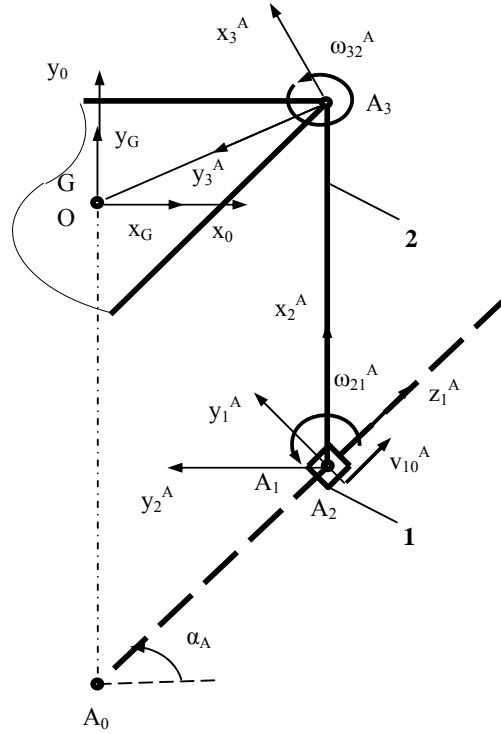

Fig. 2 Kinematical scheme of first leg *A* of the mechanism

Recently, more general approaches have been presented. Daniali et al. [16] present a study of velocity relationships and singular conditions for general planar parallel robots. Merlet [17] solved the forward pose kinematics problem for a broad class of planar parallel manipulators. Williams et al. [18] analysed the dynamics and the control of a planar three-degree-of-freedom parallel manipulator at Ohio University, while Yang et al. [19] concentrate on the singularity analysis of a class of 3-<u>*RRR*</u> planar parallel robots developed in its laboratory. Bonev, Zlatanov and Gosselin [20] describe several types of singular configurations by

studying the direct kinematics model of a 3-$\underline{R}PR$ planar parallel robot with actuated base joints.

A recursive method is introduced in the present paper, to reduce significantly the number of equations and computation operations by using a set of matrices for kinematics and dynamics models of the 3-PRR planar parallel robot.

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2. Kinematics analysis

Having a closed-loop structure, the planar parallel robot 3-PRR is a special symmetrical mechanism composed of three planar kinematical chains with identical topology, all connecting the fixed base to the moving platform (Fig. 1). The points A_0, B_0, C_0 define the summits of a fixed triangular base and the three moving revolute joints A_3, B_3, C_3 define the geometry of the moving platform. Each leg consists of two links, with one prismatic joint and two revolute joints. Together, the parallel mechanism consists of seven moving links, three prismatic joints and six revolute joints. Grübler mobility equation predicts that the device has certainly three degrees of freedom.

In a first kind of the robot $(\underline{P}RR)$ each prismatic joint is an actively controlled prismatic cylinder. Thus, all prismatic actuators can be installed on the fixed base. In the second configuration $(P\underline{R}R)$ we consider the moving platform as the output link and A_2A_3, B_2B_3, C_2C_3 as the input links of three mobile revolute actuators.

For the purpose of analysis, we attach a Cartesian frame $x_0y_0z_0(T_0)$ to the fixed base with its origin located at triangle centre O, the z_0 axis perpendicular to the base and the x_0 axis pointing along the direction C_0B_0. Another mobile reference frame $x_Gy_Gz_G$ is attached to the moving platform. The origin of this coordinate central system is located just at the centre G of the moving triangle (Fig. 2).

To simplify the graphical image of the kinematical scheme of the mechanism, in the follows we will represent the intermediate reference systems by only two axes, so as is proceed in most of robotics papers [1], [3], [9]. It is noted that the relative translation of T_k body with $\lambda_{k,k-1}$ displacement or the relative rotation with $\varphi_{k,k-1}$ angle must be always pointing about or along the direction of z_k axis.

In what follows we consider that the moving platform is initially located at a *central configuration*, where the platform is not rotated with respect to the fixed base and the mass centre G is at the origin O of fixed frame.

One of three active legs (for example leg A) consists of a prismatic joint, which is as well as a piston **1** of mass m_1 linked at the $x_1^A y_1^A z_1^A$ frame, having a rectilinear motion of displacement λ_{10}^A, velocity $v_{10}^A = \dot{\lambda}_{10}^A$ and acceleration $\gamma_{10}^A = \ddot{\lambda}_{10}^A$. Second

element of the leg is a rigid rod **2** linked at the $x_2^A y_2^A z_2^A$ frame, having a relative rotation about z_2^A axis with the angle φ_{21}^A, velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$ and acceleration $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$. It has the length l_2, mass m_2 and tensor of inertia $\hat{J}_2$. Finally, a revolute joint is introduced at a planar moving platform, which is schematised as an equilateral triangle with edge $l = r\sqrt{3}$, mass m_3 and inertia tensor $\hat{J}_3$ with respect to A_3, which rotates with the angle φ_{32}^A and the angular velocity $\omega_{32}^A = \dot{\varphi}_{32}^A$ about z_3^A.

At the central configuration, we also consider that all legs are symmetrically extended and that the angles of orientation of three edges of fixed platform are given by

$$\alpha_A = \frac{\pi}{3}, \alpha_B = \pi, \alpha_C = -\frac{\pi}{3}. \tag{1}$$

Pursuing the first leg A in the $OA_0A_1A_2A_3$ way, we obtain the following matrices of transformation [21]:

$$a_{10} = \theta_1 a_\alpha^A, \quad a_{21} = a_{21}^\varphi \theta_2 \theta_1^T, \quad a_{32} = a_{32}^\varphi \theta_2, \tag{2}$$

where

$$a_\alpha^A = \begin{bmatrix} \cos\alpha_A & \sin\alpha_A & 0 \\ -\sin\alpha_A & \cos\alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$a_{k,k-1}^\varphi = \begin{bmatrix} \cos\varphi_{k,k-1}^A & \sin\varphi_{k,k-1}^A & 0 \\ -\sin\varphi_{k,k-1}^A & \cos\varphi_{k,k-1}^A & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta_2 = \frac{1}{2}\begin{bmatrix} \sqrt{3} & 1 & 0 \\ -1 & \sqrt{3} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$a_{k0} = \prod_{j=1}^{k} a_{k-j+1,k-j}, \quad (k = 1, 2, 3). \tag{3}$$

Analogous relations can be written for other two legs of the mechanism.

Three displacements λ_{10}^A, λ_{10}^B, λ_{10}^C of the active links are the joint variables that give the input vector $\vec{\lambda}_{10} = [\lambda_{10}^A \quad \lambda_{10}^B \quad \lambda_{10}^C]^T$ of the instantaneous position of the mechanism in the first study configuration. But, in the inverse geometric problem, we can consider that the position of the mechanism is completely given by the coordinates x_0^G, y_0^G of the mass centre G of the moving platform and the orientation angle ϕ of the movable frame $x_G y_G z_G$. The orthogonal rotation matrix of the moving platform from $x_0 y_0 z_0$ to $x_G y_G z_G$ reference system is

$$R = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

Further, we suppose that the position vector of G centre $\vec{r}_0^G = [x_0^G \quad y_0^G \quad 0]^T$ and the orientation angle ϕ, which are expressed by following analytical functions

$$x_0^G = x_0^{G*}(1 - \cos\frac{\pi}{3}t), \quad y_0^G = y_0^{G*}(1 - \cos\frac{\pi}{3}t), \quad \phi = \phi^*(1 - \cos\frac{\pi}{3}t) \tag{5}$$

can describe the general absolute motion of the moving platform.

From the rotation conditions of the moving platform

$$a_{30}^{\circ T} a_{30} = b_{30}^{\circ T} b_{30} = c_{30}^{\circ T} c_{30} = R, \tag{6}$$

with, for example,

$$a_{30}^{\circ} = \theta_2 \theta_2 a_\alpha^A, \tag{7}$$

we obtain the following relations between angles

$$\varphi_{21}^A + \varphi_{32}^A = \varphi_{21}^B + \varphi_{32}^B = \varphi_{21}^C + \varphi_{32}^C = \phi. \tag{8}$$

The six variables $\lambda_{10}^A, \varphi_{21}^A, \lambda_{10}^B, \varphi_{21}^B, \lambda_{10}^C, \varphi_{21}^C$ will be determined by several vector-loop equations, as follows

$$\vec{r}_{10}^A + \sum_{k=1}^{2} a_{k0}^T \vec{r}_{k+1,k}^A + a_{30}^T \vec{r}_3^{GA} =$$

$$= \vec{r}_{10}^B + \sum_{k=1}^{2} b_{k0}^T \vec{r}_{k+1,k}^B + b_{30}^T \vec{r}_3^{GB} = \tag{9}$$

$$= \vec{r}_{10}^C + \sum_{k=1}^{2} c_{k0}^T \vec{r}_{k+1,k}^C + c_{30}^T \vec{r}_3^{GC} = \vec{r}_0^G,$$

where one denoted

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{r}_{10}^A = \vec{r}_{00}^A + (l_1 + \lambda_{10}^A) a_{10}^T \vec{u}_3$$

$$\vec{r}_{10}^B = \vec{r}_{00}^B + (l_1 + \lambda_{10}^B) b_{10}^T \vec{u}_3$$

$$\vec{r}_{10}^C = \vec{r}_{00}^C + (l_1 + \lambda_{10}^C) c_{10}^T \vec{u}_3$$

$$\vec{r}_{00}^A = l_0 [0 \quad -1 \quad 0]^T$$

$$\vec{r}_{00}^B = 0.5 l_0 [\sqrt{3} \quad 1 \quad 0]^T$$

$$\vec{r}_{00}^{\,C} = 0.5l_0 [-\sqrt{3} \quad 1 \quad 0]^T$$
$$\vec{r}_{21}^{\,i} = \vec{0}, \ \vec{r}_{32}^{\,i} = l_2 \vec{u}_3, \ \vec{r}_{3}^{\,Gi} = r\vec{u}_2 \quad (i = A, B, C). \tag{10}$$

Actually, these vector equations mean that there is only one inverse geometric solution for the manipulator:

$$(l_1 + \lambda_{10}^i) \sin \alpha_i + l_2 \sin(\varphi_{21}^i + \frac{\pi}{6} + \alpha_i) =$$

$$= y_0^G - y_{00}^i - r\cos(\phi + \frac{\pi}{3} + \alpha_i)$$

$$(l_1 + \lambda_{10}^i) \cos \alpha_i + l_2 \cos(\varphi_{21}^i + \frac{\pi}{6} + \alpha_i) =$$

$$= x_0^G - x_{00}^i + r\sin(\phi + \frac{\pi}{3} + \alpha_i)$$

$$(i = A, \ B, \ C). \tag{11}$$

We develop the inverse kinematics problem and determine the velocities and accelerations of the manipulator, supposing that the planar motion of the moving platform is known. First, we compute the linear and angular velocities of each leg in terms of the angular velocity $\vec{\omega}_0^G = \dot{\phi}\vec{u}_3$ and the centre's velocity $\vec{v}_0^G = \dot{\vec{r}}_0^G$ of the moving platform.

The motions of the component elements of each leg (for example the leg *A*) are characterized by the following skew symmetric matrices

$$\widetilde{\omega}_{k0}^A = a_{k,k-1}\widetilde{\omega}_{k-1,0}^A a_{k,k-1}^T + \omega_{k,k-1}^A \widetilde{u}_3, \ \ \omega_{k,k-1}^A = \dot{\varphi}_{k,k-1}^A, \tag{12}$$

which are *associated* to the absolute angular velocities given by the recursive relations

$$\vec{\omega}_{k0}^A = a_{k,k-1}\vec{\omega}_{k-1,0}^A + \omega_{k,k-1}^A \vec{u}_3, (k=1,2,3), \omega_{10}^A = 0. \tag{13}$$

Following relations give the velocities $\vec{v}_{k0}^A$ of the joints A_k

$$\vec{v}_{k0}^A = a_{k,k-1}\vec{v}_{k-1,0}^A + a_{k,k-1}\widetilde{\omega}_{k-1,0}^A \vec{r}_{k,k-1}^A + v_{k,k-1}^A \vec{u}_3$$

$$v_{\sigma,\sigma-1}^A = 0 \quad (\sigma = 2, 3). \tag{14}$$

Equations of geometrical constraints (8) and (9) can be derivate with respect to time to obtain the following *matrix conditions of connectivity* [22]

$$v_{10}^A \vec{u}_i^T a_{10}^T \vec{u}_3 + \omega_{21}^A \vec{u}_i^T \{a_{20}^T \widetilde{u}_3 \vec{r}_{32}^A + a_{20}^T \widetilde{u}_3 a_{32}^T \vec{r}_3^{GA}\} + \omega_{32}^A \vec{u}_i^T a_{30}^T \widetilde{u}_3 \vec{r}_3^{GA} = \vec{u}_i^T \dot{\vec{r}}_0^G, \ (i=1,2)$$

$$\omega_{21}^A + \omega_{32}^A = \dot{\phi}. \tag{15}$$

where $\widetilde{u}_3$ is a skew-symmetric matrix associated to unit vector $\vec{u}_3$ pointing in the positive direction of z_k axis. From these equations, we obtain the relative velocities $v_{10}^A, \omega_{21}^A, \omega_{32}^A$ as functions of angular velocity of the platform and velocity of mass

centre G. But, the conditions (15) give the *complete* Jacobian matrix of the manipulator. This matrix is a fundamental element for the analysis of the robot workspace and the particular configurations of singularities where the manipulator becomes uncontrollable.

Rearranging, above six constraint equations (11) of the planar robot can immediately written as follows

$$[x_0^G - x_{00}^i + r\sin(\phi + \frac{\pi}{3} + \alpha_i) - (l_1 + \lambda_{10}^i)\cos\alpha_i]^2 +$$

$$+ [y_0^G - y_{00}^i - r\cos(\phi + \frac{\pi}{3} + \alpha_i) - (l_1 + \lambda_{10}^i)\sin\alpha_i]^2 = \tag{16}$$

$$= l_2^2 \qquad (i = A, B, C),$$

where the "zero" position $x_0^{0G} = 0, \ y_0^{0G} = 0, \ \phi^0 = 0$ corresponds to the joints variables $\bar{\lambda}_{10}^0 = [0 \quad 0 \quad 0]^T$. The derivative with respect to time of conditions (16) leads to the matrix equation

$$J_{1p}\dot{\bar{\lambda}}_{10} = J_{2p}[\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{\phi}]^T \tag{17}$$

for the planar robot with *fixed prismatic actuators*.

Matrices J_{1p} and J_{2p} are, respectively, the inverse and forward Jacobian of the manipulator and can be expressed as

$$J_{1p} = diag\{\delta_{Ap} \quad \delta_{Bp} \quad \delta_{Cp}\}$$

$$J_{2p} = \begin{bmatrix} \beta_{1p}^A & \beta_{2p}^A & \beta_{3p}^A \\ \beta_{1p}^B & \beta_{2p}^B & \beta_{3p}^B \\ \beta_{1p}^C & \beta_{2p}^C & \beta_{3p}^C \end{bmatrix}, \tag{18}$$

with

$$\delta_{ip} = (x_0^G - x_{00}^i)\cos\alpha_i + (y_0^G - y_{00}^i)\sin\alpha_i +$$

$$+ r\sin(\phi + \frac{\pi}{3}) - l_1 - \lambda_{10}^i, \ (i = A, B, C)$$

$$\beta_{1p}^i = x_0^G - x_{00}^i + r\sin(\phi + \frac{\pi}{3} + \alpha_i) - (l_1 + \lambda_{10}^i)\cos\alpha_i$$

$$\beta_{2p}^i = y_0^G - y_{00}^i - r\cos(\phi + \frac{\pi}{3} + \alpha_i) - (l_1 + \lambda_{10}^i)\sin\alpha_i \tag{19}$$

$$\beta_{3p}^i = r[(x_0^G - x_{00}^i)\cos(\phi + \frac{\pi}{3} + \alpha_i) +$$

$$+ (y_0^G - y_{00}^i)\sin(\phi + \frac{\pi}{3} + \alpha_i) - (l_1 + \lambda_{10}^i)\cos(\phi + \frac{\pi}{3})].$$

The three kinds of singularities of the three closed-loop kinematical chains can be determined through the analysis of two Jacobian matrices J_{1p} and J_{2p} [23], [24].

Since λ_{10}^i is a passive variable in the second kind of the planar robot with *mobile revolute actuators*, it should be eliminated from equations (11) as follows

$$l_2 \sin(\varphi_{21}^i + \frac{\pi}{6}) = (x_{00}^i - x_0^G) \sin \alpha_i -$$
$$- (y_{00}^i - y_0^G) \cos \alpha_i - \tag{20}$$
$$- r \cos(\phi + \frac{\pi}{3}), \quad (i = A, B, C).$$

A new matrix relation is obtained by taking the derivative of equation (20) with respect to time

$$J_{1r} = diag \{\delta_{Ar} \quad \delta_{Br} \quad \delta_{Cr}\}$$

$$J_{2r} = \begin{bmatrix} \beta_{1r}^A & \beta_{2r}^A & \beta_{3r}^A \\ \beta_{1r}^B & \beta_{2r}^B & \beta_{3r}^B \\ \beta_{1r}^C & \beta_{2r}^C & \beta_{3r}^C \end{bmatrix}, \tag{21}$$

with

$$\delta_{ir} = l_2 \cos(\varphi_{21}^i + \frac{\pi}{6}), \quad (i = A, B, C)$$

$$\beta_{1p}^i = -\sin \alpha_i, \ \beta_{2r}^i = -\cos \alpha_i, \ \beta_{3r}^i = r \sin(\phi + \frac{\pi}{3}). \tag{22}$$

Now, let us assume that the robot has successively two *virtual motions* determined by the linear velocities $v_{10a}^{Av} = 1$, $v_{10a}^{Bv} = 0$, $v_{10a}^{Cv} = 0$ or the angular velocities $\omega_{21a}^{Av} = 1$, $\omega_{21a}^{Bv} = 0$, $\omega_{21a}^{Cv} = 0$. The characteristic virtual velocities are expressed as functions of the position of the mechanism by the general kinematical constraints equations (15).

As for the relative accelerations ε_{10}^A, γ_{21}^A, ε_{32}^A of the robot, the derivatives with respect to time of the equations (15) give other following conditions of connectivity [25]

$$\gamma_{10}^A \vec{u}_i^T a_{10}^T \vec{u}_3 + \varepsilon_{21}^A \vec{u}_i^T \{a_{20}^T \tilde{u}_3 \vec{r}_{32}^A + a_{20}^T \tilde{u}_3 a_{32}^T \vec{r}_3^{GA}\} + \varepsilon_{32}^A \vec{u}_i^T a_{30}^T \tilde{u}_3 \vec{r}_3^{GA} = \vec{u}_i^T \ddot{\vec{r}}_0^G -$$
$$- \omega_{21}^A \omega_{21}^A \vec{u}_i^T \{a_{20}^T \tilde{u}_3 \vec{r}_{32}^A + a_{20}^T \tilde{u}_3 \tilde{u}_3 a_{32}^T \vec{r}_3^{GA}\} - \omega_{32}^A \omega_{32}^A \vec{u}_i^T a_{30}^T \tilde{u}_3 \tilde{u}_3 \vec{r}_3^{GA} -$$
$$- 2\omega_{210}^A \omega_{32}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 a_{32}^T \tilde{u}_3 \vec{r}_3^{GA}, \quad (i = 1,2) \tag{23}$$

$$\varepsilon_{21}^A + \varepsilon_{32}^A = \ddot{\phi} \ .$$

If the other two kinematical chains of the robot are pursued, analogous relations can be easily obtained.

The following recursive relations give the angular accelerations $\vec{\varepsilon}_{k0}^{A}$ and the accelerations $\vec{\gamma}_{k0}^{A}$ of joints A_k

$$\vec{\varepsilon}_{k0}^{A} = a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{A} + \varepsilon_{k,k-1}^{A}\vec{u}_3 + \omega_{k,k-1}^{A}a_{k,k-1}\widetilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\vec{u}_3$$

$$\widetilde{\omega}_{k0}^{A}\widetilde{\omega}_{k0}^{A} + \widetilde{\varepsilon}_{k0}^{A} = a_{k,k-1}\left(\widetilde{\omega}_{k-1,0}^{A}\widetilde{\omega}_{k-1,0}^{A} + \widetilde{\varepsilon}_{k-1,0}^{A}\right)a_{k,k-1}^{T} +$$

$$\quad + \omega_{k,k-1}^{A}\omega_{k,k-1}^{A}\widetilde{u}_3\widetilde{u}_3 + \varepsilon_{k,k-1}^{A}\widetilde{u}_3 + 2\omega_{k,k-1}^{A}a_{k,k-1}\widetilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\widetilde{u}_3$$

$$\vec{\gamma}_{k0}^{A} = a_{k,k-1}\left[\vec{\gamma}_{k-1,0}^{A} + \left(\widetilde{\omega}_{k-1,0}^{A}\widetilde{\omega}_{k-1,0}^{A} + \widetilde{\varepsilon}_{k-1,0}^{A}\right)\vec{r}_{k,k-1}^{A}\right] +$$

$$\quad + 2v_{k,k-1}^{A}a_{k,k-1}\widetilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\vec{u}_3 + \gamma_{k,k-1}^{A}\vec{u}_3, \ (k=1,2,3)$$

$$\varepsilon_{10}^{A} = 0, \ \gamma_{\sigma,\sigma-1}^{A} = 0 \ (\sigma = 2,3) . \tag{24}$$

3. Dynamics simulation

In the context of the real-time control, neglecting the frictions forces and considering the gravitational effects, the relevant objective of the dynamics is to determine the input torques of forces, which must be exerted by the actuators in order to produce a given trajectory of the effectors.

There are three methods, which could provide the same results concerning these actuating torques or forces. The first one is using the Newton-Euler classic procedure [26], [27], [28], the second one applies the Lagrange's equations and multipliers formalism [29], [30] and the third one is based on the principle of virtual work [1], [9], [21], [31].

In the inverse dynamic problem, in the present paper one applies the principle of virtual work in order to establish some recursive matrix relations for the powers of the three active systems.

Three independent mechanical systems acting along the planar directions z_1^A, z_1^B, z_1^C, with the forces $\vec{f}_{10}^{A} = f_{10}^{A}\vec{u}_3$, $\vec{f}_{10}^{B} = f_{10}^{B}\vec{u}_3$, $\vec{f}_{10}^{C} = f_{10}^{C}\vec{u}_3$ or other three electric motors A_2, B_2, C_2 that generate the three couples of moments $\vec{m}_{10}^{A} = m_{10}^{A}\vec{u}_3$, $\vec{m}_{10}^{B} = m_{10}^{B}\vec{u}_3$, $\vec{m}_{10}^{C} = m_{10}^{C}\vec{u}_3$ oriented about parallel axes can control the motion of the moving platform.

The force of inertia of an arbitrary rigid body T_k^A, for example,

$$\vec{f}_{k0}^{inA} = -m_k^A\left[\vec{\gamma}_{k0}^{A} + \left(\widetilde{\omega}_{k0}^{A}\widetilde{\omega}_{k0}^{A} + \widetilde{\varepsilon}_{k0}^{A}\right)\vec{r}_k^{CA}\right] \tag{25}$$

and the resulting moment of the forces of inertia

$$\vec{m}_{k0}^{inA} = -[m_k^A\widetilde{r}_k^{CA}\vec{\gamma}_{k0}^{A} + \hat{J}_k^A\vec{\varepsilon}_{k0}^{A} + \widetilde{\omega}_{k0}^{A}\hat{J}_k^A\vec{\omega}_{k0}^{A}], \tag{26}$$

are determined with respect to the centre of joint A_k. On the other hand, the wrench of two vectors $\vec{f}_k^{*A}$ and $\vec{m}_k^{*A}$ evaluates the influence of the action of the

weight $m_k^A \vec{g}$ and of other external and internal forces applied to the same element T_k^A of the manipulator, for example:

$$\vec{f}_k^{*A} = 9.81 m_k^A a_{k0} \vec{u}_2 , \quad \vec{m}_k^{*A} = 9.81 m_k^A \tilde{r}_k^{CA} a_{k0} \vec{u}_2 \quad (k = 1,2,...,6) . \qquad (27)$$

Knowing the position and kinematics state of each link as well as the external forces acting on the robot, in that follow one apply the principle of virtual work for an inverse dynamic problem. The torques of actuators or the active forces required in a given motion of the moving platform will easily be computed using a recursive procedure.

The fundamental principle of the virtual work states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Assuming that frictional forces at the joints are negligible, the virtual work produced by all forces of constraint at the joints is zero. Applying *the fundamental equations of the parallel robots dynamics* established by Staicu [32], the following compact matrix relations results

$$f_{10}^A = \vec{u}_3^T \{ \vec{F}_1^A + \omega_{21a}^{Av} \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + \omega_{21a}^{Bv} \vec{M}_2^B + \omega_{21a}^{Cv} \vec{M}_2^C \} \qquad (28)$$

for the force of first active *fixed prismatic joint* and

$$m_{21}^A = \vec{u}_3^T \{ v_{10a}^{Av} \vec{F}_1^A + \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + v_{10a}^{Bv} \vec{F}_1^B + v_{10a}^{Cv} \vec{F}_1^C \} \qquad . \qquad (29)$$

for the torque of first active *mobile revolute joint*.

Two recursive relations generate the vectors

$$\begin{aligned} \vec{F}_k^A &= \vec{F}_{k0}^A + a_{k+1,k}^T \vec{F}_{k+1}^A \\ \vec{M}_k^A &= \vec{M}_{k0}^A + a_{k+1,k}^T \vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^T \vec{F}_{k+1}^A \end{aligned} \qquad (30)$$

where one denoted

$$\vec{F}_{k0}^A = -\vec{f}_{k0}^{inA} - \vec{f}_k^{*A} , \quad \vec{M}_{k0}^A = -\vec{m}_{k0}^{inA} - \vec{m}_k^{*A} \qquad (31)$$

The relations (28), (29), (30) and (31) represent the *inverse dynamics model* of the 3-*PRR* planar parallel manipulator.

As application let us consider a planar manipulator which has the following characteristics:

$$\begin{aligned} & x_0^{G*} = -0.025\,m , \quad y_0^{G*} = 0.025\,m , \quad \phi^* = \frac{\pi}{12} \\ & r = 0.1m, \quad l_1 = l = r\sqrt{3}, \quad l_0 = 0.3\,m \\ & l_2 = 0.2\,m, \quad \Delta t = 3\,s \\ & m_1 = 1\,kg, \quad m_2 = 1.5kg, \quad m_3 = 3\,kg \end{aligned} \qquad (32)$$

Using the MATLAB software, a computer program was developed to solve the inverse dynamics of the planar PRR parallel robot. To illustrate the algorithm, it is assumed that for a period of three second the platform starts at rest from a central configuration and rotates or moves along two orthogonal directions.
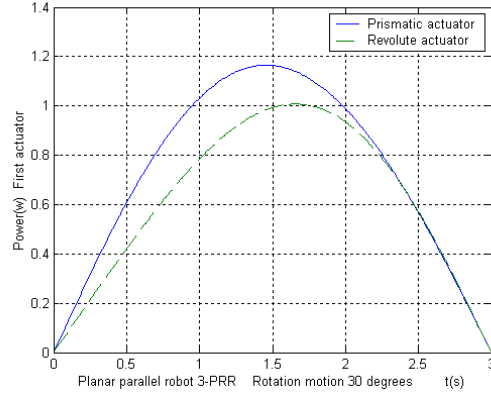

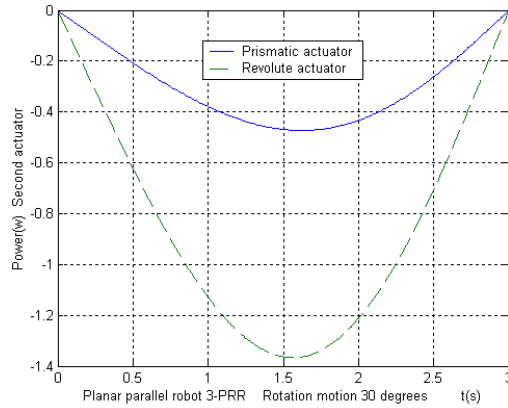Fig. 3 Powers p_{10}^{A}, p_{21}^{A} of first actuator

Fig. 4 Powers p_{10}^{B}, p_{21}^{B} of second actuator

Assuming that there is no external force and moment acting on the moving platform, a comparative study of the robots in two configurations: prismatic actuators ($\underline{P}RR$) and revolute actuators $(P\underline{R}R)$ is based on the computation of the power required by each actuator: p_{10}^{A}, p_{10}^{B}, p_{10}^{C} and p_{21}^{A}, p_{21}^{B}, p_{21}^{C} during the platform's evolution.

Following examples are solved to illustrate the simulation. For the first example we consider the *rotation motion* of the moving platform about z_0 axis with variable angular acceleration while all the other positional parameters are held equal to zero. As can be seen from p_{10}^A, p_{21}^A (Fig. 3), p_{10}^B, p_{21}^B (Fig. 4), p_{10}^C, p_{21}^C (Fig. 5) is proved to be true that for the third leg C only both actuating powers are permanently of opposite sign.
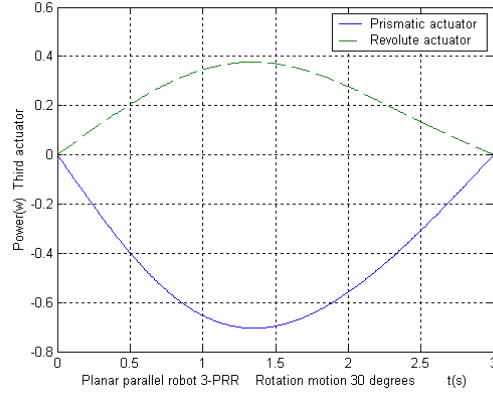

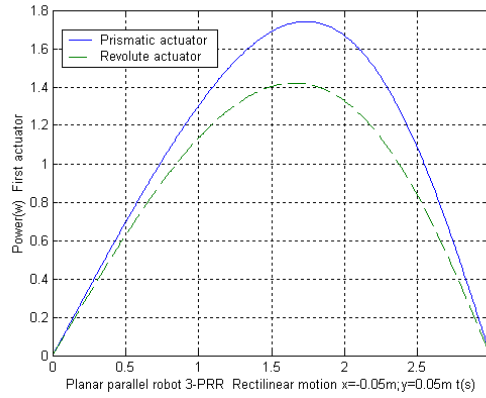Fig. 5 Powers p_{10}^C, p_{21}^C of third actuator

Fig. 6 Powers p_{10}^A, p_{21}^A of first actuator

If the platform's centre G moves along a *rectilinear planar trajectory* without rotation of platform, the powers required by the actuators A_1, B_1, C_1 are calculated by the program and plotted versus time as follows: Fig. 6, Fig. 7 and Fig. 8.

The simulation through the MATLAB program certify a permanent equality of the total power of the three actuators in the two configurations and that one of the major advantages of the current matrix recursive formulation is a reduced number
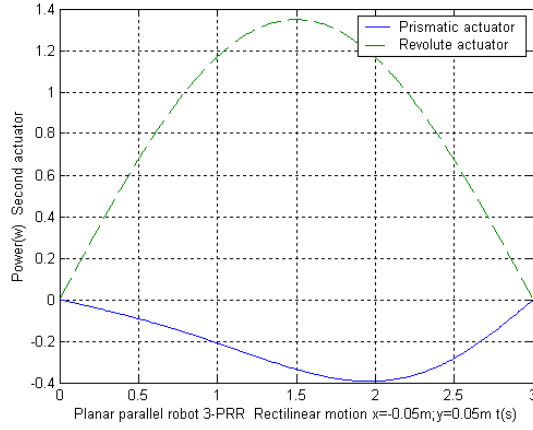

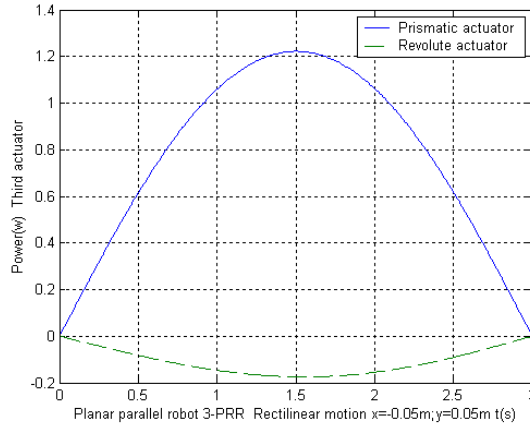Fig. 7 Powers p_{10}^{B}, p_{21}^{B} of second actuator

Fig. 8 Powers p_{10}^{C}, p_{21}^{C} of third actuator

of additions or multiplications and consequently a smaller processing time of numerical computation. Also, the proposed method can be applied to various types of complex robots, when the number of components of the mechanism is increased.

4. Conclusions

Most of dynamical models based on the Lagrange formalism neglect the weight of intermediate bodies and take into consideration only the active forces or moments and the wrench of applied forces on the moving platform. The number of relations given by this approach is equal to the total number of the position variables and Lagrange multipliers inclusive. The commonly known Newton-Euler method, which takes into account the free-body-diagrams of the mechanism, leads to a large number of equations with unknowns among which are also the connecting forces in the joints.

Within the inverse kinematics analysis some exact relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in present paper. The dynamics model takes into consideration the masses and forces of inertia introduced by all component elements of the parallel mechanism. The new approach based on the principle of virtual work can eliminate all forces of internal joints and establishes a direct determination of the time-history evolution of powers required by the actuators.

R E F E R E N C E S

[1]. T*sai, L-W.,* Robot analysis: the mechanics of serial and parallel manipulators, John Wiley & Sons, Inc., New York, 1999

[2]. *Stewart, D.,* A Platform with Six Degrees of Freedom, Proc. Inst. Mech. Eng., 1, 15, 1965

[3]. *Merlet, J-P.,* Parallel robots, Kluwer Academic Publishers, 2000

[4]. *Parenti-Castelli, V., Di Gregorio,* R., A new algorithm based on two extra-sensors for real-time computation of the actual configuration of generalized Stewart-Gough manipulator, Journal of Mechanical Design, 122, 2000

[5]. *Clavel, R.,* Delta: a fast robot with parallel geometry, Proceedings of 18th International Symposium on Industrial Robots, Lausanne, 1988

[6]. *Staicu, Şt., Carp-Ciocardia, D.C.,* Dynamic analysis of Clavel's Delta parallel robot, Proceedings of the IEEE International Conference on Robotics & Automation ICRA'03, Taipei, Taiwan, pp. 4116-4121, 2003

[7]. *Tsai, L-W., Stamper, R.,* A parallel manipulator with only translational degrees of freedom, ASME Design Engineering Technical Conferences, Irvine, CA, 1996

[8]. *Hervé, J-M., Sparacino, F.,* Star. A New Concept in Robotics, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, 1992

[9]. *Angeles, J.,* Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms, Springer-Verlag, New York, 2002

[10]. *Gosselin, C., Gagné, M.,* Dynamic models for spherical parallel manipulators, Proceedings of the IEEE International Conference on Robotics & Automation, Milan, Italy, 1995.

[11]. *Wang, J., Gosselin, C.,* A new approach for the dynamic analysis of parallel manipulators, Multibody System Dynamics, 2, 3, 317-334, 1998

[12]. *Aradyfio, D.D., Qiao, D.,* Kinematic Simulation of Novel Robotic Mechanisms Having Closed Chains", ASME Mechanisms Conference, Paper 85-DET-81, 1985

[13]. *Gosselin, C., Angeles, J.,* The optimum kinematic design of a planar three-degree-of-freedom parallel manipulator, ASME journal of Mechanisms, Trans .and Automation in Design., 110, 1, pp. 35-41, 1988

[14]. *Pennock, G.R., Kassner, D.J.*, Kinematic Analysis of a Planar Eight-Bar Linkage: Application to a Platform-type Robot, *ASME Mechanisms Conference*, Paper DE - 25, pp. 37-43, 1990

[15]. *Sefrioui, J., Gosselin, C.*, On the quadratic nature of the singularity curves of planar thre-degree-of-freedom parallel manipulators, Mechanism and Machine Theory, 30, 4, pp. 533-551, 1995

[16. *Mohammadi-Daniali, H., Zsombor-Murray, P., Angeles, J.*, Singularity Analysis of Planar Parallel Manipulators, Mechanism and Machine Theory, 30, 5, pp. 665-678, 1995

[17. *Merlet, J-P.,* Direct kinematics of planar parallel manipulators, Proc. of the IEEE International Conference on Robotics & Automation, Minneapolis, Minnesota, pp. 3744-3749, 1996

[18]. *Williams II,R.L., Reinholtz, C.F.,* Closed-Form Workspace Determination and Optimization for Parallel Mechanisms, The 20th Biennal ASME Mechanisms Conference, Kissimmee, Florida, DE, Vol. 5-3, pp. 341-351, 1988

[19]. *Yang, G., Chen, W., Chen, , I-M.*, A Geometrical Method for the Singularity Analysis of 3-RRR Planar Parallel Robots with Different Actuation Schemes, Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, Lausanne, Switzerland pp. 2055-2060, 2002

[20]. *Bonev, I., Zlatanov, D., Gosselin, C.*, Singularity analysis of 3-DOF planar parallel mechanisms via screw theory, Journal of Mechanical, 25, 3, pp. 573-581, 2003

[21. *Staicu, S., Liu, X-J., Wang, J.,* Inverse dynamics of the HALF parallel manipulator with revolute actuators, Nonlinear Dynamics, Springer Press, 50, 1-2, pp. 1-12, 2007

[22]. *Staicu, S., Zhang, D.*, A novel dynamic modelling approach for parallel mechanisms analysis, Robotics and Computer-Integrated Manufacturing, Pergamon-Elsevier, 24, 1, pp. 167-172, 2008

[23]. *Pashkevich, A., Chablat, D., Wenger, P.,* Kinematics and workspace analysys of a three-axis parallel manipulator: the Orthoglide, *Robotica*, Cambridge University Press, 24, 1, 2006

[24]. *Liu, X-J., Tang, X., Wang, J.*, A Kind of Three Translational-DOF Parallel Cube-Manipulator" Proceedings of the 11th World Congress in Mechanism and Machine Science, Tianjin, China, 2004

[25]. *Staicu, S.,* Planetary Gear Train for Robotics, Proceedings of the IEEE International Conference on Mechatronics ICM'2005, Taipei, Taiwan, 2005

[26]. *Li, Y-W., Wang, J., Wang, L-P., Liu, X-J.*, Inverse dynamics and simulation of a 3-DOF spatial parallel manipulator, Proceedings of the IEEE International Conference on Robotics & Automation ICRA'2003, Taipei, Taiwan, pp. 4092-4097, 2003

[27]. *Dasgupta, B., Mruthyunjaya, T.S.*, A Newton-Euler formulation for the inverse dynamics of the Stewart platform manipulator, Mechanism and Machine Theory, 34, 1998

[28]. *Guegan, S., Khalil, W., Chablat, D., Wenger, P.*, Modélisation dynamique d'un robot parallèle à 3-DDL: l'Orthoglide, Conférence Internationale Francophone d'Automatique, Nantes, France, 8-10 Juillet 2002

[29]. *Geng, Z., Haynes, L.S., Lee, J.D., Caroll, R.L.*, On the dynamic model and kinematic analysis of a class of Stewart platforms, Robotics and Autonomous Systems, 9, 1992

[30]. *Miller, K., Clavel, R.,* The Lagrange-Based Model of Delta-4 Robot Dynamics, Robotersysteme, 8, pp. 49-54, 1992

[31]. *Zhang, C-D. Song, S-M.*, An efficient nethod for inverse dynamics of manipulators based on the virtual work principle, Journal of Robotic Systems, 10, 5, 1993

[32]. *Staicu, S.*, Relations matricielles de récurrence en dynamique des mécanismes, Revue Roumaine des Sciences Techniques - Série de Mécanique Appliquée, 50, 1-3, pp. 15-28, 2005