ALGORITHM TO COMPUTE THE DOUBLE-FLOW IN A POLY-DISK MATRIX HEAT EXCHANGER

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This paper examines the mathematical model of a compact double-flow poly-disk heat exchanger in order to determine the heat capacity, temperature profiles and possible optimization of developed design. The heat exchanger has a unique patented design, which ensures its remarkable compactness for specific operating conditions in a cascade of cryogenic distillation columns of hydrogen isotopes when the heat transfer coefficients of the flows might vary significantly from one another. Such situations can be observed when the gas flows participating in the heat exchange have small mass flow rates, differing from each other by several times.

Keywords: Matrix Heat Exchanger, mathematical model, hydrogen, heat transfer, temperature profile

1. Introduction

Matrix Heat Exchangers (MHE), according to the literature [1],[2] and lately reports from experimental test and computational simulation [3],[4] have excellent heat exchange properties, ensuring small temperature differences between the inlet and outlet flows with small pressure drops. Such a heat exchanger (MHE) consists in a large number of perforated plates, made of a metal material with high thermal conductivity, parallel arranged in the form of a circular matrix, interspersed with spacers of low thermal conductivity material. Two gas flows pass axially through the heat exchanger, in separate enclosures, in counter-current. The heat is transferred first by thermal convection, from the gas to the metal material, by thermal conduction, in the transverse direction inside the perforated plate, and then also by thermal convection, from the metal material to the other gas flow [3].

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Differences between the mass flow rates of the flows by a factor of 1.5 to 2.5 can lead to corresponding differences in the heat transfer coefficients between the flow and a heat exchanger's channel wall along which it flows. The smaller of the heat transfer coefficient values has a decisive effect on the thermal resistance and, accordingly, on the heat exchange surface area. In such a situation, increasing the heat exchange surface by finning on the side of the flow with a lower heat transfer coefficient is a classic solution.

In heat exchangers widely used in cryogenic systems, such as Linde or Giauque-Hampson [5], finning one of the heat exchange surfaces is a rather difficult and sometimes impossible task. Where it is possible to increase the heat exchange surface by finning, as, say, in the Collins heat exchanger (Joy Manufacturing Company) [5], it is difficult to achieve heat exchanger compactness. The MHE-type heat exchanger is intended for use in cryogenic distillation systems for hydrogen isotopes, where, in the case of tritium, additional considerations must be taken into account beyond the requirements for a small, compact, and highly efficient heat exchanger. These include the need to implement special industrial safety and protection measures, such as accounting for the potential permeation of tritium. As a result, the construction materials must be carefully selected, processed, and assembled.

We have proposed an original design for a heat exchanger that does not use fins on one of the two heat exchange surfaces. Fluid flows go through channels made in a material with a high thermal conductivity coefficient.

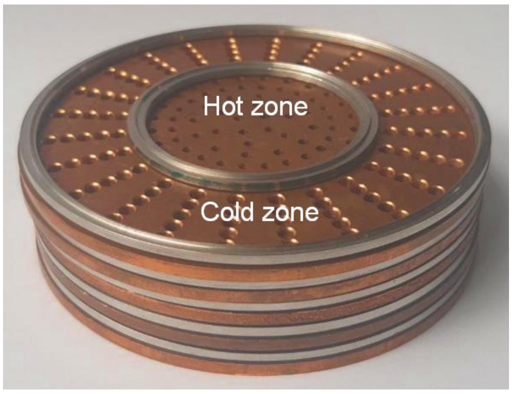


Fig. 1 The poly-disk heat exchanger

Fig. 1 shows a photograph of the poly-disk heat exchanger structural element - a perforated copper disk with such through channels. It is technically possible to separate the flows in the heat exchanger so that the flow with a lower heat transfer coefficient goes through a larger number of channels in the disk. In this way, it is possible to control the heat exchange surface for each flow by varying the number and sizes of channels available for the flow. Under steady-state conditions, all the heat from the hot flow is transferred to the cold flow through the body of the disk with a high thermal conductivity coefficient with minimal losses.

To reduce the axial heat flow, the height of a single disk should be limited, and to ensure the required heat exchange area, we use several sequentially located disks. This design ensures both the efficiency of heat exchange and the compactness of the heat exchanger itself.

2. The heat exchanger geometry

The MHE under examination is made to exchange heat between two gas flows in a variety of cryogenic systems [6] in an effective manner. The following characteristics set this MHE apart:

- minimal mass flow rates, between 0.001 and 0.005 kg/sec, which lead to low "flow-surface" heat transfer coefficient values;
- hydrogen is a gas that flows at hot and cold sides;
- the temperature of both hot and cold flows varies throughout the heat exchange process across a large range (the hot flow inlet temperature is around 300 K, while at the cold side is around 20 K);
- the mean pressure values of the flows are within around 0.1 MPa.

To guarantee the well operating conditions for the heat exchanger under these circumstances, we created a novel “poly-disk” design and filed for patent on it (see Fig. 1). The disk is made of copper, and its dimensions are as follows: height (thickness) h_D= 0.003 m, diameter $D_D =$ 0.106 m. All disks have perforated through channels of circular cross-section, according to one template. We arrange the disk channels in strict order, forming concentric rows - each plate has five rows for hot zone and another five rows for cold zone with different number of perforations. Sealing ring gaskets have dual function. First, they provide a constant gap between adjacent disks equal to the thickness (height) of the gaskets. Such a design eliminates any possibility of thermal contact between adjacent disks, reducing the magnitude of parasitic axial heat flows from the MHE hot zone to the cold one. The latter is crucial in the case of cryogenic heat exchangers, when the temperature difference along the height (length) of a heat exchanger can reach several hundred degrees. Second, the sealing ring gaskets create two internal volumes (inter-disk volumes) between adjacent disk surfaces, isolating them from each other and the environment, as shown in fig. 1. These volumes serve as collectors for gas flows that pass through the disk channels. These collectors spatially separate the groups of channels in the disks. This creates two distinct gas flows that are unable to mix as they pass through the heat exchanger but interact strongly thermally through the disk body.

3. A mathematical description of heat transfer processes in a separate perforated disk of the MHE

The model should be capable of performing a thermal calculation to ascertain a separate disk temperature field distribution, estimate the heat flow from the “hot” zone to the “cold” zone, and compute the temperatures of the hot and cold gas flows in the outlet inter-disk collectors.

For a mathematical description of heat transfer processes in one separate disk of the MHE, we will approach several *simplifying assumptions* regarding the nature of the processes occurring in it, such as: stationary mode of operation; the intensive mixing of the gaseous H_2 flow in spacer rings which ensures that the temperature of the hot and cold flows in this area is equal to the temperature at the inlets to the channels of the corresponding MHE zones, invariable regardless of the location of the channel (from the adjacent hot or cold zone) (fig. 2); the double-flows are evenly distributed across the channels; uniformity of the disk temperature field in the axial direction; the MHE adiabatically isolated to the environment.

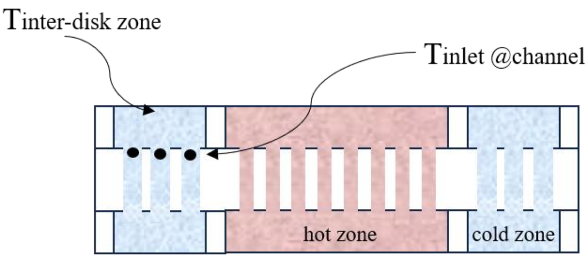


Fig. 2 Intensive gas mixing at inter-disk zone

A radial temperature gradient develops in the disk as a result of heat exchange with gaseous H_2 flows in the channels. The center of the disk is where the temperature reaches its greatest $T_D^{max}(r = 0)$, and its periphery is where it drops to its lowest $T_D^{min}(r = R_D)$. According to Fourier law [7], a heat flow of linear density q_l, equation (1), passes through the disk isothermal surface of radius r:

$$q_l = \lambda(T) \frac{dT}{dr} \cdot 2\pi r \tag{1}$$

The magnitude of this heat flow q_l depends on the rate $q_V(r)$, equation (2), of heat generation per unit volume of the disk with its radius:

$$q_l(r) = q_V(r) \cdot \pi r^2 \tag{2}$$

The differential equation (3) for the stationary process of heat conduction of a homogeneous and isotropic disk in a cylindrical coordinate system [7] with temperature changing only with radius together with boundary conditions of the second order will have the following form:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q_V(r)}{\lambda(T)} = 0 \;,\; \frac{dT}{dr}(r = 0) = 0 \;,\; \frac{dT}{dr}(r = R_D) = 0 \qquad (3)$$

Obviously, $q_V(r)$ depends on many parameters, including: temperatures of the hot and cold H_2 flows at the inlet to the channels of a given disk; mass flow rates; temperature distribution along the disk radius itself $T(r)$, etc. By writing a system with 10 equations based on equation (3) (one equation for each plate), and calculating by the iterative method and approximations of the derivatives by Taylor Theorem [8], we obtain the temperature profile along the length of the heat exchanger and the thermal power of each plate and of the entire MHE.

4. Heat balances for one separate channel

It is obvious that a mathematical model with a high degree of accuracy for describing heat transfer processes in the poly-disk heat exchanger (assembled from a number of disks connected in series) is an extremely difficult task. Above, we have made several simplifying assumptions about the geometry of the MHE disks and the nature of the flow in the inter-disk space. A holistic, internally consistent mathematical model based on these assumptions should "predict" the temperatures of the gaseous H_2 flows at the outlet of the heat exchanger T_H^{outlet}, T_C^{outlet} for given input temperatures T_H^{inlet}, T_C^{inlet}, the number of disks N_D of the MHE, the geometry of the disks, the features of the disk perforation, and their mutual arrangement (the dimensions of the inter-disk space).

For a distinct channel within a l-th row of *hot and cold flow* channels with the radius $r[l]$, the system of equations is as follows:

$$Q_H = \alpha\left(\dot{m}_H, d_H, \frac{T_H^{in} + T_H^{out}}{2}\right)\pi d_H h\left(\frac{T_H^{in} + T_H^{out}}{2} - T_d(r[l])\right) \qquad (4)$$

$$Q_C = \alpha\left(\dot{m}_C, d_C, \frac{T_C^{in} + T_C^{out}}{2}\right)\pi d_C h\left(T_d(r[l]) - \frac{T_C^{in} + T_C^{out}}{2}\right)$$

where:

α is heat transfer coefficient «H_2 flow – channel’s wall», W/m^{2}K;

$\dot{m}_H$, $\dot{m}_C$ are the hot, cold H_2 mass flow rates in a separate channel, kg/s;

d_H, d_C are the internal diameters of the hot, cold channels, m;

h is the height of the disk (= the height of the through channel), m;

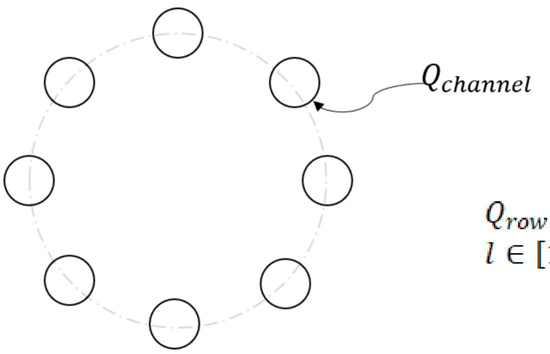
C_p is the specific heat capacity of the gaseous H_2, J/kg•K;

$T_d(r[l])$ is the disk temperature calculated for the radius $r[l]$ of the channels l-th row under consideration.

Considering the T^{out} approximation as a known value and applying the iterative calculation method until a calculation accuracy of 0.1 K is obtained, we calculate the amount of heat transfer between the flow and the channel wall Q_H (Q_C) using equation (4).

5. The volumetric heat source distribution throughout the MHE disk volume

In Section 4, we have considered the procedure for calculating the amount of heat transferred in the process of heat exchange "flow H_2-channel wall" for a *separate channel* of any row of channels of "hot" or "cold" disk zones. Here we will describe the procedure for the transition from the thermal power of a separate channel of a row to the distribution of volumetric power of heat sources and sinks $q_V(r[i])$, W/m^3, along the radius of the disk. The total amount of heat $Q_{H\_row}[l]$ or cold $Q_{C_{row}}[l]$ generated by all channels of one row (fig. 3) is equal to the product of the thermal power of one individual channel of this row and the number of channels in the row $n_{row}[l]$:



$$Q_{row}[l] = Q_{channel}\big[r[l]\big] \cdot n_{row}[l], \quad (5)$$
$$l \in [1..N_{row}]$$

Fig. 3 One channel thermal power production according to a row thermal power

The heat flow with power $Q_{N_{row}}$ is distributed over the allocated volumes of the disk adjacent to the outermost row of channels as follows:

$$Q_{N_{row}} = \frac{1}{4}V_{N-1} \cdot q_{V_{N-1}} + \left(\frac{3}{4}V_{N-1} + V_N + \frac{1}{2}\left((1+\varepsilon_1)V_{N+1} + \varepsilon_1 V_{N+2}\right)\right) q_{V\_N} \tag{6}$$

We applied the matrix method for solving systems of linear equations and obtained the values of the volumetric heat sources and heat sinks q_{V_i} in a number equal to the number of rows in the disk.

6. Results and conclusions of the double-flow poly-disk thermal calculations

We have implemented the mathematical model of thermal processes in the poly-disk MHE previously presented as a computer numerical model using the Python 3.12 programming language and input data from Table 1.

Table 1

Input data for the polydisk MHE thermal calculations

MHE's flows		
flow's parameter	**Hot**	**Cold**
$\dot{m}$, [g/sec]	1.77	4.57
T_{in}, [K]	296.0	26.0
p_{in}, [MPa]	0.1	0.1
MHE disk		
N_{disk}	10	

The results of the MHE's numerical simulation are presented both in fig. 4 where shows the screenshot of the graphic representation of the calculated temperature and thermal parameters of the poly-disk MHE, which is composed of *10* disks.

It is important to mention that the mathematical model does not include the presence of the sealing caps (included in the real model) or the chamfers of the flow channels.

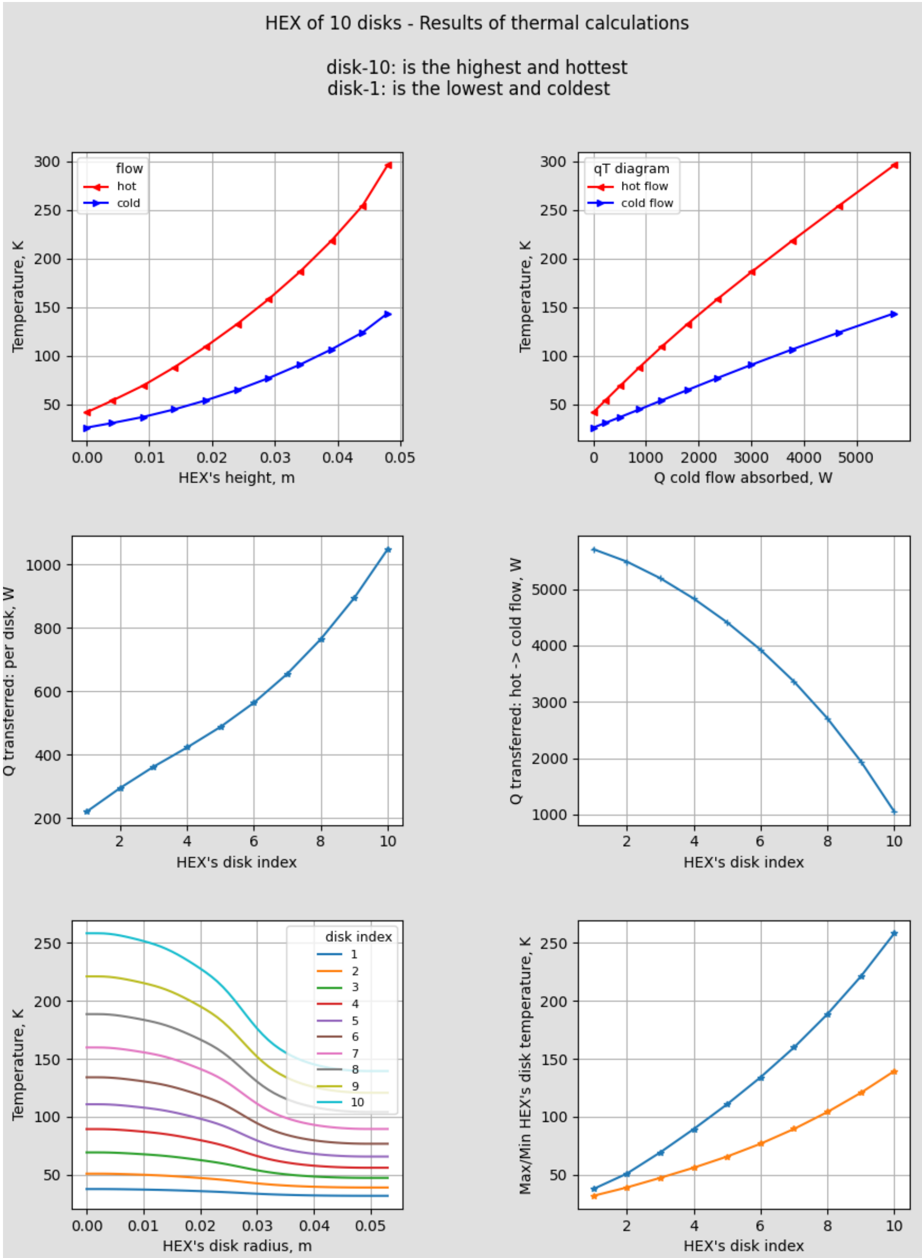


Fig. 4 Screenshot of the graphical interpretation of the results of numerical simulation of the poly-disk MHE of 10 disks

For this case, the difference between the mass rates of the flows is $\dot{m}_c/\dot{m}_H$ = 2.6. The flows temperature at the outlet ports of the heat exchanger is T_H^{out}=144.75 K and T_c^{out}=38.41 K. This means that the MHE with $N_{disk} = 10$ demonstrate a large temperature difference between the hot and cold flows along the height of the heat exchanger (12.41 K for coldest disk (disk 1) and 151.25 K for hottest disk (disk 10)), and a lot of heat is transferred in each disk, up to 1120 W.

The power transferred from the hot to the cold flow is 5.77 kW, with temperature differences at the hot and cold ends of the heat exchanger of 12.41 K and the minimum recorded hot gas temperature of 38.41 K for minimum flow of 1.5 l/min on the hot side and 3.7 l/min on the cold side. In the ideal case, the minimum temperature difference between the hot and cold flows on the hot side of the heat exchanger is $\Delta T \rightarrow 148.7$ K, and on the cold side is $\Delta T \rightarrow 0$ K. Since all the disks in the heat exchanger considered are identical, such temperature differences must inevitably lead to the thermal asymmetry observed in this case. The steady-state temperature distribution in the heat exchanger disks when $N_{disk} = 10$ represented in fig. 4 shows that only a few rows of hot and cold flow channels, which are in close proximity to each other in the *transition zone*, are effectively involved in the heat exchange process. In our case, we can assume that during the heat transfer from the hot flow to the cold one, the channels outside the transition zone thermally “shield” each other in some way, resulting in ineffective heat transfer between the flows. Such an effect can be caused by the strict distribution of channels in concentric rows with a “two-zone” flow pattern through the disk.

The yield of the thermal process of the 10-disk MHE heat exchanger calculated in accordance with the data obtained is 95 %.

Adding 2-3 disks in the MHE structure, which is sufficient for the given inlet conditions for reaching the maximum yield (further increasing the number of plates does not make much difference), we cool down the hot fluid outlet at $T_H^{out} = T_C^{in} = 26$ K while the cold flow can be heated up to the $T_c^{out} = 148.7 < T_H^{in} = 296$ K. Simultaneously, the MHE can transfer a maximum of 6000 W of heat power. Also, it was observed from fig. 4 that with a decrease in the number of channels in the outer rows and an increase in their number in the two "border" ones situated at the boundary of the warm and cold zones of the disk will increase the heat exchanger efficiency.

One can speculate that such "tailoring" of a poly-disk MHE for specific heat exchange conditions can be especially effective in a case similar to the one considered above, i.e., when the flows exchanging heat have significantly different maximum heat contents. This claim, however, is still purely theoretical at this point and needs the MHE numerical model adjustment, more computer simulations and experimental testing.

All the above reasoning and recommendations rely on the results of *numerical simulation* of the poly-disk heat exchanger. Actually, the calculation program is based on the mathematical model using a number of simplifying approximations and assumptions, which can, in some way, distort the true picture of heat transfer processes in the heat exchanger disks. As a result, we should treat the above as speculations that require experimental confirmation of their adequacy.

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