U.P.B. Sci. Bull., Series D, Vol. 77, Iss. 3, 2015 ISSN 1454-2358

MATHEMATICAL MODEL FOR DETERMINATION OF THE MELTING VELOCITY OF THE COATED ELECTRODES

Dumitru Titi CICIC[1], Corneliu RONTESCU[2], Gabriel IACOBESCU[3], Oana Roxana CHIVU[4], Cătălin Gheorghe AMZA[5]

In this paper, there are presented the mathematical relationships derived through mathematical simulation of coated electrodes' melting rate used within the process of manual welding with coated electrode. There are also presented the results from the verification of mathematical model obtained using Ficher criterion.

Keywords: melting rate, mathematical simulation, Fisher criterion

1. Introduction

In designing technological processes of welding, it is necessary to choose the most appropriate welding procedures, filler materials necessary to lay down the manner of preparation and the assembly of welded components, to provide the welding equipment and to establish the elements of welding. Also, within the activities for designing the technology, it is necessary to establish the methodologies for controlling, the removal or reduction of deformations and stresses. Often, in the development of welding, it is necessary to approach the geometric configuration necessary to ensure the welding cord, as well as to guarantee the conditions for obtaining the physical-mechanical properties and the quality of the required welded joint, [1].

The importance of determining the melting velocity derives from the necessity to properly choose the length of the electrode according to the chosen welding parameters. Furthermore, it is a well-known fact that the melting velocity of electrodes influences to a great extent the depositing rate, and, consequently, the productivity of the used welding process.

[1] Lecturer , Dept. of Materials Technology and Welding, University POLITEHNICA of Bucharest, Romania, e-mail: dumitru.cicic@gmail.com
[2] Lecturer , Dept. of Materials Technology and Welding, University POLITEHNICA of Bucharest, Romania, e-mail: corneluts@yahoo.com
[3] Prof , Dept. of Materials Technology and Welding, University POLITEHNICA of Bucharest, Romania, e-mail: gabiiacobescu@yahoo.com
[4] Lecturer , Dept. of Materials Technology and Welding, University POLITEHNICA of Bucharest, Romania, e-mail: virlan_oana@yahoo.co.uk
[5] Associate Professor , Dept. of Materials Technology and Welding, University POLITEHNICA of Bucharest, Romania, e-mail: acata1@amza.camis.pub.ro

During the working processes of the active elements used in the automotive industry the wear process may occur due to the degradation of the surface layers of the elements of the friction couples, which is characterized either by material loss, or by the plastic deformation of the contact surfaces.

If equipment parts are damaged during operation, they can be replaced or reconditioned so that they can be brought to their initial dimensional value or so that they can regain their initial mechanical properties. If the replacement of the parts is expensive, they are reconditioned [2].

During the reconditioning by welding process it is necessary to take into account the fact that the heat cycle the base material and the deposited material are subject to (heating – melting – cooling – solidification) determines changes in the chemical composition caused by the dilution phenomenon as well as changes in the functional and technological properties, as they are the result of the structural modification in the three distinct zones (base material, heat affected zone and deposited material). Following the reconditioning process the reconditioned element needs to have the proper dimension and the required properties according to the respective duty cycle [3, 4].

The use of the direct current welding process with reverse or direct polarity influences the geometric characteristics of the beads as well as process dilution. In certain situations, such as weld deposits, the penetration should be as shallow as possible and the heat affected zone of the bead should be reduced [5].

The geometric simulation of the cord obtained by using the process of manual welding with coated electrode is the result, in addition to the base material and filler, of the interactions of the following speeds: welding speed, the speed of advance of the electrode and the melting speed.

Of these, only the speed which does not depend on the welder, is the speed of melting and that influences the welding time, in the sense that the longer the welding time, the smaller the time necessary to achieve the merging.

Rendered melting rate/speed depends on the electrode through its diameter, on the type of coating, etc. but also on the main welding technology parameter of the current I_s.

Currently, there are very few analytical relations for determining the characteristic quantities involved in the setting of welding, except those that relate to the thermal phenomena and heat transfer in the welded parts.

At the moment, there are no mathematical ratios for determining the electrode melting rate. For mathematical simulation, we can use mathematical models of linear order (1) or first-order (2)

$$y = b_o + b_1 x_1 + b_2 x_2 + b_3 x_3 + ... = b_0 + \sum_{l=1}^{n} b_i x_i \qquad (1)$$

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_i^2 + \sum_{i=1}^{n-1} \sum_{j=1}^{n} b_{ij} x_i x_j \quad (2)$$

where: b_0, b_i are numerical linear coefficients, x_i are independent variables, b_{ii} quadric coefficients and b_{ij} are numerical quadric coefficients.

In order to determine the experimental model coefficients and the statistical analysis of these models, we use statistical programming experience. In this respect, it was found that for such cases, the most rational programs are factorial programs, which are based on the analysis by active experiment.

Each variable *i* is assigned certain amounts (levels of variation), which is obtained by combining different variants of experimentation. Noting with *n,* the number of levels selected for each variable change and *k,* the number of variables, the resulting number *N* is the number of experiments to be carried out, and the relation 3:

$$N = n^k \quad (3)$$

Often, two levels of variation are used, a maximum variation which usually is recorded with (+1), a minimum one, labelled (-1), compared with an average value of variables that are recorded with zero.

2. Mathematical simulation input data

In order to establish the mathematical relation to the melting rate, an electrode E42 4 B 42 H5 has been chosen.

Table 1

Dependent parameters between I_s and d_e for electrode E42 4 B 42 H5 [6]

Electrode brand	d_e (mm)								
	2.5			3.25			4		
	I_s								
	Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.
E7018	90	95	100	120	130	140	160	175	190

Those two parameters, the intensity of the welding current I_s and electrode diameter d_e, in operation of modelling are considered as independent variables.

3. The elaboration of the mathematical model

The first stage in the development of the mathematical model for the electrodes' melting rate is the codification of the independent variables. For I_s we are going to use the encoding x_1 and for d_e the codification x_2 .

After encoding, we are setting the maximum and the minimum values of I_s and d_e and the table 2 is drawn.

Table 2

Values x_1 and x_2.

Experimental level	x_1 (I_s)		x_2 (d_e)
	Min.	Max.	
Superior level (+ 1)	160	190	4
Basic level (0)	120	140	3.25
Lower/inferior level (-1)	90	100	2.5

The value of x_i can be calculated with the formula in relation 4:

$$x_i = \frac{z_i - z_0}{\Delta z_i} \quad (4)$$

where: z_0 – the basic level and Δz_i - the variation range

Substituting this into the relationship (4), we can calculate the values of x_1 and x_2, the relations (5) and (6):

$$x_1 = \frac{z_1 - z_0}{\Delta z} \quad (5)$$

$$x_2 = \frac{z_2 - z_0}{\Delta z} \quad (6)$$

Substituting this into the relation (5) and (6), the values in table 2 results:

$$x_1 = \frac{z_1 - 140}{50} \quad (7)$$

$$x_2 = \frac{z_2 - 3.25}{0.75} \quad (8)$$

The mathematical model chosen is the nonlinear- order, relation 2, which by replacement becomes relation 9:

$$\overline{y_1} = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2 + b_{11}\left(x_1^2 - \frac{2}{3}\right) + b_{22}\left(x_2^2 - \frac{2}{3}\right) \quad (9)$$

The number of experiments needed was determined with the relation (3), in which the number of levels of variation, *n,* is 3, and the number of independent variables, *k*, is 2.

Substituting this into the relation (3), the values indicated above, results:

$$N=3^2=9 \text{ experiments} \quad (10)$$

Following the electrodes coating with the values shown in Tables 1 and 2, resulted the melting values rate indicated in table 3.

Table 3

Melting values rate y_{exp} depending on I_s obtained in the experiments

No. exp.	x_0	x_1	x_2	$x_1 * x_2$	$x_1^2-(2/3)$	$x_2^2-(2/3)$	$y_{exp\ (sec)}$
1	1	1	1	1	1/3	1/3	24
2	1	1	-1	-1	1/3	1/3	17
3	1	-1	1	-1	1/3	1/3	26
4	1	-1	-1	+1	1/3	1/3	20
5	1	1	0	0	1/3	-2/3	21
6	1	-1	0	0	1/3	-2/3	24
7	1	0	1	0	-2/3	1/3	25
8	1	0	-1	0	-2/3	1/3	19
9	1	0	0	0	-2/3	-2/3	23

4. The determination of nonlinear mathematical model coefficients

Mathematical relationships that formed the basis for the calculation of the coefficients are written below [7, 8]:

$$b_i = \frac{\sum_{u=1}^{N} x_{iu} \cdot y_{iu}}{\sum_{u=1}^{N} x_{iu}^2} \tag{11}$$

$$b_{ij} = \frac{\sum_{u=1}^{N} x_{iu} \cdot x_{ju} \cdot y_{iu}}{\sum_{u=1}^{N} x_{iu} \cdot x_{ju}} \tag{12}$$

$$b_{ii} = \frac{\sum_{u=1}^{N} x'_{iu} \cdot y_{iu}}{\sum_{u=1}^{N} x'^2_{iu}} \tag{13}$$

$$b_0 = \frac{\sum_{u=1}^{N} x_{0u} \cdot y_{iu}}{\sum_{u=1}^{N} x_{0u}^2} \tag{14}$$

By using the above mathematical relation and substituting the values in Table 2, it results:

$$b_1 = \frac{24\times1+17\times1+26\times(-1)+20\times(-1)+21\times1+24\times(-1)}{6} = \frac{-8}{6} = -1.33;$$

$$b_{12} = \frac{1\times1\times24+1\times(-1)\times17+(-1)\times1\times26+(-1)\times(-1)\times20+1\times0\times21+(-1)\times0\times24+0\times1\times25+0\times(-1)\times19+0\times0\times23}{(1\times1)^2+(1\times(-1))^2+((-1)\times1)^2+((-1)\times(-1))^2+(1\times0)^2+((-1)\times0)+(0\times1)^2+(0\times(-1))^2+(0\times0)^2} =$$

$$= \frac{24-17-26+20}{4} = 0.25$$

$$b_{11} = \frac{\frac{1}{3}\times(24+17+26+20+21+24)+(-\frac{2}{3})\times(25+19+23)}{(\frac{1}{3})^2+(\frac{1}{3})^2+(\frac{1}{3})^2+(\frac{1}{3})^2+(\frac{1}{3})^2+(\frac{1}{3})^2+(-\frac{2}{3})^2+(-\frac{2}{3})^2+(-\frac{2}{3})^2} =$$

$$= \frac{44-44,66}{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{4}{9}+\frac{4}{9}} = \frac{-0,66}{2} = -0.33$$

$$b_0 = \frac{1\times24+1\times17+1\times26+1\times20+1\times21+1\times24+1\times25+1\times19+1\times23}{9} = \frac{199}{9} = 22.11;$$

$$b_2 = \frac{24\times1+17\times(-1)+26\times1+20\times(-1)+21\times0+24\times0+25\times1+19\times(-1)+23\times0}{6} = \frac{19}{6} = 3.166$$

$$b_{22} = \frac{\frac{1}{3}(24+17+26+20+19+25)+(-\frac{2}{3})(21+24+23)}{(\frac{1}{3})^2+(\frac{1}{3})^2+(\frac{1}{3})^2+(\frac{1}{3})^2+(-\frac{2}{3})^2+(-\frac{2}{3})^2+(-\frac{2}{3})^2+(\frac{1}{3})^2+(\frac{1}{3})^2} =$$

$$= \frac{43,66-45.33}{1,88} = \frac{-1,67}{2} = -0.835$$

Substituting the determined values of the coefficients b_i, b_{ii}, b_{ij} into the relation (9), it results the representing function of the melting variation rate in relation to I_s and d_e relation (15):

$$\overline{y_1}=22{,}11+(-1{,}33)\cdot(\frac{z_1\text{-}140}{50})+(3{,}166)\cdot(\frac{z_2-3{,}25}{0{,}75})+0{,}25\cdot(\frac{z_1-140}{50})\cdot(\frac{z_2-3{,}25}{0{,}75})+$$

$$+(-0{,}33)\cdot(\frac{z_1\text{-}140}{50})^2-0{,}835\cdot(\frac{z_2\text{-}3{,}25}{0{,}8})^2=$$

$$=22.11-0.0266z_1+3.7240+4.2213z_2-13.7193+0.0066\left(z_1z_2-3.25z_1-140z_2+455\right)-$$

$$-0.0001(z_1^2-280z_1+19600)-1.3046(z_2^2-6.5z_2+10.5625)=22.11-0.0266z_1+3.7240+ \qquad (15)$$

$$+4.2213z_2-13.7193+0.0066z_1z_2-0.0214z_1-0.924z_2+3.003-0.0001z_1^2+0.0280z_1+$$

$$-1.96-1.3046z_2^2+8.4799z_2-13.7798=(22.11+3.7240-13.7193+3.003-1.96-13.7798)-$$

$$-0.0001z_1^2-1.3046z_2^2+z_1(-0.0266-0.0214+0.0280)+z_2(4.2213-0.924+8.4799)+0.0066z_1z_2=$$

$$=-0.5621-0.0001z_1^2-1.3046z_2^2-0.02z_1+11.7772z_2+0.0066z_1z_2$$

$$\overline{y_1}=-0.5621-0.0001z_1^2-1.3046z_2^2-0.02z_1+11.7772z_2+0.0066z_1z_2$$

By replacing the values known in relation 15, it results the values shown in table 4.

Table 4

The melting rate values obtained experimentally y_{exp}, the melting rate determined with the mathematical model resulted y_n

No.	y_{exp} [s]	y_n [s]	No.	y_{exp} [s]	y_n [s]	No.	y_{exp} [s]	y_n [s]
1	24	23.2791	4	20	19.3772	7	25	24.6091
2	17	16.4522	5	21	20.5995	8	19	18.2772
3	26	25.3131	6	24	23.0790	9	23	23.0790

5. The verification of the resulted non-linear mathematical model

The nonlinear mathematical model resulting from the calculations was verified using the Fisher criterion, in order to establish its correctness and especially if the mathematical model of order 2 is suitable for this application [7,8].

For verification we calculated a value F_c as a result of the application of the criterion, with the help of relation16:

$$F_c=\frac{S_{conc}^2}{S_0^2} \qquad (16)$$

where: S_{conc}^2 - represents the dispersion caused by the model calculated, with relation 17 and S_0^2 - the reproducibility of results dispersion, based on the relation 18

$$S^2_{conc} = \frac{\sum_{u=1}^{N}\left(y_u - y_{\exp}\right)^2}{N-K^{'}} \tag{17}$$

where:

y_u – the calculated values of melting rate with non-linear model obtained in the experiments u.

y_{exp} – the values melting rate, resulted experimentally under the experiment u;

K' - the number of coefficients; for the case K' = 6 (b_0, b_1, b_2, b_{11}, b_{22}, b_{12});

$N-K'$ - number of degrees of freedom representing the difference between number N of tests and number k' of coefficients in the regression equation (including bo);

u – the number of the experiment;

$$S^2{}_0 = \frac{\sum_{u=1}^{N}(y_u - \overline{y_u})^2}{n-1} \tag{18}$$

where:

n - number of parallel tests;

y – the average obtained in the result of n parallel tests;

u – the number of the experiment;

The mathematical model obtained is valid if it complies with relation (19):

$$F_c < F_{0.05;}\upsilon_1 > \upsilon_2 \tag{19}$$

where:

υ_1 - the number of degrees of freedom for S^2_{conc}, calculated with relation 20 has the value 3;

υ_2 - the number of degrees of freedom for $S^2{}_0$, calculated with relation 21, has the value 2.

$$\upsilon_1 = N - K' \tag{20}$$

$$\upsilon_2 = n - 1 \tag{21}$$

The value for $F_{0.05;}3.2$ is 19. 6.

To calculate the experimental errors we are using information in Table 5.

Table 5

Calculation of experimental error y_{exp} , of the melting rate

Nr. exp	y_n	$\bar{y}$	$y_n - \bar{y}$	$(y_n - \bar{y})^2$	V_2=n-1
1	25	21.56	3.44	11.8144	3-1=2
2	19		-2.56	6.5679	
3	23		1.44	2.0656	

In order to calculate the fluctuation due to the nonlinear model, we prepared Table 6.

Table 6

Melting rate values obtained experimentally y_{exp} and melting rate determined with the mathematical model results y_n

No. exp	y_{exp}	y_u	$y_u - y_{exp}$	$(y_u - y_{exp})^2$
1	24	23.2791	-0.7209	0.5197
2	17	16.4522	-0.5478	0.3001
3	26	25.3131	-0.6869	0.4718
4	20	19.3772	-0.6229	0.3879
5	21	20.5995	-0.4005	0.1604
6	24	23.0790	-0.9210	0.8483
7	25	24.6091	-0.3909	0.1528
8	19	18.2772	-0.7228	0.5225
9	23	23.0790	0.0790	0.0062

Substituting in relation 17 and 18, it results:

$$S^2_{conc} = \frac{2.2466}{3} = 1.1233$$

$$S^2_0 = \frac{11.8144 + 6.5679 + 2.0656}{2} = 10.2239$$

The 2 values obtained are introduced in relation 16, resulting in the value of F_c:

$$F_C = \frac{S^2_{conc}}{S^2_0} = \frac{1.1233}{10.2239} = 0.1098$$

As we can see, the condition is met, so the linear 2[nd] order model is valid under the conditions set out in the current paper (the values of I_s and d_e).

19.6 >0.1098 => $F_{0,05;3;2}$> F_c

6. Conclusions

After analyzing the results, the following conclusions may be drawn:

- the non-linear mathematical model of 2[nd] order can be used to establish mathematical relationships to determine the electrodes melting rate;

- the differences between experimental values and those calculated with 2nd order nonlinear model of melting rate values were in the range (-0.9210 - 0.079s), acceptable values in the development of welding technology;

- from the experimental values of the analysis, it can be seen that the rate of melting is directly proportional to the intensity of welding. This means that if it is necessary to increase the productivity of deposit/filling by increasing the rate of melting, the intensity of the welding current I_s should also be increased.

Acknowledgement

The work has been funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU/159/1.5/S/134398.

R E F E R E N C E S

[1]. *Dumitru Titi Cicic*, Informatizarea şi optimizarea proceselor de sudare (Computerization and optimization of welding process) Editura Politehnica Press, 2014 (in Romanian).

[2] *G.M. Dumitru, C. Radu, B. Dumitru*, Recondiționarea şi repararea produselor (Reconditioning and repair products), Editura Printech, Bucharest, 2010 (in Romanian).

[3] *Gh. Amza, D. Dobrotă, Z. Apostolescu, V. Petrescu, R.G. Albulescu*, Influence of use of ultrasound on metallographic structure of plated pieces by welding in ultrasonic field, Metalurgija 54 (2015) 3, 523-526

[4] *D.T. Cicic, Gh. Solomon, G. Iacobescu, C. Rontescu* - Method of establishing the optimum technique for reconditioning by welding, taking into account the consumption of filler metals and electrical power, U.P.B. Sci. Bull. Series D, Vol.73, No.1/ 2011, pp. 85…98

[5] *C. Rontescu, D.T. Cicic, G. Iacobescu, C. Gh. Amza*, The influence of the welding current polarity on the geometric configuration of the bead, Advanced Materials Research Vol. 1088 (2015) pp 797-801

[6]. *** ESAB Welding Handbook.

[7]. *D. Taloi,* Optimizarea proceselor metalurgice, (Optimization of metallurgical process) Editura Tehnică, Bucureşti, 1984 (in Romanian).

[8]. *Gabriel Iacobescu, Ionelia Voiculescu,* A mathematical model for humidifying the electrodes coats, U.P.B. Sci. Bull., Series D, Vol. 68, No. 4, 2006, 51-58