MODELING OF THE INDUCTION MOTOR VENTILATION BY ELECTRIC CIRCUITS

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Lucrarea prezintă o metodă noua de analiză a debitelor fluidului de răcire din maşinile electrice. Metoda se bazează pe analogia dintre rețeaua de rezistențe aerodinamice a sistemului de ventilație a unei maşini electrice şi un circuit electric neliniar echivalent, care conține rezistoare neliniare controlate în curent. Pentru analiza circuitului rezistiv neliniar echivalent s-a utilizat metoda nodală modificată.

The paper presents a new method to analyze the cooling medium flow in electrical machines. The method is based on the analogy between the equivalent air-dynamic scheme and a nonlinear resistive electrical circuit by which the ventilation system can be modeled. To perform the analysis, the modified nodal method is used.

Keywords: Induction Motor, Ventilation System, Cooling Medium, Air-Dynamic Scheme, Electric Modeling

1. Introduction

In general, the ventilation system design of electrical machines is based on approximate global values of pressure drops of the cooling medium. In order to estimate better the pressure drops, it is necessary to keep in mind that the losses depend on the load of the electrical machine and on the cooling medium flow, which must transport the corresponding heat of the machine. The knowledge of the cooling medium flow is necessary in order to determine the cooling medium velocities and the coefficient of the heat transfer by convection α, defined by the following relation:

$$\alpha = \alpha_0 \left(1 + k\sqrt{v}\right), \tag{1}$$

where: α_0 – is the coefficient of the heat transfer by convection corresponding to the zero velocity, k – represents a constant, depending on the roughness of the

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ventilated surfaces, and v – is the velocity of the cooling medium at the convection surface.

2. Modeling ventilation system by air - dynamic resistances

In the induction motor whose ventilation system is presented in Fig. 1, the air is absorbed by the fan and is exhausted both inside and outside the frame. After that the air goes out from the air gap and from the axial channels in the rotor, it blows the frontal surfaces of the windings and the magnetic core, then being exhausted from the machine. In the outside of the machine the air blows the exterior surface of the frame, which may cool fins to increase the convection surface.

By traveling the ventilation path, the cooling medium takes several kinds of pressure drops, each of them being able to be simulated by air-dynamic resistances (see Fig. 7).

As shown in [1 - 3] the pressure drop due to flow through a section is expressed by the following relation:

$$\Delta p = R_a \cdot Q^2, \tag{2}$$

where: Δp – is the pressure drop, Q – represents the cooling medium flow and R_a – is the air-dynamic resistance.

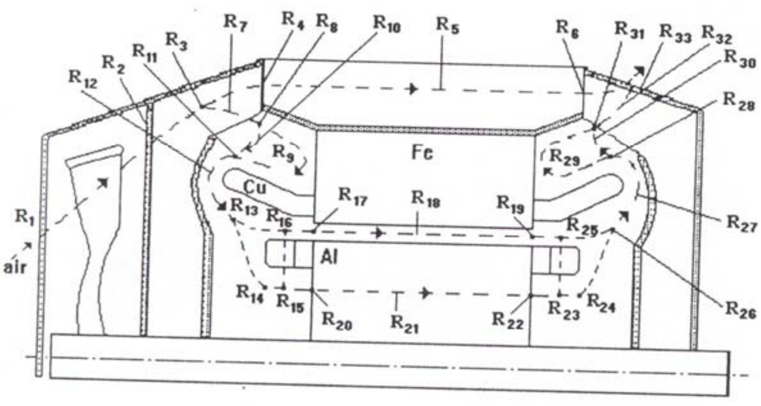

Fig. 1. Ventilation system of an induction motor

The air-dynamic resistance is computed according to the relation

$$R_a = \frac{\zeta}{S^2}, \tag{3}$$

in which: ζ - is a resistance factor, depending on the type of the pressure drop and S – is the area of the cross-section through which flows the cooling medium.

The resistance factor for *friction by flow in a duct or pipe* with the length L and cross section area S (Fig. 2) is

$$\zeta_f = \lambda \frac{L}{D} \frac{\gamma}{2g} \tag{4}$$

where: λ - is the friction factor; L – is the length of the duct; D – is the equivalent diameter of the duct; g – is the acceleration of the gravitation, in m/s^2; and γ - is the specific weight of the cooling medium, in N/m^3.

The friction factor λ depends on the Reynolds number of the fluid and on the relative roughness ε/D of the duct walls, ε being the value of the roughness, and D the hydraulic diameter. The last is defined as

$$D = 4 \times \frac{cross - \sec tion\ \ area}{wetted\ \ perimeter} \tag{5}$$

For circular ducts, the hydraulic diameter reduces to D. The flow of the cooling medium in the electrical machines has Reynolds numbers in the range $R_e \in (3000,\ 20000)$. For the ratio values D/ε, specific for the usual processing technologies of the ventilation channel surfaces and of the magnetic core exterior surfaces ($40 < D/\varepsilon < 10000$), [1] recommends to estimate the friction factor for pressure drop by friction, according the diagram from Fig. 3.

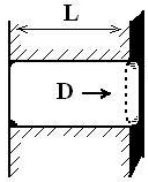

Fig.2. Friction in a channel (duct).

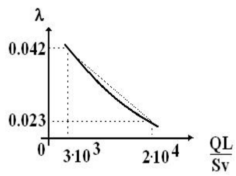

Fig. 3. Variation of the friction factor λ.

The sudden change in flow section (Fig. 4) causes a pressure drop. As shown in [1], the corresponding air-dynamic resistance may be calculated with the resistance factor

$$\zeta_f = \xi_S \frac{\gamma}{2g} \tag{6}$$

where: ξ_S is the sudden change coefficient, which depends on the area ratio; S is the cross-section area of the big section of the pipe, in m^2.

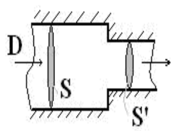

Fig. 4 - Sudden change of the flow section.

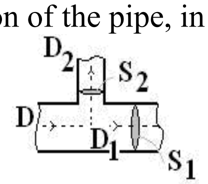

Fig. 5 - Deviation of the cooling medium.

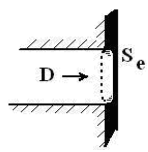

Fig. 6 – Exit of the cooling medium

The change in flow direction, branch or confluence of the cooling medium (Fig. 5) produces a pressure drop and the corresponding air-dynamic resistances can be calculated by the following expressions, [1 - 6]:

$$R_{d1} = \xi_{d1} \frac{\gamma}{2g} \frac{1}{S_1^2}, \quad R_{d2} = \xi_{d2} \frac{\gamma}{2g} \frac{1}{S_2^2} \tag{7}$$

where: ξ_{d1}, ξ_{d2} are coefficients of pressure drop by flow direction change or branch, which depend on the angle of direction change or branch; S_1, S_2 are the cross-section areas of the ducts, in m^2.

Remark. When it is difficult to estimate the different cross sections of the branches, they may be considered equal to the initial cross section divided by the number of branches.

The entrance or the exit of the cooling medium (Fig. 6) causes a pressure drop, and the corresponding air-dynamic resistance is calculated with the expression given in [1, 2]

$$R_e = \xi_e \frac{\gamma}{2g} \frac{1}{S_e^2} \tag{8}$$

where: $\xi_e = 1$; S_e is the cross-section area of the duct, in m^2.

The main pressure source is a fan (blower), attached to the rotor. The pressure-flow characteristic of the fan with radial blades has the form

$$p = p_0 \left(1 - \left(Q / Q_{\max} \right)^2 \right), \tag{9}$$

where p_0 is the pressure for null flow, and Q_{max} is the maximum flow, for null pressure. This characteristic is equivalent to an ideal fan, giving the pressure p_0 and a series connected “internal” air-dynamic resistance

$$R_v = \frac{p_0}{Q_{max}^2}. \tag{10}$$

The blades attached to the rotor short-circuiting ring have also a ventilating effect, whose static pressure may be estimated by

$$p_r = 0.25\gamma \left(v_1^2 - v_2^2 \right) \tag{11}$$

where γ is the specific weight of the cooling medium, in N/m^3, and v_1, v_2 are the outer and inner peripheral velocities of the blades.

3. Modeling by electric circuits the ventilation system

Using the analogy between the equivalent air-dynamic circuits and the electric circuits, the methods used in the electric circuit analysis may be applied in order to obtain the flows of the cooling medium in different part of the air-dynamic circuit of the electrical machines. The analogy between the two types of circuits is presented in table 1.

Table 1

Electric circuits	Air-dynamic circuits
Voltage, v [V]	Pressure drop of the cooling medium, Δp [Pa]
Electromotive force (emf), e [V]	Cooling medium pressure generated by the fan or by the coil ends, p [Pa]
Intensity of the electric current, i [A]	Flow of the cooling medium, Q [m^3/s]

Current controlled nonlinear resistor, having the characteristic: $v=f(i)=k \cdot i^2$.	Nonlinear air-dynamic resistor controlled in cooling medium flow, having the characteristic: $\Delta p=R_a \cdot Q^2$.

Example 1: The equivalent electric scheme of the ventilation system from Fig. 1 is presented in Fig. 7. The air-dynamic resistances of this scheme are due to the causes outlined in Fig. 1. Three sources corresponding to the three fans were represented: the main fan, with the pressure p_0 and inner resistance R_v, and two fans due to the ventilating action of the blades on the rotor short-circuiting rings, with pressure p_r and without inner resistance. In the circuit from Fig. 7 all resistors are nonlinear resistors, with quadratic voltage-current characteristics.

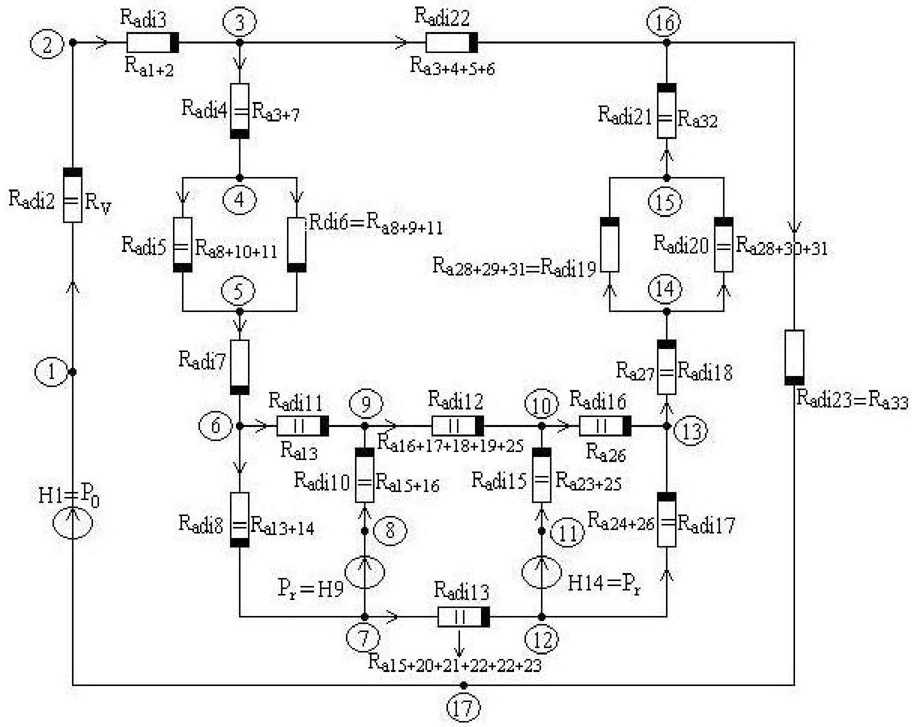

Fig. 7. Air-dynamic equivalent circuit.

In order to analyze the nonlinear resistive circuit in Fig. 7 we use the modified nodal method (MNM), [7 – 10]. The voltage vector of the $n-1$ independent nodes - $\boldsymbol{v}_{n\text{-}1}$ and the current vector $\boldsymbol{i}_m$ – so called the controlling currents (the currents corresponding to the circuit elements which are incompatible with the classical nodal method), [2, 5 - 8] are considered as independent variables. The characteristics v - i of the current-controlled (c.c.) nonlinear resistors are approximated by continuous piecewise-linear curves. The constitutive equations of the c.c. nonlinear resistors corresponding to the arbitrary segment combination s have the form

$$v_{ik} = R_{dik}(s) i_{ik} + e_{ik}(s),\quad i^{-}_{ik}(s) \le i_{ik} \le i^{+}_{ik}(s) \tag{12}$$

and the modified nodal equations in matrix form are

$$\begin{bmatrix} \boldsymbol{G}_{dn-1,n-1}(s) & \boldsymbol{B}_{n-1,m} \\ \boldsymbol{A}_{m,n-1} & \boldsymbol{R}_{dm,n-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_{n-1} \\ \boldsymbol{i}_{m} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_{sc,n-1}(s) \\ \boldsymbol{e}_{m}(s) \end{bmatrix} \tag{13}$$

where: $\boldsymbol{G}_{dn-1,n-1}(s)$ - is the (n–1)x(n–1) matrix of differential (dynamic) nodal conductances corresponding to the n – 1 independent circuit nodes; $\boldsymbol{B}_{n-1,m}$ - represents the (n–1)xm matrix with inputs: –1, 0, 1 and the current transfer factors (current gains) of the current-controlled current sources; $\boldsymbol{A}_{m,n-1}$ - is a mx(n–1)x matrix, which contains the elements: –1, 0, 1 and the voltage transfer factors (voltage gains) of the voltage-controlled voltage sources; $\boldsymbol{R}_{dm,m}(s)$ - represents the square mxm matrix having as inputs: the transfer resistances of the current-controlled voltage sources and the differential (dynamic) resistances of the c.c. nonlinear resistors; $\boldsymbol{i}_{sc,n-1}(s)$ - is the short-circuit current vector injected in the n-1 independent nodes (including the currents resulted by simulating the voltage-controlled nonlinear resistors), and $\boldsymbol{e}_{m}(s)$ - represents the emf vector corresponding to the branches made up only of ideal independent voltage sources and those from the approximation of the c.c. nonlinear resistors by continuous piecewise-linear curves.

Using the ECSAP program [9] to analyze the nonlinear circuit shown in Fig. 7, the following results were obtained:

Pressure drops [Pa]		Flows [m^3/s]	
Δp1=-300.000000	Δp13=5.702740	Q1=0.014742	Q13=0.002635
Δp2=84.269812	Δp14=-200.000000	Q2=0.014742	Q14=0.030760
Δp3=84.269812	Δp15=196.019394	Q3=0.014742	Q15=0.030760
Δp4=0.372284	Δp16=3.446908	Q4=0.005315	Q16=0.033440
Δp5=0.319629	Δp17=-0.533698	Q5=0.000299	Q17=-0.028125
Δp6=0.319629	Δp18=18.500849	Q6=0.005016	Q18=0.005315
Δp7=18.500849	Δp19=0.319629	Q7=0.005315	Q19=0.005016
Δp8=3.635998	Δp20=0.319629	Q8=0.033373	Q20=0.000299
Δp9=-200.000000	Δp21=0.372284	Q9=0.030738	Q21=0.005315
Δp10=195.863278	Δp22=47.190563	Q10=0.030738	Q22=0.009427
Δp11=-0.500724	Δp23=84.269812	Q11=-0.028058	Q23=0.014742
Δp12=5.858857		Q12=0.002680	

These results were compared with the ones obtained by Spice simulator and Motor CAD program.

Example 2: In figure 8,a is represented the longitudinal section by the generator rotor. In figure 8, b is shown the air-dynamic equivalent circuit corresponding to the rotor cooling.

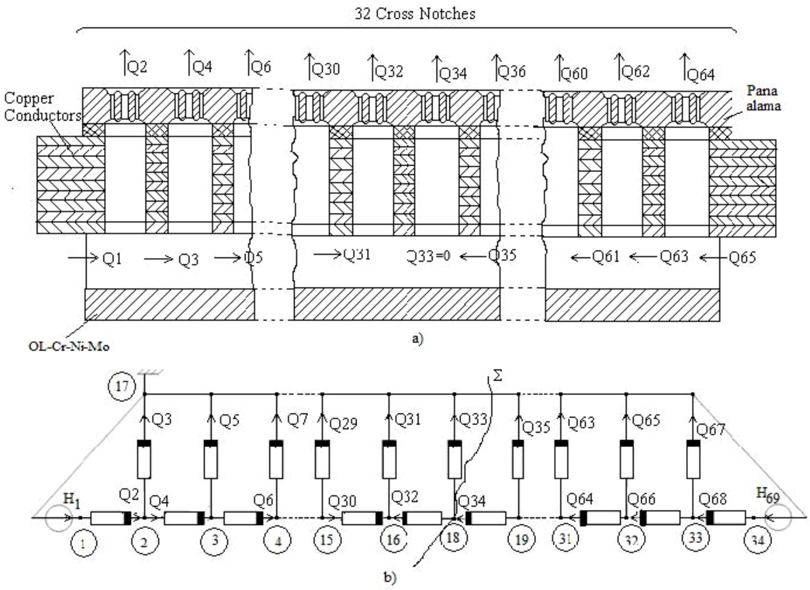

Fig. 8. a) Longitudinal section by the physical model of the generator rotor;
b**)** Air-dynamic equivalent circuit

Using the ECSAP program to analyze the nonlinear circuit shown in Fig. 8, we obtain the following results:

Pressure drops [Pa]	Pressure drops [Pa]	Flows [m^3/s]	Flows [m^3/s]
Δp1=-1236.312000	ΔP17=396.732001	Q1=0.112988	Q17=0.006795
ΔP2=769.676796	ΔP18=5.862756	Q2=0.112988	Q18=0.068627
ΔP3=466.635204	ΔP19=390.869245	Q3=0.005149	Q19=0.007333
ΔP4=13.097295	ΔP20=5.330620	Q4=0.107838	Q20=0.061294
ΔP5=453.537909	ΔP21=385.538625	Q5=0.004554	Q21=0.007843
ΔP6=12.129500	ΔP22=4.119383	Q6=0.103284	Q22=0.053451
ΔP7=441.408409	ΔP23=381.419242	Q7=0.004870	Q23=0.008499
ΔP8=11.263375	ΔP24=3.571529	Q8=0.098414	Q24=0.044952
ΔP9=430.145033	ΔP25=377.847713	Q9=0.005188	Q25=0.008768
ΔP10=10.080175	ΔP26=3.043882	Q10=0.093226	Q26=0.036184
ΔP11=420.064858	ΔP27=374.803830	Q11=0.005559	Q27=0.008964
ΔP12=8.858362	ΔP28=2.213094	Q12=0.087667	Q28=0.027220
ΔP13=411.206497	ΔP29=372.590737	Q13=0.005975	Q29=0.009131
ΔP14=7.667973	ΔP30=1.347620	Q14=0.081691	Q30=0.018088
ΔP15=403.538524	ΔP31=371.243116	Q15=0.006269	Q31=0.009042
ΔP16=6.806523	ΔP32=371.243116	Q16=0.075422	Q32=0.009047

These results were compared with the ones obtained experimental on the physical model, and the ones obtained on Spice simulator and Motor CAD program.

4. Conclusions

The proposed method for determining the cooling medium flow in different sections can be applied to all the types of electrical machines with similar ventilation systems as the induction motor.

Of course, a more rigorous connection between the mathematical model and the physical one is a more complex and difficult problem. That is, because the estimation of the friction surfaces, the working temperatures of the cooling medium, the deviation angles, and the turbulences are known with a certain level of error, and it is difficult to come down below this level.

The mathematical model proposed for the ventilation system modeling allows the determination with a greater accuracy of the cooling medium flows in different parts of the machine, and consequently of the coefficients of the heat transfer by convection.

Modeling the ventilation system by equivalent electric circuits, and using the modified nodal analysis in association with the Newton-Raphson algorithm, offer a suitable tool for a simple formulation of the nonlinear algebraic circuit equations and for their efficient numerical solving.

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