GENERALIZED MATHEMATICAL MODELS OF THE ELECTROMAGNETIC FIELD IN NONLINEAR MEDIA FOR THE STATIONAR, QUASISTATIONARY AND TIME VARIABLE REGIMES

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În lucrare sunt prezentate şi analizate modele matematice generalizate ale câmpului electromagnetic în diverse regimuri. Pentru aceste modele matematice sunt formulate teoreme de existenta si unicitate. Ecuațiile de câmp sunt scrise ca distribuție. Considerăm că relația constitutivă poate fi neliniară.

This paper presents and analyzes generalized mathematical models of the electromagnetic field in different regimes. Existence and uniqueness theorems are formulated for these mathematical models. Field equations are written as a distribution. We consider that the constitutive relationship may be nonlinear.

Keywords: existence and uniqueness theorems, stationary electromagnetic field, periodic quasistationary regime, variable regime

1. Introduction

The correct formulation of problems of the electromagnetic field analysis involves, in addition to defining the basic phenomenology of the problem, establishing a phenomenological model which relates to the field system, identifying sources of field and models of the material, obtaining a mathematical model of electromagnetic field, which plays an important role in this issue [1]. The mathematical model consists of an operational equation; the unknown (primary size) is a state of the field size or other associated scalar or vector. In addition to the above, and depending on the type of operational equation satisfied by the primary size, supplementary conditions necessary for the uniqueness of the solution equation may be enumerated.

For the correctness of the mathematical model, there must be demonstrated existence and uniqueness theorems and stability of the solution, which should ensure the narrowing of the solution deviations in case of data corruption.

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For simplicity of presentation, we consider the boundary conditions zero and in addition into the steady-state conditions, we consider the current density and charge zero.

Uniqueness theorems have been proposed and demonstrated long before, by Răduleț R., Al. Timotin, and A. Țugulea [2], [3]. However these theorems imposed the restriction according to which the dimensions of the electromagnetic field belong to $\boldsymbol{C}^1$ class on subdomains, while at their borders conditions of transition are established. Obviously, imposing conditions of transition requires, in turn, restrictions on surfaces.

In case of nonlinear media, must be imposed restrictions that all constitutive relations must be $\boldsymbol{C}^1$ class. From mathematical point of view, the legitimate question arises, whether it is possible for the aforementioned restrictions not to allow any solutions at all. So it can be expected that expanding the class of the functions in which vector fields are defined, may prove useful. We propose for the generalized magnetic field to be an ordered pair (**B**, **H**) $\in$ $\boldsymbol{S}$, where $\boldsymbol{S} = \boldsymbol{L}^2(\Omega)\times\boldsymbol{L}^2(\Omega)$. Obviously, at this point the risk of the uniqueness of solution getting lost might occur. In this case, the derivatives must be treated in a general way:

$$\left(-\langle div\boldsymbol{Z},\varphi\rangle\right) = \langle \boldsymbol{Z}, grad\varphi\rangle,\ (\forall)\ \ \varphi \in K \tag{1}$$

$$\left(-\langle rot\boldsymbol{Z},\boldsymbol{\Psi}\rangle\right) = \langle \boldsymbol{Z}, rot\boldsymbol{\Psi}\rangle,\ (\forall)\ \ \Psi \in \boldsymbol{K} \tag{2}$$

where *rot* and *div* take the meanings of operators. K and $\boldsymbol{K}$ are spaces of the scalar functions (vectors) indefinitely derivable and having compact support in Ω.

Equations of stationary regime of the magnetic field (assuming we also have a magnetic charge ρ) are:

$$div\ \mathbf{B} = \rho,\ rot\ \mathbf{H} = \mathbf{J},\ \mathbf{B} = F\ (\mathbf{H})$$

where ρ and **J** are distributions, and $F:\ \boldsymbol{L}^2(\Omega)\rightarrow\boldsymbol{L}^2(\Omega)$ is a nonlinear relation of material. Relations (1), (2) are local forms (“differentials”) of the equations of the magnetic field.

1.1. Notations and definitions

Let be the domain $\Omega \subset R^3$, bounded by the closed area $\partial\Omega$ and let be the interval $[0,T] \subset R$.
We note $\Pi = \Omega \times \left]0,T\right[$ and $\Pi_t = \Omega \times \left]0,t\right[$, with $t \in \left]0,T\right[$.

Let E^3 be the Euclidean space, and $\boldsymbol{L}^2(\Omega)$ the Hilbert space of the function $\boldsymbol{x} : \Omega \to E^3$, with inner product $\langle \boldsymbol{x}, \boldsymbol{y} \rangle_{L^2(\Omega)} = \int_\Omega \boldsymbol{x} \cdot \boldsymbol{y}\, dv$; we consider $\boldsymbol{C}^k(\Omega)$ the space of the continuous function $\boldsymbol{x} : \Omega \to E^3$, having k-continuous Frechet differentiable on Ω, $0 \le k \le \infty$ (so they have continuous partial derivatives up to ordinal k); $\boldsymbol{K}(\Omega)$, the space of the function belonging to $\boldsymbol{C}^\infty(\Omega)$ and having compact support Ω.

The spaces $\boldsymbol{C}^k(\Omega)$ and $\boldsymbol{K}(\Omega)$ are dense in $\boldsymbol{L}^2(\Omega)$. We define similarly the spaces $\boldsymbol{L}^2(\Pi)$, $\boldsymbol{C}^k(\Pi)$, $\boldsymbol{K}(\Pi)$.

We use the notations:

$$\frac{\partial \boldsymbol{x}}{\partial t} = \dot{\boldsymbol{x}}, \qquad \int_0^t \boldsymbol{x}\, d\tau = \underset{\bullet}{\boldsymbol{x}}$$

$\underset{\bullet}{\boldsymbol{L}}^2(\Pi)$ is the space of the functions $\underset{\bullet}{\boldsymbol{x}}$, where $\boldsymbol{x} \in \boldsymbol{L}^2(\Pi)$ and similarly $u\underset{\bullet}{\boldsymbol{L}}^2(\Pi)$, $\underset{\bullet}{C}^k(\boldsymbol{\Pi})$ etc.

We denote $u\boldsymbol{L}^2(\Pi)$ the completed space $\boldsymbol{C}^1(\boldsymbol{\Pi})$, in the average uniform topology. Obviously, $u\boldsymbol{L}^2(\Pi) \subset \boldsymbol{L}^2(\Pi)$.

Theorem 1: The operator $\left(\int_0^t (\bullet) d\tau \right) : \boldsymbol{L}^2(\Pi) \to u\boldsymbol{L}^2(\Pi)$ is continuous.

If E^3 is replaced by R, we define similarly the space $\mathrm{L}^2(\Omega)$, K(Ω), $\mathrm{L}^2(\Pi)$, etc. Where the function defined on Ω (or Π) are values in R.

Let $\boldsymbol{H}$ a Hilbert space. We note a material relationship as a continuous function $\hat{F} : \boldsymbol{H} \to \boldsymbol{H}$. The relationship $\hat{F}$ is lipschitzian and uniformly monotone if:

$$sup_{\boldsymbol{x},\boldsymbol{y} \in \boldsymbol{H}} \frac{\left\| \hat{F}(\boldsymbol{x}) - \hat{F}(\boldsymbol{y}) \right\|}{\left\| \boldsymbol{x} - \boldsymbol{y} \right\|} = \Theta < \infty \tag{3}$$

and

$$inf_{\boldsymbol{x},\boldsymbol{y} \in \boldsymbol{H}} \frac{\left\langle \hat{F}(\boldsymbol{x}) - \hat{F}(\boldsymbol{y}), \boldsymbol{x} - \boldsymbol{y} \right\rangle}{\left\| \boldsymbol{x} - \boldsymbol{y} \right\|^2} = \theta > 0 \tag{4}$$

We say that $\hat{F} \in \mathcal{U}(\boldsymbol{H})$.

Basically, the application $\hat{F} \in \mathcal{U}\ (\boldsymbol{L}^2(\Omega))$ or ($\hat{F} \in \mathcal{U}\ (\boldsymbol{L}^2(\Pi))$), can be obtained from a local material relationship F: $\boldsymbol{\Omega} \times E^3 \to E^3$ (sau F: $\boldsymbol{\Omega} \times [0, T] \times E^3 \to E^3$).

2. The generalized mathematical model of the stationary field [4] [5]

We call a generalized stationary magnetic field the ordered pair [s] = (**B**, **H**), where $\mathbf{B}, \mathbf{H} \in \boldsymbol{L}^2(\Omega)$. We denote $\mathcal{SF}$ the vector space of pairs [s].

We say that a series $([\mathbf{s}_n])_{n\geq 1}$ is convergent if $(\mathbf{B}_n)_{n\geq 1}$ and $(\mathbf{H}_n)_{n\geq 1}$ are convergent in $\boldsymbol{L}^2(\Omega)$.

Let be $S_h \subset \partial\Omega$ consisting of a finite number (k) of disjoint surfaces ($S_h = \bigcup_{i=1}^{k} S_{hi}$ and $S_{hi} \cap S_{hj} = 0$ for $i \neq j$).

We denote $\mathcal{SF}_F$ the set of pairs [s] with $\mathbf{B}, \mathbf{H} \in \boldsymbol{C}^1(\boldsymbol{\Omega})$, where [3]:

$$(\mathbf{n} \times \mathbf{H}(P)) \times \mathbf{n} = 0, \quad \text{for } P \in S_h \tag{5}$$

$$\mathbf{n} \times \mathbf{B}(P) = 0, \quad \text{for } P \in S_b = \partial\Omega \setminus S_h \tag{6}$$

$$\int_{S_{hi}} \mathbf{B}(P) \cdot \mathbf{n} \cdot \mathrm{d}A = 0, \quad \text{for } i = 1, 2, \ldots, k\text{-}1 \tag{7}$$

$$div\, \mathbf{B} = 0,\ rot\, \mathbf{H} = 0 \tag{8}$$

Let be $\mathbf{H}, \mathbf{B} \in \boldsymbol{C}^2(\Omega)$ with properties $rot\,\mathbf{H}=0$, $div\,\mathbf{B}=0$ and null conditions on the border. The fields **B,H** may be nonzero because the constitutive relation may be nonlinear and $F(\mathbf{H}) \neq 0$. We say $(\mathbf{B},\mathbf{H}) \in \mathcal{SF}_F$.

Let be $\overline{\mathcal{SF}}_F$ the completed space in $\mathcal{SF}_F$ topology. $\overline{\mathcal{SF}}_F$ is a subspace of $\mathcal{SF}$.

Equations (1), (2) are local forms of the stationary regime equations. It should also be interesting to find an integral formulation for the equations of the quasistationary system. Affiliation to the $\boldsymbol{L}^2(\Omega)$ spaces implies the existence of Lebesque sets of null measure on which the vector fields may not be defined. These sets can be surfaces or curves. For this reason, the formulation of the integral forms must resort to Fubini theorem.

Let there be $(S_u, u \in [0,1] \subset R^2)$ a family of surfaces, who can be represented by the family of sheets $\mathbf{r} = \hat{r}(u, \xi, \eta)$, where **r** is the position vector and $\hat{r}$: $[0,1]^3 \to E^3$ is $\boldsymbol{C}^1$ class. The family of surfaces has the property (PS): $\exists\ c > 0$

so that $\forall u \in [0,1]$ and $\forall\ P \in S_u$, $\left(\dfrac{\partial \mathbf{r}}{\partial u}, \mathbf{n}\right) \geq c$, where **n** is the normal to surface S_u. So the family of surfaces fills a $\boldsymbol{\omega}$ domain.

Let be $\mathbf{B} \in \boldsymbol{L}^2(\Omega)$. For almost any $u \in [0, 1]$ there are the $\Phi = \int_{S_u} \mathbf{Bn}\mathrm{dA}$ fluxes (in the Lebesque's sense). Let be a series $(\mathbf{B_n})_{n\geq 1}$ who converges to **B**, then the average fluxes $\widetilde{\Phi}_n = \int_0^1 \mathrm{du} \int_{S_u} \mathbf{B}_n \mathbf{n} \mathrm{dA}$, will converge for $\widetilde{\Phi}$.

The integral form of the magnetic flux law: Let there be $(\sigma_u, u \in [0,1])$ a family of closed surfaces with (PS) property and [s] $\in \overline{\mathscr{B}}_F$. Then $\widetilde{\Phi} = 0$ and for almost any $u \in [0,1]$, $\Phi_{\sigma_u} = 0$.

The integral form of the *div* **B =** 0 relation for [s] $\in \overline{\mathscr{B}}_F$ field**:** Let be $\left(C_M, M \in [0,1]^2 \subset R^2\right)$ a family of curves which can be represented by a family of roads $\mathbf{r} = \hat{r}(u,\xi,\eta)$, ξ and η being the coordinates of point M and $\hat{r} : [0,1]^3 \to E^3$ is $\boldsymbol{C}^1$ class. The family of curves has the (PC) property: $\exists\ c > 0$ so that $\forall M$ and $\forall\ u$

$$\left(\frac{\partial \mathbf{r}}{\partial \xi} \times \frac{\partial \mathbf{r}}{\partial \eta}\right) \left(\frac{\partial \mathbf{r}}{\partial u} \Big/ \left|\frac{\partial \mathbf{r}}{\partial u}\right|\right) \geq c \tag{9}$$

Let be $\mathbf{H} \in \boldsymbol{L}^2(\Omega)$ [4]; for almost any $M \in [0,1]^2$ there is the circulation (in the Lebesque's sense) $\mathrm{U}_{C_M} = \int_{C_M} \mathbf{H}\mathrm{d}\mathbf{r}$. Let be a series $(\mathbf{H}_n)_{n\geq 1} \subset \boldsymbol{L}^2(\Omega)$ who converges to **H**, then average circulations $\widetilde{\mathrm{U}}_\mathrm{n} = \int_{[0,1]^2} \mathrm{dA}_M \int_{C_M} \mathbf{H}\,\mathrm{d}\mathbf{r}$ converge for $\widetilde{\mathrm{U}}_{C_M}$.

$$\left|\widetilde{\mathrm{U}}_{C_M} - \widetilde{\mathrm{U}}_n\right| \leq \frac{1}{c}\sqrt{v(\omega)}\ \left\|\mathbf{H} - \mathbf{H}_n\right\|_{\boldsymbol{L}^2(\omega)} \tag{10}$$

where $v(\omega)$ is the volume of the domain ω.

The integral form of the *rot* **H =** 0 relation for [s] $\in \overline{\mathscr{B}}_F$ field**:** Let be $\left(\gamma_M, M \in [0,1]^2\right)$ a family of closed curves with (PC) property and [s] $\in \overline{\mathscr{B}}_F$. Then $\mathcal{V}\gamma_M = 0$ and for almost any $M \in [0,1]^2$, $\mathrm{U}\gamma_M = 0$.

Theorem 2 (uniqueness and existence): Let be $\hat{\mathrm{U}} \in \mathscr{U}(\boldsymbol{L}^2(\Omega))$; there is one and only one field [s] $\in \overline{\mathscr{B}}_F$ where $\mathbf{H} = \hat{\mathrm{U}}(\mathbf{B})$.

3. Generalized mathematical model for quasistationary electromagnetic field [6]

Let be $\mathcal{QF}$ the vectorial space of the ordered quartets $[q]=(\boldsymbol{e}, \boldsymbol{b}, \boldsymbol{h}, \boldsymbol{j}) \in C^2\overline{\mathcal{QF}}$ where:

$$rot\ \boldsymbol{e} = -\dot{\boldsymbol{b}},\ rot\ \boldsymbol{h} = \boldsymbol{j},\ div\ \boldsymbol{b} = 0 \tag{11}$$

We say that $([\boldsymbol{q}_n])_{n\geq 1}$ is a Cauchy series, if $(\boldsymbol{e}_n)_{n\geq 1}$, $(\boldsymbol{j}_n)_{n\geq 1}$ in $\boldsymbol{L}^2(\Pi)$, and $(\boldsymbol{b}_n)_{n\geq 1}$, $(\boldsymbol{h}_n)_{n\geq 1}$ are Cauchy into $u\boldsymbol{L}^2(\Pi)$.

Let be $\overline{\mathcal{QF}}$ the completed $\mathcal{QF}$ space of this topology. Let be $[q] \in \overline{\mathcal{QF}}$

We name periodic quasistationary electromagnetic field any $[q] \in \overline{\mathcal{QF}}$ element.

The local equations of the quasistationary electromagnetic field [3]

Let there be $[q] \in \overline{\mathcal{QF}}$; $\forall\, \varphi,\ \psi \in \boldsymbol{K}(\Pi)$ and $\forall\ \chi \in K(\Pi)$; the following relations are valid:

$$\langle \boldsymbol{e}, rot\varphi \rangle_{\boldsymbol{L}^2(\Pi)} = \langle \dot{\boldsymbol{b}}, \varphi \rangle_{\boldsymbol{L}^2(\Pi)} \quad \text{- the induction law} \tag{12}$$

$$\langle \boldsymbol{h}, rot\psi \rangle_{\boldsymbol{L}^2(\Pi)} = \langle \boldsymbol{j}, \psi \rangle_{\boldsymbol{L}^2(\Pi)} \quad \text{- the magnetic circuit law} \tag{13}$$

$$\langle \boldsymbol{b}, grad\chi \rangle_{\boldsymbol{L}^2(\Pi)} = 0 \quad \text{- the magnetic flux law} \tag{14}$$

Let be $S_e \in \partial\Omega$ and $S_h \in \partial\Omega / S_e$ .We denote QF_F the set of fields $[q] = QF$ wherefore:

$$(\boldsymbol{n} \times \boldsymbol{e}(P)) \times \boldsymbol{n} = 0 \quad \text{for } P \in \sum e = S_e \times [0,T] \tag{15}$$

$$(\boldsymbol{n} \times \boldsymbol{h}(P)) \times \boldsymbol{n} = 0 \quad \text{for } P \in \sum h = S_h \times [0,T] \tag{16}$$

$$\boldsymbol{b}|_{t=0} = 0 \tag{17}$$

Let $\overline{\mathcal{QF}}_F$ be the completed space of $\mathcal{QF}_F$ in $\overline{\mathcal{QF}}$ topology $\overline{\mathcal{QF}}$. $\overline{\mathcal{QF}}_F$ is a $\overline{\mathcal{QF}}$ subspace.

Theorem 3 **:** Let be $\hat{A}$, $\hat{B}$ two positive and liniar operators and $\hat{A}$ is hermitian. If $[q] \in \overline{\mathcal{QF}}_F$ fulfils the relation $\boldsymbol{e} = \hat{A}\ \boldsymbol{j}$ and $\boldsymbol{b} = \hat{B}\ \boldsymbol{h}$, then $[q] = 0$.

Corollary: Let be $\boldsymbol{s} \in \boldsymbol{L}^2(\Pi)$ and , $\boldsymbol{i} \in u\boldsymbol{L}^2(\Pi)$ $\hat{A}$, $\hat{B}$ two positive operators and $\hat{A}$ hermitian; there is at most $[q] \in \overline{\mathcal{QF}}_F$ field where $\boldsymbol{e} = \hat{A}\,\boldsymbol{j} + \boldsymbol{s}$ and $\boldsymbol{b} = \hat{B}\,\boldsymbol{h} + \boldsymbol{i}$.

Theorem 4 **(**of existence in $\overline{\mathcal{QF}}_F$)

We presume fulfilled the following condition: $\forall\, \varphi \in K(\Pi)$, the equation

$$\hat{M}\,\boldsymbol{x} = rotrot\,\dot{\boldsymbol{x}} + \boldsymbol{x} = \varphi \tag{18}$$

has a solution $\boldsymbol{x} \in C^2(\Pi)$ so that

$$(\boldsymbol{n} \times \boldsymbol{x}(P)) \times \boldsymbol{n} = 0, \quad \text{for } P \in \sum e \tag{19}$$

$$(\boldsymbol{n} \times rot\boldsymbol{x}(P)) \times \boldsymbol{n} = 0, \text{ for } P \in \sum h \tag{20}$$

Equation (18) has unique solution because:

$\left\langle \boldsymbol{x}, \hat{M}\boldsymbol{x} \right\rangle_{L^2(\Pi_t)} = \frac{1}{2}\left\| rot\,\dot{x}(t) \right\|^2_{L^2(\Omega)} + \left\| x \right\|^2_{L^2(\Pi_t)}$ and the operator $\hat{M}$ is therefore defined positive.

Theorem 5**:** Let be $\boldsymbol{s} \in \boldsymbol{K}(\Pi)$ and $\boldsymbol{i} \in \dot{\boldsymbol{L}}^2(\Pi)$. There is a only one field $[q] \in \overline{\mathcal{QF}}_{F,}$ with $\boldsymbol{e}, \boldsymbol{j} \in u\boldsymbol{L}^2(\Pi)$ and $\boldsymbol{b}, \boldsymbol{h} \in \dot{\boldsymbol{L}}^2(\Pi)$, where $\boldsymbol{e} = \rho\,\boldsymbol{j} + \boldsymbol{s}$, $\dot{\boldsymbol{b}} = \mu\,\dot{\boldsymbol{h}} + \dot{\boldsymbol{i}}$ (and $\boldsymbol{b} = \mu\ \boldsymbol{h} + \boldsymbol{i}$) and $\rho,\ \mu > 0$. The function $\hat{\Psi} : \boldsymbol{L}^2(\Pi) \rightarrow \boldsymbol{L}^2(\Pi)$ defined by $\hat{\Psi}(\dot{\boldsymbol{i}}) = (\dot{\boldsymbol{b}})$ is a contraction.

4. Generalized mathematical model of the electromagnetic field in variable regime [7]

Let $\mathcal{GF}$ be the vectorial space of the ordered quintets $[g] = (\boldsymbol{e}, \boldsymbol{j}, \boldsymbol{d}, \boldsymbol{b}, \boldsymbol{h})$, with $\boldsymbol{e}, \boldsymbol{j}, \boldsymbol{d}, \boldsymbol{b}, \boldsymbol{h} \in C^2(\boldsymbol{\Pi})$, where $rot\,\boldsymbol{e} = -\,\dot{\boldsymbol{b}}$, $rot\,\boldsymbol{h} = \boldsymbol{j} + \dot{\boldsymbol{d}}$, $div\,\boldsymbol{b} = 0$ (21)

We say that $([g_n])_{n\geq 1}$ is a Cauchy series if $(\boldsymbol{e}_n)_{n\geq 1}$, $(\boldsymbol{d}_n)_{n\geq 1}$, $(\boldsymbol{b}_n)_{n\geq 1}$, $(\boldsymbol{h}_n)_{n\geq 1}$, are Cauchy in $u\boldsymbol{L}^2(\Pi)$ and $(\boldsymbol{j}_n)_{n\geq 1}$ is Cauchy in $\boldsymbol{L}^2(\Pi)$**.**

Let be $\overline{\mathcal{GF}}$ the completed space of the $\mathcal{GF}$ in this topology.

We call generalized electromagnetic field any element $[g] \in \overline{\mathcal{GF}}$.

Generalized local forms of the electromagnetic field equations

Let be $[g] = (\boldsymbol{e}, \boldsymbol{j}, \boldsymbol{d}, \boldsymbol{b}, \boldsymbol{h}) \in \overline{\mathcal{GF}}$; $\forall\, \varphi, \psi \in \boldsymbol{K}(\Pi)$ and $\forall\, \chi \in \mathrm{K}(\Pi)$ the following relationships are valid:

$$\langle \boldsymbol{e}, rot\varphi \rangle_{\boldsymbol{L}^2(\Pi)} = \langle \boldsymbol{b}, \dot{\varphi} \rangle_{\boldsymbol{L}^2(\Pi)} \quad \text{- the induction law} \tag{22}$$

$$\langle \boldsymbol{h}, rot\psi \rangle_{\boldsymbol{L}^2(\Pi)} = \langle \boldsymbol{j}, \psi \rangle_{\boldsymbol{L}^2(\Pi)} - \langle \boldsymbol{d}, \dot{\psi} \rangle_{\boldsymbol{L}^2(\Pi)} \text{ – the magnetic circuit law} \tag{23}$$

$$\langle \boldsymbol{b}, grad\chi \rangle_{\boldsymbol{L}^2(\Pi)} = 0 \quad \text{- the magnetic flux law} \tag{24}$$

Let be $S_e \subset \partial\Omega$ and $S_h \in \partial\Omega / S_e$. We denote $\mathcal{GF}_F$ the set of fields $[g] = \mathcal{GF}$ for which:

$$\boldsymbol{n} \times \boldsymbol{e}(P) = 0, \quad \text{for } P \in \sum e = S_e \times [0,T] \tag{25}$$

$$\boldsymbol{n} \times \boldsymbol{h}(P) = 0, \quad \text{for } P \in \sum h = S_h \times [0,T] \tag{26}$$

$$\boldsymbol{b}\big|_{t=0} = 0 \tag{27}$$

$$\boldsymbol{d}\big|_{t=0} = 0 \tag{28}$$

We denote $\overline{\mathcal{GF}}_F$ the completed space of the $\mathcal{GF}_F$ in this topology $\overline{\mathcal{GF}}$. $\overline{\mathcal{GF}}_F$ is a $\overline{\mathcal{GF}}$ subspace**.**

Let be $\hat{B}$, $\hat{C} \in \mathcal{F}(\boldsymbol{L}^2(\Omega))$.

We presume the following condition is met:

$\forall\, \varphi \in K(\Pi)$ and $\sigma > 0$, $(\exists)\, \boldsymbol{x} \in \boldsymbol{C}^2(\Pi)$ the solution of equation:

$$\hat{N}\,\boldsymbol{x} = \sigma\boldsymbol{x} + rot\left(\hat{B}\, rot\, \underset{\bullet}{\boldsymbol{x}}\right) + \hat{C} \cdot \dot{\boldsymbol{x}} = \varphi \tag{29}$$

with

$$\boldsymbol{n} \times \boldsymbol{x}(P) = 0, \text{ for } P \in \sum e \tag{30}$$

$$\boldsymbol{n} \times rot\boldsymbol{x}(P) = 0, \text{ for } P \in \sum h \tag{31}$$

$$\boldsymbol{x}\big|_{t=0} = 0 \tag{32}$$

In the previous condition we considered that $\boldsymbol{x} \in C^2(\Pi)$, which implies restrictions on operators $\hat{B}$, $\hat{C}$ (these restrictions could be partially avoided, considering that $\boldsymbol{x}$ is an element of a Sobolev-type space of norm:

$$\left(\|\boldsymbol{x}\|^2_{\boldsymbol{L}^2(\Pi)} + \|rot\,\boldsymbol{x}\|^2_{\underset{\bullet}{\boldsymbol{L}}^2_B(\Pi)} + \|\boldsymbol{x}\|^2_{\boldsymbol{L}^2_C(\Pi)}\right)^{\frac{1}{2}}. \tag{33}$$

The $\hat{N}$ operator is defined positive and the equation solution (29) is unique.

Let $W^2(\Pi)$ be the Sobolev space of norm [7]

$$\left(\|\boldsymbol{x}\|^2_{\boldsymbol{L}^2(\Pi)} + \|rot\boldsymbol{x}\|^2_{\boldsymbol{L}^2(\Pi)}\right)^{\frac{1}{2}} \tag{34}$$

Then whatever $\boldsymbol{s} \in \boldsymbol{L}^2(\Pi)$, $\boldsymbol{p} \in \underset{\bullet}{\boldsymbol{L}}^2(\Pi)$, $\boldsymbol{m} \in W^2(\Pi)$, there is a field

$[g] \in \overline{\mathfrak{G}\mathfrak{F}}_F$, with $\boldsymbol{e} = \rho\,\boldsymbol{j} + \boldsymbol{s}$, $\boldsymbol{h} = \hat{B}\,\boldsymbol{b} - \boldsymbol{m}$, $\boldsymbol{d} = \hat{C}\,\boldsymbol{e} + \boldsymbol{p}$.

Theorem 6.(of existence): Let there be $\boldsymbol{p} \in \underset{\bullet}{\boldsymbol{L}}^2(\Pi)$, $\boldsymbol{m} \in W^2(\Pi)$, $\hat{C}$, $\hat{B} \in \mathfrak{F}(\boldsymbol{L}^2(\Omega))$ and $\hat{\mathrm{U}}$, which for any t applies Lipschitzian $\boldsymbol{L}^2(\Pi_t)$ in $\boldsymbol{L}^2(\Pi_t)$; additionally $\hat{U}$ is causal, namely, for any t, y belongs in $[0, t]$, it is determined only by $\boldsymbol{x}$ in the $[0, t]$ interval (through $y = \hat{U}(\boldsymbol{x})$). There is $[g] \in \overline{\mathfrak{G}\mathfrak{F}}_F$ with $\boldsymbol{j} = \hat{U}(\boldsymbol{e})$, $\boldsymbol{h} = \hat{B}\,\boldsymbol{b} - \boldsymbol{m}$, $\boldsymbol{d} = \hat{C}\,\boldsymbol{e} + \boldsymbol{p}$.

6. Conclusions

For the fairness of the mathematical model, there must be demonstrated theorems of the existence, uniqueness and stability of the solution, which should ensure the narrowing of the solution deviations in case of data corruption.

Taking into consideration the local forms of the equations of the magnetic field, is interesting to find integral formulations for them. Affiliation to the $\boldsymbol{L}^2(\Omega)$ spaces implies the existence of Lebesque sets of null measure on which the vector fields may not be defined. These sets can be surfaces or curves. For this reason, the formulation of the integral forms must resort to Fubini theorem.

The conditions for the solution of the electromagnetic field in order to ensure its uniqueness may be too restrictive and the existence of the solution might be unsafe, especially when the constitutive relation is nonlinear. Because of these restrictions, the components of the electromagnetic field are described by functions with continuous derivatives on subdomains so that local forms may be written for the field equations. In order there should be a solution which preserves, at the same time, its uniqueness, lighter restrictions will therefore be adopted for the solution of the field $\boldsymbol{L}^2$ (Ω).

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