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CONTROLLABILITY AND LONGITUDINAL MOVEMENT STABILITY OF AN INSECT-TYPE MICRO AIR VEHICLE

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This paper presents some studies concerning the controllability and the stability of the longitudinal movement of an insect-type micro air vehicle (MAV) by using a non-dimensional linearized dynamic model. The model's controllability studies are based on the modal transformation.

For different combinations of control variables, the stabilization is realized by repositioning the model's matrix eigenvalues, respectively by optimal linear quadratic control. The theoretical results are validated by numerical simulations for a specific case study.

Keywords: ***Eigen-mode, eigenvalues, controllability, stability***

1. Introduction

Scientific literature presents the results of some studies and researches concerning the motion dynamics of insect-type MAV's, as, for example, in [1], [2], [3], respectively of the robots built based on biological insect models (MFI- Micro-mechanical Flying Insect) [4], [5], [6]. MAV's dynamical models are built based on the aerospace vehicles general motion equations, [7], using the flapping (vibrating) wing's forces and moments. Stability and command derivatives are calculated based on the values obtained during wind-tunnel experiments, for different insect types [8], [9].

MAV's longitudinal dynamics linearized models, described by equation with dimensional or non-dimensional state variables, have, in many cases, unstable matrix eigenvalues (unstable eigenmodes). For these models stabilization it is compulsory to reposition their matrix eigenvalues. Therefore, in this paper one determines the main (observable) state values for each eigen-mode, as well as the command variables for main state values' control. Furthermore, for each eigen-mode's control, one has to analyze the controllability and which one of the control variables is effective for that

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eigen-mode. One uses the modal transformation and a transformation matrix is built using the longitudinal dynamics model's matrix eigenvectors' components. For different control variables combinations, the MAV model's control (stabilization) law is designed, by repositioning the model's matrix eigenvalues. The linear quadratic optimal control law is also designed. In order to estimate the MAV dynamical model's state, one may use a state estimator [10]. Theoretical results are validated by the time characteristics obtained by numerical simulations.

2. Studies concerning the controllability of an insect-type MAV dynamic linear non-dimensional model

MAV's longitudinal motion model is described by the state equation:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}\,, \tag{1}$$

with the non-dimensional state variables' vector $\boldsymbol{x} = \left[\Delta V_x^* \;\; \Delta V_z^* \;\; \Delta\omega_y^* \;\; \Delta\theta\right]^T$ and the command vector $\boldsymbol{u} = \left[\Delta\Phi \;\; \Delta\alpha_1 \;\; \Delta\bar{\phi} \;\; \Delta\alpha_2\right]^T$; $\Delta\alpha_1$ and $\Delta\alpha_2 -$ wing's attack angle's symmetrical and asymmetric variations; $\Delta\Phi -$ flapping angle's amplitude's variation and $\Delta\bar{\phi} -$ flapping angle's mean value's variation;

$$\Delta V_x^* = \frac{\Delta V_x}{V_0}, \Delta V_z^* = \frac{\Delta V_z}{V_0}, \; \Delta\omega_y^* = \frac{\Delta\omega_y}{f}, \Delta\theta^* \equiv \Delta\theta, \tag{2}$$

where V_x and V_z are MAV's velocities with respect to the ox and oz axis of the $oxyz -$ frame (solidary with the MAV, $ox -$ longitudinal axis, $oy -$ lateral axis, $oz -$ completes the right rectangular frame), $f -$ flapping frequency. The matrices A and B have the following general and particular forms [9], [11]:

$$A = \begin{bmatrix} \frac{X_u^*}{m^*} & \frac{X_w^*}{m^*} & \frac{X_q^*}{m^*} & -g^* \\ \frac{Z_u^*}{m^*} & \frac{Z_w^*}{m^*} & \frac{Z_q^*}{m^*} & 0 \\ \frac{M_u^*}{J_{yy}^*} & \frac{M_w^*}{J_{yy}^*} & \frac{M_q^*}{J_{yy}^*} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{X_\Phi^*}{m^*} & \frac{X_{\alpha_1}^*}{m^*} & \frac{X_{\bar{\phi}}^*}{m^*} & \frac{X_{\alpha_2}^*}{m^*} \\ \frac{Z_\Phi^*}{m^*} & \frac{Z_{\alpha_1}^*}{m^*} & \frac{Z_{\bar{\phi}}^*}{m^*} & \frac{Z_{\alpha_2}^*}{m^*} \\ \frac{M_\Phi^*}{J_{yy}^*} & \frac{M_{\alpha_1}^*}{J_{yy}^*} & \frac{M_{\bar{\phi}}^*}{J_{yy}^*} & \frac{M_{\alpha_2}^*}{J_{yy}^*} \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

One has chosen the morphological and cinematic parameters for MAV (insect type) from [11]: $m = 27.3\,\text{mg}, r_2 = 5.6\text{mm}, c = 2.2\;\text{mm}, S_a = 41\text{mm}^2, \Phi = 90\text{deg}$, $f = 160\text{Hz}, t_w = 1/f = 6.25\text{ms}, J_{yy} = 1.84\cdot 110^{-10}\text{kgm}^2, \rho = 1.25\text{kgm}^{-3}, V_0 = 2\Phi f r_2$.

One uses the notations:

$X^* = \frac{X}{\rho \frac{V_0^2}{2} S_a}, Z^* = \frac{Z}{\rho \frac{V_0^2}{2} S_a}, M_y^* = \frac{M_y}{\rho \frac{V_0^2}{2} S_a c}, t^* = \frac{t}{t_w}, m^* = \frac{m}{\rho \frac{V_0^2}{2} S_a t_w}, t^* = \frac{t}{t_w},$

$m^* = \frac{m}{\rho \frac{V_0^2}{2} S_a t_w}, J_{yy}^* = \frac{J_{yy}}{\rho \frac{V_0^2}{2} S_a c t_w^2}, g^* = \frac{g t_w}{V_0}$; X, Y and Z are the components of the resultant forces after MAV axes and $M_y -$ the pitch moment. With these, one expresses: $X_u^* = \frac{\partial X^*}{\partial V_x^*}, X_w^* = \frac{\partial X^*}{\partial V_z^*}, X_q^* = \frac{\partial X^*}{\partial \omega_y^*}, Z_u^* = \frac{\partial Z^*}{\partial V_x^*}, Z_w^* = \frac{\partial Z^*}{\partial V_z^*}, Z_q^* = \frac{\partial Z^*}{\partial \omega_y^*},$

$M_u^* = \frac{\partial M_y^*}{\partial V_x^*}, M_w^* = \frac{\partial M_y^*}{\partial V_z^*}, M_q^* = \frac{\partial M_y^*}{\partial \omega_y^*}$ and $X_\Phi^* = \frac{\partial X^*}{\partial \Phi}, X_{\alpha_1}^* = \frac{\partial X^*}{\partial \alpha_1}, X_{\bar{\phi}}^* = \frac{\partial X^*}{\partial \bar{\phi}},$

$X_{\alpha_2}^* = \frac{\partial X^*}{\partial \alpha_2}, Z_\Phi^* = \frac{\partial Z^*}{\partial \Phi}, Z_{\alpha_1}^* = \frac{\partial Z^*}{\partial \alpha_1}, Z_{\bar{\phi}}^* = \frac{\partial Z^*}{\partial \bar{\phi}}, Z_{\alpha_2}^* = \frac{\partial Z^*}{\partial \alpha_2}, M_\Phi^* = \frac{\partial M_y^*}{\partial \Phi},$

$M_{\alpha_1}^* = \frac{\partial M_y^*}{\partial \alpha_1}, M_{\bar{\phi}}^* = \frac{\partial M_y^*}{\partial \bar{\phi}}, M_{\alpha_2}^* = \frac{\partial M_y^*}{\partial \alpha_2}$. For the calculation of non-dimensional stability and control derivatives one uses the values resulted from the characteristics given in [11]; using these, by linear interpolation, one obtains the characteristics whose slopes are the non-dimensional stability and command derivatives; it results: $X_u^* = -2.320, Z_u^* = -0.050, M_u^* = 2.240, X_w^* = 0.0092, Z_w^* = -1.240,$ $M_w^* = 0.0092, X_q^* = -0.0459, Z_q^* = -0.0080, M_q^* = -0.6857, X_\Phi^* = 0.00075, Z_\Phi^* = -1.5625,$ $M_\Phi^* = 0.0000, X_{\alpha_1}^* = 0.00046, Z_{\alpha_1}^* = -2.9536, M_{\alpha_1}^* = 0.0000, X_{\bar{\phi}}^* = -0.0716, Z_{\bar{\phi}}^* = -0.0242,$ $M_{\bar{\phi}}^* = -3.0773, X_{\alpha_2}^* = -2.9536, Z_{\alpha_2}^* = 0.0000, M_{\alpha_2}^* = 0.5670$. The matrices (3) become:

$$A = \begin{bmatrix} -0.0115 & -0.0015 & -0.0111 & -0.0230 \\ -0.0040 & 0 & 0 & 0 \\ 0.1989 & -0.0926 & -0.0661 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.00001 & 0.00046 & -0.03759 & -0.0415 \\ -0.0242 & -0.04463 & -0.03748 & 0 \\ 0 & 0 & -0.3096 & 0.0480 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (4)$$

Regarding the control of each one of the natural modes (flight controllability), one has to know which one of these modes is controllable and if the answer is positive, which inputs are effective. It can be realized using the modal decomposing method; dynamical system's model is transformed into a system with modal coordinates (state variables). Analyzing this new system, one can identify which modes could be controlled. The couple of complex-conjugated eigenvectors, corresponding to the complex-conjugated eigenvalues $\lambda_{1,2}$, are denoted as $\eta_1 \pm \eta_2 i$ and the real eigenvectors, corresponding to the real eigenvalues λ_3 and λ_4, as η_3 and η_4 (table 1).

Transformation matrix T is built up with the A-matrix (in (4)) eigenvectors' components.

$$T = [2\eta_1 \quad 2\eta_2 \quad \eta_3 \quad \eta_4] = \begin{bmatrix} 2(-0.0702) & 2(0.1263) & 0.1149 & 0.4106 \\ 2(-0.00239) & 2(0.00289) & 0.0024 & 0.8818 \\ 2(0.05534) & 2(0.14171) & -0.1844 & 0.00043 \\ 2(0.9786) & 0 & 0.9760 & -231702 \end{bmatrix}. \tag{5}$$

Changing the coordinates (state variables) as

$$\boldsymbol{x} = T\xi, \xi = [\xi_1 \quad \xi_2 \quad \xi_3 \quad \xi_4]^T, \tag{6}$$

where $\xi_i, i = \overline{1,4}$ are the modal coordinates, the system in (1) becomes

$$\dot{\xi} = \overline{A}\xi + \overline{B}\mathrm{u}, \overline{A} = T^{-1}AT, \overline{B} = T^{-1}B, \tag{7}$$

or

$$[\dot{\xi}_1 \quad \dot{\xi}_2 \quad \dot{\xi}_3 \quad \dot{\xi}_4]^T = \overline{A}[\xi_1 \quad \xi_2 \quad \xi_3 \quad \xi_4]^T + \overline{B}[\Delta\Phi \quad \Delta\alpha_1 \quad \Delta\phi \quad \Delta\alpha_2]^T; \tag{8}$$

this system represents the modal form of (1)-system. Analyzing the A-matrix elements, one observes that its first two lines contain values which are approximately equal to the real parts and to the imaginary parts of $\lambda_{1,2}$ eigenvalues. The third and the fourth line contains eigenvalues approximately equal to λ_3 and λ_4. State variables ξ_1 and ξ_2 are the modal coordinates of the unstable oscillating mode, while ξ_3 and ξ_4 are state variables of the rapid mode, respectively of the slow mode.

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \end{bmatrix} = \begin{bmatrix} 0.0578 & 0.1433 & -0.0000 & -0.0001 \\ -0.1466 & 0.0554 & -0.0001 & 0.0001 \\ -0.0019 & 0.0032 & -0.1889 & 0.0001 \\ 0.0019 & -0.0007 & -0.0000 & -0.0018 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_{\xi_1} \end{bmatrix} + \\ + \begin{bmatrix} -0.0166 & -0.0311 & -0.3238 & 0.1046 \\ 0.0239 & 0.0449 & -0.5504 & -0.0076 \\ 0.0276 & 0.0502 & 0.6386 & -0.22093 \\ -0.0279 & -0.0512 & -0.0424 & 0.0012 \end{bmatrix} \begin{bmatrix} \Delta\Phi \\ \Delta\alpha_1 \\ \Delta\overline{\phi} \\ \Delta\alpha_2 \end{bmatrix}. \tag{9}$$

In order to assure a MAV stable plane flight, one has to stabilize the unstable oscillating mode and to increase the stabilization of the stable slow mode (λ_4 must be displaced to the left side of the complex plan, on the negative side of the real axis, which means a $|\lambda_4|$ growing). Although this mode is a stable one, it is slowly convergent, so $|\lambda_4|$ should be increased, for example $\lambda_4 = -0.15$. ξ_1 and ξ_2 modal coordinates control (unstable oscillating mode coordinates) could be made by the first two lines of the B-matrix (9). As long as the elements in 3rd and 4th columns of those lines are ten times greater than the elements in 1st and 2nd columns of the same matrix, it results that the oscillating mode's stability could be controlled by $\Delta\overline{\phi}$ and/or $\Delta\alpha_2$ (elements in columns 1-3 of the 4th line in matrix B are one order greater than the element in 4th column of the same line); because $\Delta\overline{\phi}$ controls the oscillating mode, it shall be omitted for the coordinate's control.

Table 1

System's matrix eigenvectors

State variable	Mode 1 (oscillating) - $\eta_1 \pm \eta_2 i$	Mode 2 (rapid) - η_3	Mode 2 (slow) - η_4
ΔV_x^*	$-0.0702 \pm 0.1263i$	0.1149	0.4106
ΔV_z^*	$-0.00239 \pm 0.00289i$	0.0024	0.8818
$\Delta\omega_y^*$	$0.05534 \pm 0.1417i$	-0.1844	0.00043
$\Delta\theta$	0.9786	0.9760	-0.231702

Table 2

Eigenvectors components' amplitudes and phases

State variable	Mode 1 (oscillating)	Mode 2 (rapid)	Mode 2 (slow)
ΔV_x^*	0.1445 (119.06^0)	0.1149 (0^0)	0.4106 (0^0)
ΔV_z^*	0.0038 (129.59^0)	0.0024 (0^0)	0.8818 (0^0)
$\Delta\omega_y^*$	0.1521 (68.66^0)	0.1844 (180^0)	0.00043 (0^0)
$\Delta\theta$	0.9786 (0^0)	0.9760 (0^0)	0.2317 (180^0)

As table 2 shows, ΔV_x^*, $\Delta\omega_y^*$, and $\Delta\theta$ are the main variables for the unstable oscillating mode and ΔV_z^* is the observable variable of the aperiodic slow mode. Therefore, the unstable oscillating mode can be controlled by $\Delta\overline{\phi}$ and/or $\Delta\alpha_2$, with feedback gains after ΔV_x^*, $\Delta\omega_y^*$ $\Delta\theta$ and the aperiodic slow mode can be controlled by $\Delta\Phi$ and/or $\Delta\alpha_2$.

3. Insect-type MAV linear non-dimensional model motion stability by two or more command variables

3.1. Control by the variables $\Delta\Phi$ and $\Delta\overline{\phi}$

The variable $\Delta\overline{\phi}$ is expressed as:

$$\Delta\overline{\phi} = k_2 \Delta V_x^* + k_3 \Delta\omega_y^* + k_4 \Delta\theta. \tag{10}$$

Consequently, the aperiodic slow mode could be controlled by $\Delta\Phi$ with ΔV_z^* feedback: $\Delta\Phi = k_1 \Delta V_z^*$. From the systems (1) and (3) one separates the ΔV_z^* variable's equation, but omitting the terms containing the state variables $\Delta V_x^*, \Delta\omega_y^*, \Delta\theta$, as well as the command variables $\Delta\alpha_1$, $\Delta\overline{\phi}$ and $\Delta\alpha_2$; it results the following equation, which is built only with the terms containing $\Delta\phi$ and ΔV_z^*

variables with the values in (4),

$$\Delta \dot{V}_z^* = \frac{Z_w^*}{m^*}\Delta V_z^* + \frac{Z_\Phi^*}{m^*}\Delta\Phi = 0\cdot\Delta V_z^* - 0.0242\Delta\Phi. \tag{11}$$

Introducing $\Delta\Phi = k_1\Delta V_z^*$ in (11), it becomes:

$$\Delta \dot{V}_z^* = -0.0242 k_1 \Delta V_z^*. \tag{12}$$

One chooses k_1 so $|\lambda_4| = 0.15 = 0.0242k_1;\ k_1 = 6.2.$

Considering equation (10), one extract from (1) and (3) the first, the third and the fourth equation, but omitting the term which contains $\Delta V_z^* -$ variable and the terms containing the command variables $\Delta\bar{\phi}, \Delta\alpha_1, \Delta\alpha_2$; one obtains the equation:

$$\begin{bmatrix}\Delta\dot{V}_x^*\\ \Delta\dot{\omega}_y^*\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{X_u^*}{m^*} & \frac{X_q^*}{m^*} & -g^*\\ \frac{M_u^*}{J_{yy}^*} & \frac{M_q^*}{J_{yy}^*} & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta V_x^*\\ \Delta\omega_y^*\\ \Delta\theta\end{bmatrix} + \begin{bmatrix}\frac{X_{\bar{\phi}}^*}{m^*}\\ \frac{M_{\bar{\phi}}^*}{J_{yy}^*}\\ 0\end{bmatrix}\Delta\bar{\phi}; \tag{13}$$

Eliminating $\Delta\bar{\phi}$ between equations (11) and (13), one obtains:

$$\left[\Delta\dot{V}_x^* \quad \Delta\dot{\omega}_y^* \quad \Delta\dot{\theta}\right]^T = M\left[\Delta V_x^* \quad \Delta\omega_y^* \quad \Delta\theta\right]^T; \tag{14}$$

$$M = \begin{bmatrix}m_{11} & m_{12} & m_{13}\\ m_{21} & m_{22} & m_{23}\\ m_{31} & m_{32} & m_{33}\end{bmatrix} + \begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}\begin{bmatrix}k_2\\ k_3\\ k_4\end{bmatrix}^T = \begin{bmatrix}m_{11}^{'} & m_{12}^{'} & m_{13}^{'}\\ m_{21}^{'} & m_{22}^{'} & m_{23}^{'}\\ m_{31}^{'} & m_{32}^{'} & m_{33}^{'}\end{bmatrix}, \tag{15}$$

$$\begin{aligned} &m_{11}^{'} = m_{11} + b_1k_1, m_{12}^{'} = m_{12} + b_1k_3, m_{13}^{'} = m_{13} + b_1k_4,\\ &m_{21}^{'} = m_{21} + b_2k_2, m_{22}^{'} = m_{22} + b_2k_3, m_{23}^{'} = m_{23} + b_2k_4,\\ &m_{31}^{'} = m_{31} + b_3k_2, m_{32}^{'} = m_{32} + b_3k_3, m_{13}^{'} = m_{33} + b_3k_4.\end{aligned} \tag{16}$$

The eigenvalues of the matrix M are the solutions of the characteristic equation $\det(\lambda I - M) = 0$, which has the form:

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0; \tag{17}$$

$$
\begin{aligned}
a &= -\left(m_{11} + m_{22} + m_{33}\right) - \left(b_1 k_2 + b_2 k_3\right),\\
b &= \left[\left(m_{11}m_{22} + m_{22}m_{33} + m_{33}m_{11}\right) - \left(m_{12}m_{21} + m_{23}m_{32} + m_{31}m_{13}\right)\right] +\\
&\quad + \left(m_{21}b_1 + m_{33}b_1 - m_{12}b_2\right)k_2 + \left(m_{11}b_2 + m_{33}b_2 - m_{21}b_1\right)k_3 - \left(m_{31}b_1 + m_{32}b_2\right)k_4,\\
c &= \left[\left(m_{11}m_{23}m_{32} + m_{12}m_{21}m_{33} + m_{13}m_{31}m_{22}\right) - \left(m_{12}m_{23}m_{31} + m_{13}m_{32}m_{21} + m_{11}m_{22}m_{33}\right)\right] +\\
&\quad + \left[\left(m_{23}m_{32}b_1 + m_{12}m_{33}b_2\right) - \left(m_{13}m_{32}b_2 + m_{22}m_{33}b_1\right)\right]k_2 +\\
&\quad + \left[\left(m_{33}m_{21}b_1 + m_{13}m_{31}b_2\right) - \left(m_{23}m_{31}b_1 + m_{11}m_{33}b_2\right)\right]k_3 +\\
&\quad + \left[\left(m_{11}m_{32}b_1 + m_{22}m_{31}b_1\right) - \left(m_{12}m_{31}b_2 + m_{32}m_{21}b_1\right)\right]k_4.
\end{aligned}
\tag{18}
$$

In order to calculate the coefficient k_2, k_3, k_4 one has to determine the (17) - equation's coefficient a, b, c using the Viette formulas:

$$\lambda_1 + \lambda_2 + \lambda_3 = -a, \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3 = b, \lambda_1\lambda_2\lambda_3 = -c, \tag{19}$$

where λ_i, $i = \overline{1,3}$ are imposed. Therefore, imposing the new eigenvalues:

$$\lambda_{1,2} = -0.15 \pm 0.15i, \lambda_3 = -0.15, \tag{20}$$

where $m_{i,j}, \overline{i,j} = \overline{1,3}$ are deduced from the (15)- equation's coefficient, identified with (3). It results the system's solution:

$$n_{11}k_2 + n_{12}k_3 + n_{13}k_4 = a', n_{21}k_2 + n_{22}k_3 + n_{23}k_4 = b', n_{31}k_2 + n_{32}k_3 + n_{33}k_4 = c', \tag{21}$$

where $n_{i,j}, \overline{i,j} = \overline{1,3}$ are the coefficient of $k_2, k_3, k_4 -$ arguments in (18) and a', b', c' are the terms in (18) which are not containing these arguments. For the imposed (20) eigenvalues one obtains $k_2 = 0.0645, k_3 = 1.3950, k_4 = 0.2387$.

Usually, MAV's command (stabilization) log law has the form:

$$\boldsymbol{u} = -K\boldsymbol{x}. \tag{22}$$

According to (10), this log law may be expressed as:

$$\boldsymbol{u} = \begin{bmatrix} \Delta\Phi \\ \Delta\alpha_1 \\ \Delta\overline{\phi} \\ \Delta\alpha_2 \end{bmatrix} = -K\boldsymbol{x} = \begin{bmatrix} 0 & k_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_2 & 0 & k_3 & k_4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V_x^* \\ \Delta V_z^* \\ \Delta\omega_y^* \\ \Delta\theta \end{bmatrix}. \tag{23}$$

Fig. 1 presents the structure of MAV's motion stabilization system.

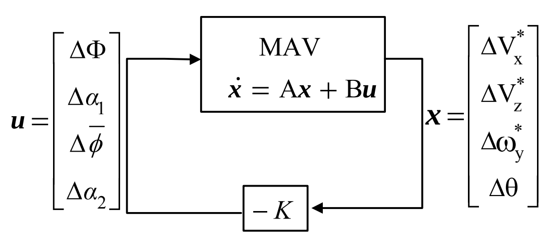


Fig. 1. Block diagram of system stabilization longitudinal movement of MAV

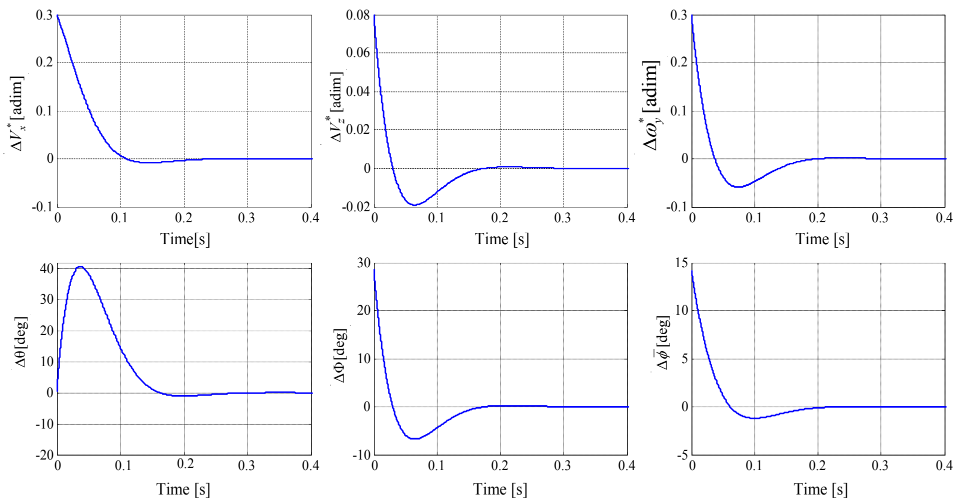


Fig. 2. Adimesionale time characteristics of the MAV, with variable control $\Delta\Phi$ and $\Delta\bar{\phi}$

Repositioning the eigenvalues in the left complex semi-plane, by introducing feedbacks after state variables $(\lambda_{1,2} = -0.15 \pm 0.15i, \lambda_3 = \lambda_4 = -0.15)$, it results: oscillating mode's period T_0^* decreases from 40 to 30 (flaps), the dumping coefficient is very good ($\xi = 0.707$) and the override is very small (4.3%); transient regime's duration (time) $t_r = 28\,\text{ms}$ (it takes approximately 23 flaps) and the half-life time $t_{1/2} = 28\,\text{ms}$ (it takes approximately 5 flaps). For the stable aperiodic eigen-modes (identical) $T_3 = T_4 = 4.17$ ms (it takes $T_3^* = T_4^* = 6.66 \cong 7$ flaps; $T_{3,4}^* = 1/|\lambda_{3,4}|$). The eigenvalues of the system presented in fig. 1 are $\lambda_{1,2} = -0.1365 \pm 0.1282i$, $\lambda_{3,4} = -0.1638 \pm 0.0479i$.

In fig. 2 there are plotted the characteristics $\Delta V_x^*(t), \Delta V_z^*(t), \Delta\omega_y^*(t)$, $\Delta\theta(t), \Delta\phi(t)$ and $\Delta\bar{\phi}(t)$ for the system in fig. 1, obtained by using Matlab, starting from initial conditions $\Delta V_x^*(0) = 0.3, \Delta V_z^* = 0.08, \Delta\omega_y^* = 0.3, \Delta\theta = 0.01\,\text{rad}$.

3.2. Control by the variables $\Delta\Phi$ and $\Delta\alpha_2$

If the control is realized by the variables $\Delta\Phi$ and $\Delta\alpha_2$, then

$$\Delta\Phi = k_1 \Delta V_z^*, \Delta\alpha_2 = k_2 \Delta V_x^* + k_3 \Delta\omega_y^* + k_4 \Delta\theta. \tag{24}$$

Equations (11) and (12) are still valid and $k_1 = 6.2$. Equations (13) and (15) will be modified, because the second term in the sum becomes

$$\begin{bmatrix} \frac{X^*_{\alpha_2}}{m^*} & \frac{M^*_{\alpha_2}}{J^*_{yy}} & 0 \end{bmatrix}^T \Delta\alpha_2 = \begin{bmatrix} b_1 & b_2 & 0 \end{bmatrix}^T \begin{bmatrix} k_2 & k_3 & k_4 \end{bmatrix} \begin{bmatrix} \Delta V^*_x & \Delta\omega^*_y & \Delta\theta \end{bmatrix}^T. \tag{25}$$

In (16) only the second term in the sum has modified, as follows:

$$\begin{bmatrix} b_1 & b_2 & 0 \end{bmatrix}^T \begin{bmatrix} k_2 & k_3 & k_4 \end{bmatrix} = \begin{bmatrix} -0.0230 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} k_2 & k_3 & k_4 \end{bmatrix}. \tag{26}$$

In the system (21) only b_1 and b_2 are modifying, resulting the values:

$$k_2 = 12.2783, k_3 = 7.8618, k_4 = -0.2445. \tag{27}$$

Equation (23) has also been modified (by interchanging the matrix's lines 3 and 4);

$$\boldsymbol{u} = \begin{bmatrix} \Delta\Phi \\ \Delta\alpha_1 \\ \Delta\overline{\phi} \\ \Delta\alpha_2 \end{bmatrix} = -K\boldsymbol{x} = \begin{bmatrix} 0 & k_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_2 & 0 & k_3 & k_4 \end{bmatrix} \begin{bmatrix} \Delta V^*_x \\ \Delta V^*_z \\ \Delta\omega^*_y \\ \Delta\theta \end{bmatrix}. \tag{28}$$

The eigenvalues of the matrix $\left(\widetilde{A} = A - BK\right)$ are not given by (20), because of the approximations in (11) and (24); a couple of the complex conjugated eigenvalues have positive real parts and the closed loop system in fig. 1 becomes unstable.

3.3. Control by the variables $\Delta\alpha_1$ and $\Delta\overline{\phi}$

With $\Delta\overline{\phi}$ given by (17) and $\Delta\alpha_1 = k_1 \Delta V^*_z$, it successively results:

$$\Delta\dot{V}^*_z = \frac{Z^*_w}{m^*}\Delta V^*_z + \frac{Z^*_{\alpha_1}}{m^*}\Delta\alpha_1 = 0 \cdot \Delta V^*_z - 0.04463\Delta\alpha_1, \tag{29}$$

$$\Delta\dot{V}^*_z = -\left(0 + 0.0453 k_1\right)\Delta V^*_z = -\lambda_4 \Delta V^*_z. \tag{30}$$

Choosing $\lambda_4 = -0.15$, it results $k_1 = 3.36$. Concerning the $\Delta\overline{\phi}$-variable, given by (10), the equations $(13) \div (21)$ still remain valid. The values for the k_2, k_3, k_4 coefficient are the same in 3.1. Consequently, the command law is

$$\boldsymbol{u} = \begin{bmatrix} \Delta\Phi \\ \Delta\alpha_1 \\ \Delta\overline{\phi} \\ \Delta\alpha_2 \end{bmatrix} = -K\boldsymbol{x} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & 0 \\ k_2 & 0 & k_3 & k_4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V^*_x \\ \Delta V^*_z \\ \Delta\omega^*_y \\ \Delta\theta \end{bmatrix}. \tag{31}$$

Similar to case 3.1, because of the assumed approximations, closed loop system's eigenvalues are a little different from the ones imposed in (20), but approximately the same as in the case 3.1: $\lambda_{1,2} = -0.1361 \pm 0.1280\, i, \lambda_{3,4} = -0.1639 \pm 0.0484\, i$.

In fig. 3 one presents the MAV's time characteristics, determined for the same initial conditions as in the before studied cases.

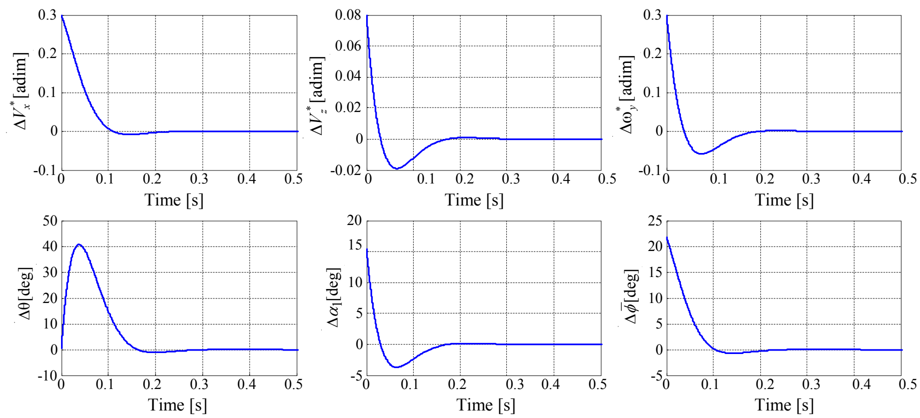


Fig. 3. Time characteristics of the MAV, controlled by variables $\Delta\alpha_1$ and $\Delta\overline{\phi}$

3.4. Control by the variables $\Delta\alpha_1$ and $\Delta\alpha_2$

The command variables $\Delta\alpha_1$ and $\Delta\alpha_2$ are chosen as:

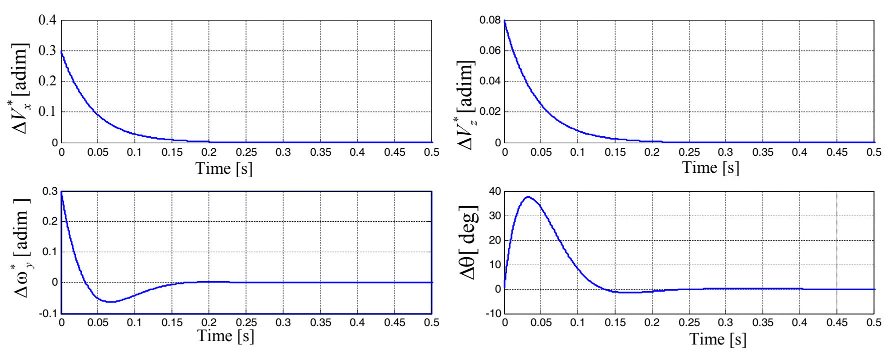
$$\Delta\alpha_1 = k_1 \Delta V_z^*, \Delta\alpha_2 = k_2 \Delta V_x^* + k_3 \Delta\omega_y^* + k_4 \Delta\theta. \quad (32)$$

Concerning the variable $\Delta\alpha_1$, the equations (29) and (30) are still valid; $k_1 = 3.36$. The values of the coefficients k_2, k_3, k_4 are the same as the ones in the case 3.2. The closed loop system becomes unstable.

3.5. Control by all the command variables

Matrix K is chosen to obtain the same eigenvalues as in (20) for the closed loop system matrix $\tilde{A} = (A - BK)$. It results matrix K and the characteristics in fig.4.

$$K = \begin{bmatrix} 0.3886 & -1.5029 & 0.2243 & 0.0208 \\ 0.7402 & -2.7629 & 0.4057 & 0.0325 \\ -1.0265 & 0.2619 & -0.6285 & -0.0522 \\ -2.4798 & -0.2399 & 0.8190 & 0.6004 \end{bmatrix}. \quad (33)$$



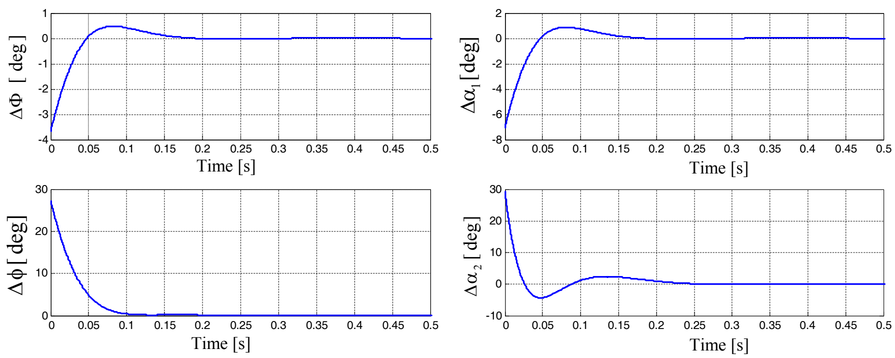


Fig. 4. Time characteristics of the MAV, controlled by variables $\Delta\Phi, \Delta\alpha_1, \Delta\bar{\phi}$ and $\Delta\alpha_2$

3.6. Liniar quadratic otimal control (LQR)

The command law has the form as in [12]:

$$\boldsymbol{u} = -K\boldsymbol{x}, K = R^{-1}B^T P, \tag{34}$$

where P is the solution of the algebraic Riccati matrix equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0; Q = 0.3\,I\,; \; R{=}0.01\,I, \tag{35}$$

with I is the unitary matrix (4x4).

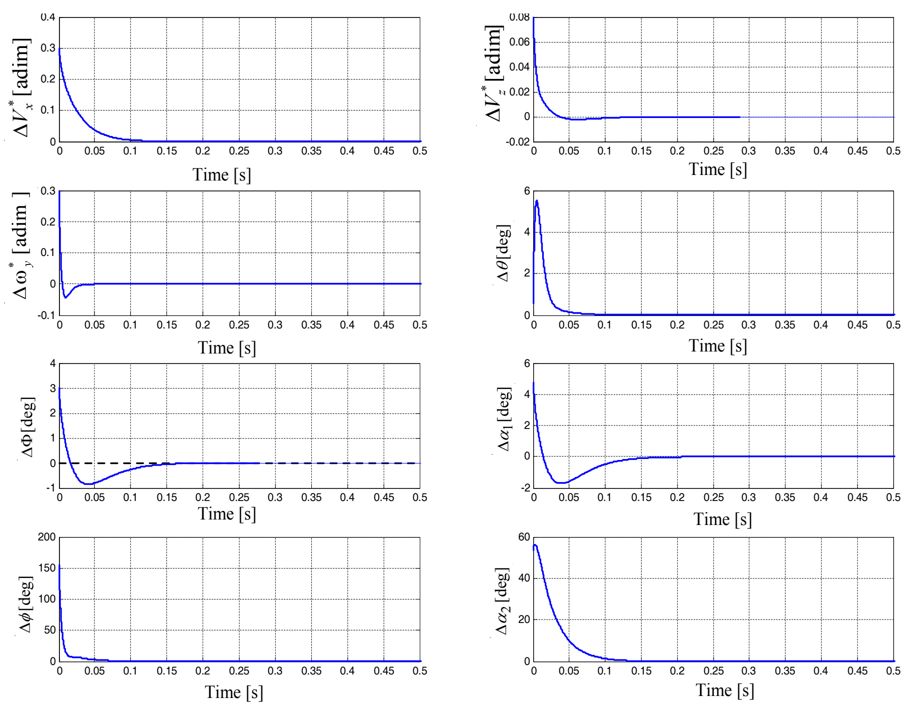


Fig. 5. Time characteristics of the MAV, the linear quadratic optimal control

$$K = \begin{bmatrix} 0.2419 & -2.7154 & 0.3053 & 0.0713 \\ 0.4925 & -4.9937 & 0.5572 & 0.1293 \\ -1.1169 & 0.0750 & -7.7387 & -5.3936 \\ -4.7642 & -0.2056 & 1.6588 & 1.0322 \end{bmatrix}. \quad (36)$$

Fig. 5 presents the time characteristics of the MAV by using the LQR control.

4. Conclusions

In this paper one studies the controllability of a system, represented by the insect-type MAV longitudinal motion model; one builds the modal variables state equation and establishes the variables' combinations options which could control the system. For each one of these possible combinations, the matrix K for the system's stabilizing was designed, by repositioning system's matrix eigenvalues, respectively for the linear quadratic optimal control. The quality of the system's dynamical regimes is emphasized by the time characteristics of the state variables and command variables, for all command variables combination options (see fig. 2, fig. 3, fig. 4, fig. 5).

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