U.P.B. Sci. Bull., Series D, Vol. 83, Iss. 4, 2021 ISSN 1454-2358

RESEARCH ON EXTREME-SHORT-TERM MOTION RESPONSE PREDICTION OF SHIPS BASED ON VOLTERRA SERIES MODEL

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Prediction of the ship's motion is of great significance to improve ship's safety and operating efficiency. Aiming at the nonlinear and chaotic characteristics of ship motion attitude under random waves, the Volterra adaptive prediction model was proposed by using the nonlinear characterization ability of Volterra series. The least square algorithm (RLS), Kalman filtering algorithm and NLMS algorithm are used to estimate the vortex core of nonlinear Volterra model. By comparing calculation results with the ship model experiment results, it can be seen that the three algorithm are consistent with the experimental results, while the least squares method has higher accuracy, which provides a solid foundation for the data analysis of extreme-short-term ship motion.

Keywords: Ship motion response prediction, Extreme-short-term, *Volterra* series, *least square* algorithm, *Kalman* filtering algorithm, *NLMS* algorithm

1. Introduction

Extreme-short-term ship motion prediction has always been the focus and difficulty of ship motion prediction. With wide application of computers, system identification technology as a relatively advanced method, can effectively identify the ship motion response [1-3]. Many scholars apply it on ship motion response identification. Researchers mainly apply wave time history and ship motion time history to build a mathematical response model. By carrying out parameter identification, they can obtain an estimation model which is equivalent to the original system. And the mathematical model of the system to be identified should be established first. In many cases, parameters of the model change dynamically, and determination of parameters is an important step in the system identification [4].

At present, the genetic algorithm [5], least square method [6], *Kalman* filtering method [7], neural network method [8] are mature parameter identification methods, among which the least square method is the most classic.

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The *AR* model as a signal identification model has a practical value in system identification by combining the least squares method with the *Kalman* filter method [9]. And it has been used in system identification in Korea.

Professor Sung from Seoul University [10] identified a ship motion response model by combining the ship motion response data with the least square method. In the study of ship maneuverability, Professor Yoon [11] used the system identification method to identify the ship's motion response model with the data from the rotary test, and finally obtained a method to determine the ship's motion response by comparing it with the theoretical calculation.

In the study of underwater vehicle movement, Professor Mahfouz [12] from Canada used the neural network method to study the parameter identification of mathematical movement model, and obtained good results. Yumori [13] once used AR model to fit real-time wave data and motion data, and finally used the measured waves to predict the ship's motion response. Lin [14] adopts a multi-parameter model through optimization of the *AR* model, and applies a filter to predict the motion response of actual wave.

With the development of control theory, researches on ship motion response go deeper, and nonlinear system for predicting ship motion response was proposed. Billiings [15] and other scholars have proposed a nonlinear model that can better describe the real situation of ship movement and contain more motion information. Therefore, the *NARMAX* nonlinear model is proposed and the ship's motion response is measured more accurately by combining relevant theoretical research and experimental data.

In this paper, aiming at the nonlinear and chaotic characteristics of ship motion attitude under the action of random waves, the *Volterra* adaptive prediction model was proposed by using the nonlinear characterization ability of *Volterra* series [16]. The least square algorithm (*RLS*), *Kalman filtering* algorithm and *NLMS* algorithm are used to estimate the vortex core of nonlinear *Volterra* model. By comparing the accuracy of motion response under different models, a appropriate calculation model was chosen to establish extreme-short-term movement response model of vessels, which lay a foundation for later researches.

2. Ship motion response model based on *Volterra* series model

The *Volterra* series is often used to describe various nonlinear systems. For ships in actual marine environment, the nonlinear system stimulated by wave is modeled using the *Volterra* series model, combined with the least square method for model parameter estimation, and an extreme-short-term motion response prediction model of the ship is established.

According to amounts of theoretical research and practical experience, *Volterra* series model can contain more signal information for the description of nonlinear systems. For input signal

$x(t)(t=1,2,\ldots,N)$, $X(t)=\left[x(t),x(t-1),\ldots,x(t-N+1)\right]$, the output signal is $\hat{x}(t+1)$, and the nonlinear system can be processed as equation (1) [17].

$$\hat{x}(t+1)=F(x(t))=h_0+\sum_{m=0}^{+\infty}h_m(t)x(t-m)+\sum_{m_1=0}^{+\infty}\sum_{m_2=0}^{+\infty}h_{m_1,m_2}(t)x(t-m_1)x(t-m_2)+\ldots+$$
$$\sum_{m_1=0}^{+\infty}\sum_{m_2=0}^{+\infty}\cdots\sum_{m_p=0}^{+\infty}h_{m_1,m_2,\cdots,m_p}(t)\times x(t-m_1)x(t-m_2)\ldots x(t-m_p)+\ldots \tag{1}$$

The adaptive prediction model established in this paper is mainly based on expression of *2-order Volterra* series. A model that can predict subsequent data based on the previous data. By solving the potential relationship between the two, the prediction is established. For input $x(t)(t=1,2,\ldots,N)$, we need to seek the following relationship as equation (2).

$$x(t+1)=f(x(t),x(t-1),\cdots,x(t-m))+\varepsilon_t\,, \tag{2}$$

Theoretically speaking, when the data is more, the corresponding error will be smaller, and it can better reflect the relationship between output and input. When a model is completely determined, ε_t should become smaller and smaller as *n* increases, and it tends to 0 when it increases to a certain extent. This is because when *n* increases, no additional error is introduced. Assuming $n= m_{\min}$ when almost no additional error increases with the data increases, then we can use the previous data information as a later data estimate, the express of *Volterra* series model as equation (3).

$$x(t+1)=f(x(t),x(t-1),\cdots,x(t-m_{\min})) \tag{3}$$

Vector expansion of second-order expression as equation (4).

$$U(t)=[1,x(t),x(t-1),\cdots,x(t-m+1),x^2(t),x(t)x(t-1),\cdots,x^2(t-m+1)]^T \tag{4}$$

The coefficient vector as equation (5).

$$H(t)=[h_0,h_0(t),h_1(t),\cdots,h_{0,0}(t),h_{0,1}(t),\cdots,h_{m-1,m-1}(t)]^T \tag{5}$$

Then the total number of coefficients after expansion as equation (6).

$$p=1+m+m(m+1)/2 \tag{6}$$

From above, $\hat{x}(t+1)$ can expressed as equation (7).

$$\hat{x}(t+1)=H^T(t)U(t) \tag{7}$$

Using the *Volterra series* model to build a prediction model, the *vortex core should be* estimated first.

Assuming $\hat{H}(t)=[\hat{h}_0,\hat{h}_o(t),\hat{h}_1(t),\cdots,\hat{h}_{0,0}(t),\hat{h}_{0,1}(t),\cdots,\ \hat{h}_{m-1,m-1}(t)]^T$ are the core estimation for a piece of input data, then prediction of the future l step are as equations (8), (9) and (10) [18].

$$\hat{x}(t+l)=\hat{h}_0+\sum_{m_1=0}^{m-1}\hat{h}_{m_1}(t)x(t-m)+\sum_{m_1=0}^{m-1}\sum_{m_2=0}^{m-1}\hat{h}_{m_1,m_2}(t)x(t-m_1)x(t-m_2) \quad \text{while } l=1 \tag{8}$$

$$\hat{x}(n+l)=\hat{H}^T(n)U_1(n) \qquad \text{while } 1<l\le m \tag{9}$$

$$\hat{x}(t+l)=\hat{H}(t+1)U_2(t) \qquad \text{while } l>m \tag{10}$$

So the prediction model is constituted.

3. The vortex core estimation based on 3 different algorithm

3.1 The vortex core estimation based on *Least square* algorithm

Least square algorithm has great advantages in *Volterra* vortex core estimation. It can be used to determine the core and related parameters at the fastest speed. The functions used for parameter calculation are as equation [19] (11)

$$\xi^d(t)=\sum_{i=0}^{t}\lambda^{t-i}e^2(i)=\sum_{i=0}^{t}\lambda^{t-i}[d(t)-U^T(i)H(t)]^2 \tag{11}$$

When matrix inputs as equation (12), the outputs are as in equation (13),

$$P(t)=[U(m),U(m+1),\cdots,U(t-1)]^T \tag{12}$$

$$Y(t)=[y(m),y(m+1),\cdots,y(t-1)]^T \tag{13}$$

Then the core estimation of *Volterra* can be calculated in equation (14).

$$\hat{H}(t)=[P^T(t)P(t)]^{-1}P^T(t)Y(t) \tag{14}$$

In the process of identification, when the data input continuously, $P(t)$ will change continuously. With the calculation amount increases quickly, timeliness of identification is affected. Therefore, *Least square* algorithm can be selected for identification using to reduce the data processing amount [20].

The system input matrix is $P(t+1)$ in equation (15) at time $t+1$, and corresponding output vectors are $Y(t+1)$ in equation (16).

$$P(t+1)=[U(m),U(m+1),\cdots,U(t+1)]^T \tag{15}$$

$$Y(t+1)=[y(m),y(m+1),\cdots,y(t+1)]^T \tag{16}$$

Then the $\Phi(t)$ can be expressed in equation (17).

$$\Phi_t = [P^T(t)P(t)]^{-1} \tag{17}$$

The core of *Volterra* series can be further calculated. We assume that $t = m+1, \cdots, N$, N is the number of sample data, then $\hat{H}(t+1)$, $K(t+1)$ and $\Phi(t+1)$ can be calculated in equation [21] (18) (19) and (20).

$$\hat{H}(t+1) = \hat{H}(t) + K(t+1)[x(t+1) - U^T(t+1)\hat{H}(t)] \tag{18}$$

$$K(t+1) = \frac{\Phi_t U(t+1)}{1 + U^T(t+1)\Phi_t U(t+1)} \tag{19}$$

$$\Phi_{t+1} = \Phi_t - K(t+1)U^T(t+1)\Phi_t \tag{20}$$

3.2 The vortex core estimation based on *Kalman filter* algorithm

When using *Kalman* filter algorithm to estimate the core of *Volterra* series, the parameter estimation process is regarded as a non-stationary process and introduced process noise *v*(*n*) for model parameters change with time. Assume that state equation is a random walk model shown in equation (21).

$$H(t+1) = H(t) + v(t) \tag{21}$$

As $\hat{H}(t) = \hat{H}(t \mid x_t)$ is the $p \times 1$ vector estimation of *U(t)* at time. It can be obtained from the recursive formula of *Kalman* algorithm shown in equation (22) and (23).

$$\hat{H}(t) = \hat{H}(t-1) + g(t)\alpha(t) \tag{22}$$

$$g(t) = K(t-1)U^T(t)[U^T(t)K(t-1)U(t) - \sigma_\varepsilon^2]^{-1} \tag{23}$$

The core estimation of *Volterra* series based on *Kalman* filter algorithm deduced as shown in equation (24) (25) and (26).

$$\alpha(t) = x(t) - U^T(t)\hat{H}(t-1) \tag{24}$$

$$K(t) = K(t, t-1) - g(t)U^T(t)K(t, t-1) \tag{25}$$

$$K(t+1, t) = K(t) + Q(t) = K(t) + qI \tag{26}$$

3.3 The core estimation based on *NLMS* algorithm

The standard method for deriving *NLMS* algorithm is to use estimated value of the mean square error (*MSE*), which is defined in equation [22] (27).

$$F[e(t)] = \xi(t) = E[e^2(t)] = E[d^2(t) - 2d(t)y(t) + y^2(t)] \tag{27}$$

The instantaneous square error is defined in equation (28).

$$e^2(t) = d^2(t) - 2d(t)y(t) + y^2(t) \tag{28}$$

Generally speaking, in order to control the imbalance, a fixed convergence factor μ is introduced in the update formula. In addition, in order to avoid large steps with small $U^T(t)U(t)$, a parameter γ is also introduced. Then the equation is updated in equation (29).

$$H(t+1) = H(t) + \frac{\mu}{\gamma + U^T(t)U(t)} e(t)U(t) \tag{29}$$

At the same time, in order to control the offset, we adopt different convergence factors to the first and second order terms of the *NLMS Volterra* filter. In this condition, the core estimation equations are updated in equation (30) and (31).

$$h_m(t+1) = h_m(t) + \frac{\mu_1}{\gamma + U^T(t)U(t)} e(t)x(t-m) \tag{30}$$

$$h_{m_1,m_2}(t+1) = h_{m_1,m_2}(t) + \frac{\mu_2}{\gamma + U^T(t)U(t)} e(t)x(t-m_1)x(t-m_2) \tag{31}$$

3. Comparison of vortex core forecast results of *Volterra* series model based on 3 different algorithm

These three forecasting algorithms are used to predict extreme-short-term motion based on time history of pitching and rolling motions of a ship under different working conditions. The comparison curve between the model prediction results and the measured values is given. The *VR* represents the *least squares* algorithm, the *VK* represents the *Kalman filtering* algorithm, while the *VNLMS* represents *Volterra core* estimation model based on *NLMS* algorithm. There are 200 modeling data and the sampling period is 0.5274s, which correlated to the random ship's motions average period.

The main dimensions of ship model are shown in Table 1.

Table 1

Main dimensions of ship model

Main dimension	Real ship	Ship model
Scale ratio	1:1	1:20
Loa (m)	400	20
B(m)	58.6	2.93
D (m)	33.5	1.76
Lop (m)	386	19.3
T(m)	16	0.8
Displacement(t)	259628	32.45
LCG(m)	192.391	9.62
VCG(m)	24.258	1.21

And the wave elongation data is shown in Fig.1, corresponding to the 5 Beaufort Sea-State condition.

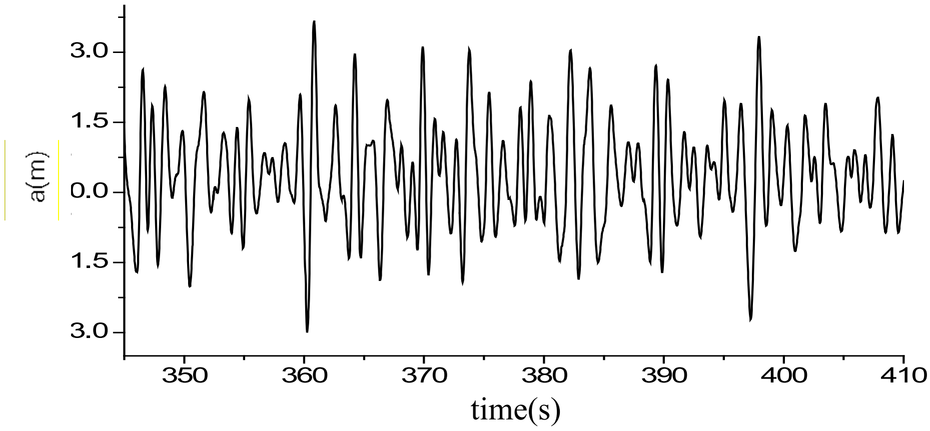
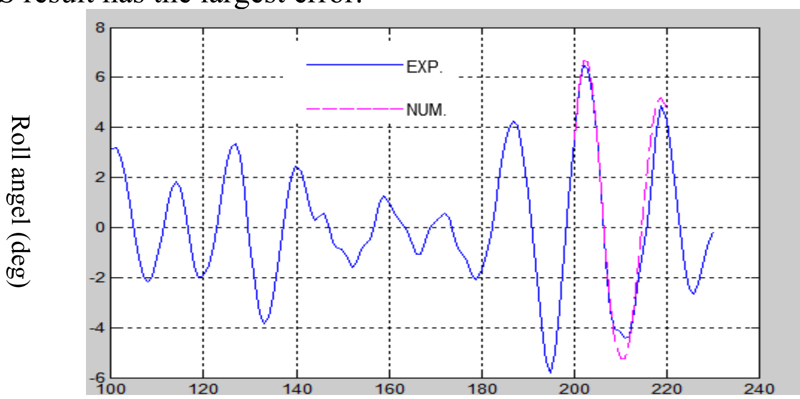


Fig. 1. The wave height data

The extreme-short-term prediction results of roll angles and pitch angles based on three algorithms under different sea conditions are calculated. In order to intuitive analysis accuracy of trend prediction, prediction results are compared with ship model tests using curve graph.

The curve comparison between prediction results of roll angle with ship model test under beam waves, ship speed 0kn, sea condition level 5 are shown in Fig. 2-4.

By comparing the three numerical algorithms with experimental results, it can be seen that all the results of the three algorithms are consistent with the trend of ship motion experiment results. The *VR* result has the smallest error and the *VNLMS* result has the largest error.



Time steps (time interval is 0.5274s)

Fig. 2. Comparison of extreme-short-term predicted results of *VR* with experimental values

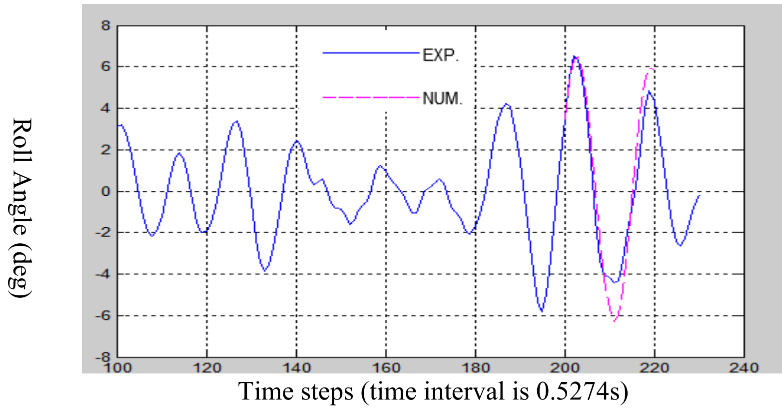


Fig. 3. Comparison of extreme-short-term predicted results of *VK* with experimental values

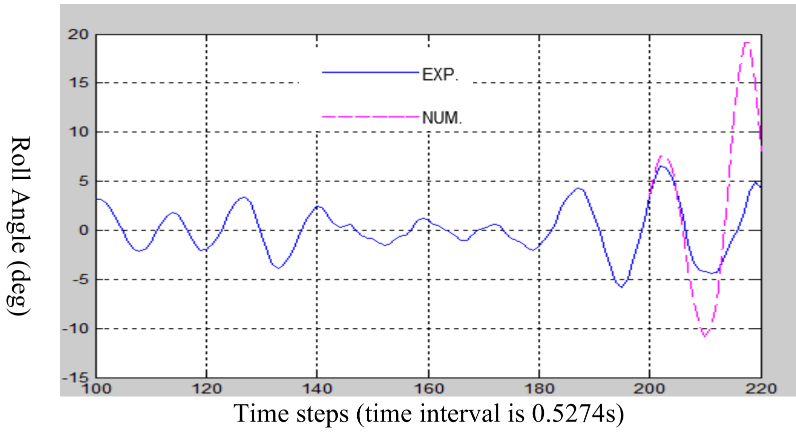


Fig. 4. Comparison of extreme-short-term predicted results of *VNLMS* with experimental values

As can be seen from the comparison between Figure 2 and Figure 4, the simulation results obtained by *VR* and *VK* algorithms are in good agreement with the experimental results. At the beginning of the simulation, *VNLMS* algorithm has good consistency with the experimental results, with the development of time, the prediction of rolling Angle has serious distortion. It indicates that the mathematical model of ship's roll movement is not accurate enough, leading to the recursive evolution of ship's inclination, the previous deviation is constantly magnified, and the final simulation results are distorted.

Further, Errors under different algorithms are calculated and compared to test results to estimate the accuracy of different algorithms.

The relative error comparison of roll angle under working condition beam waves, ship speed 0kn, sea condition level 5 are shown in Tab. 2.

Table 2

Relative error comparison of 3 different algorithms and ship model tests

Time (s)	Experimental value (deg)	Predicted value by *VR* (deg)	error	Predicted value by *VK* (deg)	error	Predicted value by *VNLMS* (deg)	error
202.45	5.52	5.55	0.57%	5.30	3.91%	6.19	12.05%
202.98	6.52	6.68	2.40%	6.33	2.98%	7.56	15.97%
203.51	6.33	6.63	4.80%	6.48	2.39%	7.63	20.54%
204.04	5.35	5.59	4.59%	5.78	8.00%	6.23	16.53%
206.67	-4.05	-4.50	11.1%	-4.29	5.83%	-4.31	6.44%
207.20	-4.19	-5.22	24.7%	-5.70	36.1%	-8.48	102.4%
207.73	-4.42	-5.23	18.1%	-6.29	42.1%	-13.07	195.8%
208.26	-4.33	-4.48	3.39%	-5.96	37.7%	-9.49	119.2%
211.95	4.85	5.17	6.66%	5.94	22.4%	11.69	141.1%
212.48	4.30	4.72	9.82%	5.82	35.2%	11.09	157.9%

The error analysis shows that under zero ship speed, the maximum error of the *VR* result calculation is 9.82%, the maximum error of *VK* is 42.1%, and the maximum error of *VNLMS* is 195.8%. The error analysis and comparison can also prove that the *VR* calculation results have the highest accuracy.

The curve comparison between prediction results of roll angle with ship model test under beam waves, ship speed 12kn, sea condition level 5 are shown in Fig. 5-7.

Through comparison, it can be seen that as speed increases, the stability of *VR* result is still good, and the calculation result of *VK* has the largest error both in periodicity and calculation accuracy.

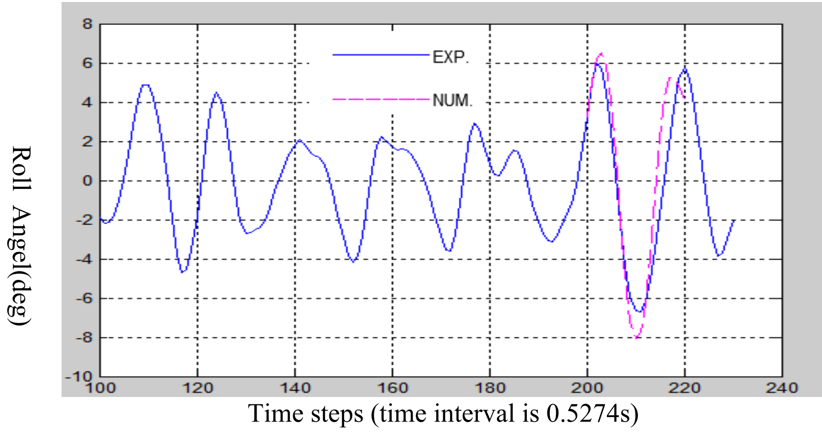


Fig. 5. Comparison of extreme-short-term predicted results of *VR* with experimental values

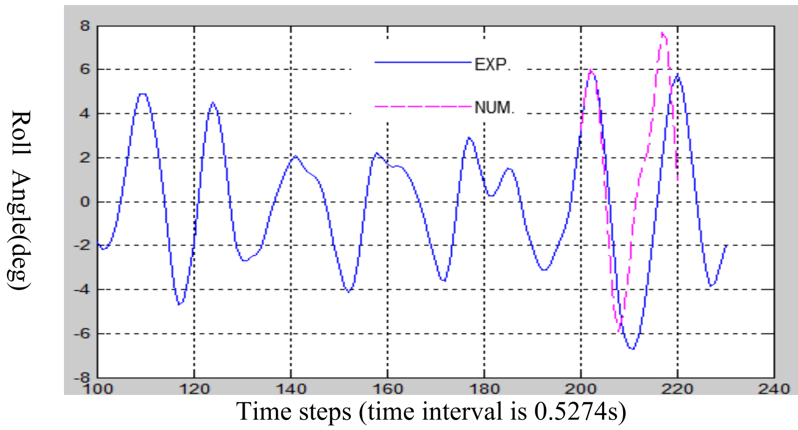


Fig. 6. Comparison of extreme-short-term predicted results of *VK* with experimental values

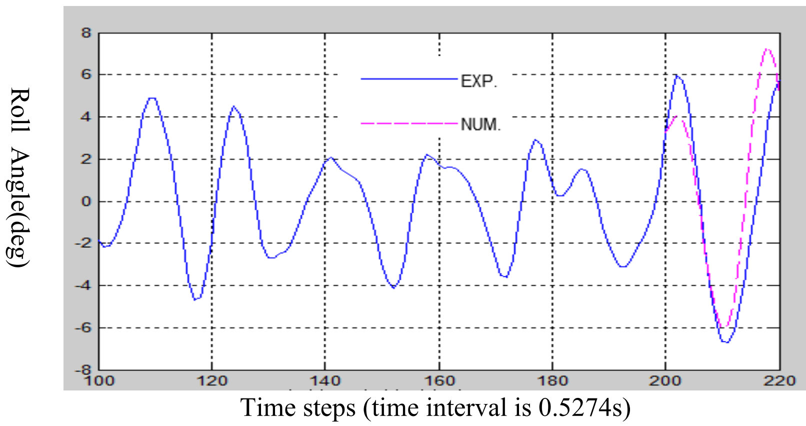


Fig. 7. Comparison of extreme-short-term predicted results of *VNLMS* with experimental values

Further, Errors under different algorithms are calculated and compared to test results to estimate the accuracy of different algorithms. The relative error comparison of roll angle under working condition beam waves, ship speed 12 kn, sea condition level 5 are shown in Tab. 3.

Table 3

Relative error comparison of 3 different algorithms and ship model tests (roll angle)

Time(s)	Experimental value (deg)	Predicted value by *VR*(deg)	error	Predicted value by *VK*(deg)	error	Predicted value by *VNLMS*/(deg)	error
203.03	5.02	5.07	1.06%	5.04	0.40%	3.70	26.38%
203.56	5.95	6.24	4.83%	6.01	1.00%	4.11	30.91%
204.08	5.73	6.50	13.42%	5.71	0.23%	3.85	32.71%
204.61	4.49	5.79	28.99%	4.05	9.79%	2.93	34.61%
206.72	-4.36	-5.04	15.43%	-5.92	35.59%	1.46	42.91%
207.25	-5.91	-7.17	21.25%	-5.16	12.74%	-0.38	259.83%
207.78	-6.67	-8.00	20.93%	-3.00	54.98%	-2.29	4.63%
208.30	-6.72	-7.75	15.33%	-0.62	90.70%	-4.08	6.47%
208.83	-6.17	-6.29	2.01%	0.95	115.4%	-5.43	8.04%
209.36	-5.07	-3.85	24.16%	1.63	132.2%	-6.09	8.65%
212.15	5.179	4.83	6.72%	4.43	6.03%	5.83	12.3%
212.68	5.77	4.11	28.74%	4.56	84.14%	6.12	6.06%

It can be seen that, as ship speed increases, the error of the forecast results increases. The maximum error of the *VR* result calculation is 28.99%, the maximum error of *VK* is 132.2%, and the maximum error of *VNLMS* is 259.83%. At the same time, both the maximum error and the error variance show that the calculation results of VR are better than the other two algorithms.

Also, the roll angle comparison between 0kn and 12kn are shown in Fig 8. Which shows under beam sea condition, the ship's speed has a reduced influence on the roll motion, so that the maximum roll angles are expected in the same range as in the case of the ship with 12 kn speed.

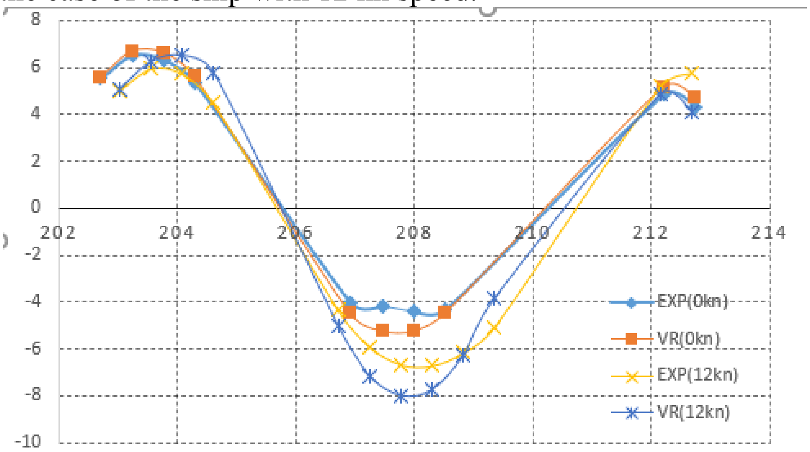
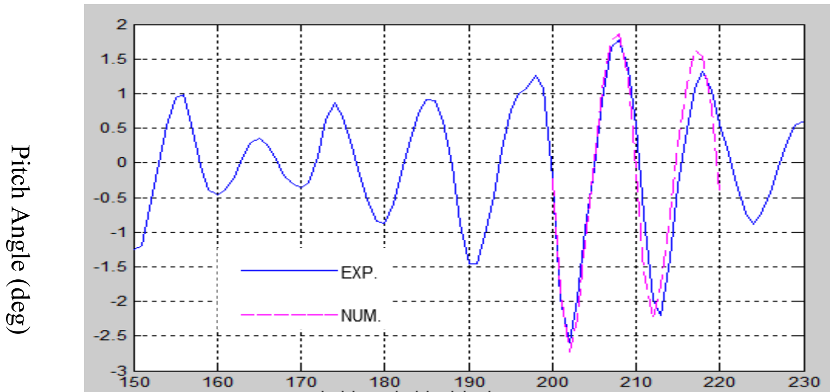


Fig. 8. Roll angle comparison between 0kn and 12kn

The curve comparison between prediction results of pitch angle with ship test under bow waves, ship speed 12kn, sea condition level 5 are shown in Fig. 9-11.

By comparing the forecast results with ship model test results, it can be seen that the forecast results of the *VK* and *VNLMS* algorithms have some time delays compared with the test results. Therefore, the *VR* algorithm performs better than the other two in terms of trend consistency and accuracy.



Time steps (time interval is 0.5274s)

Fig. 9. Comparison of extreme-short-term predicted results of *VR* with experimental values

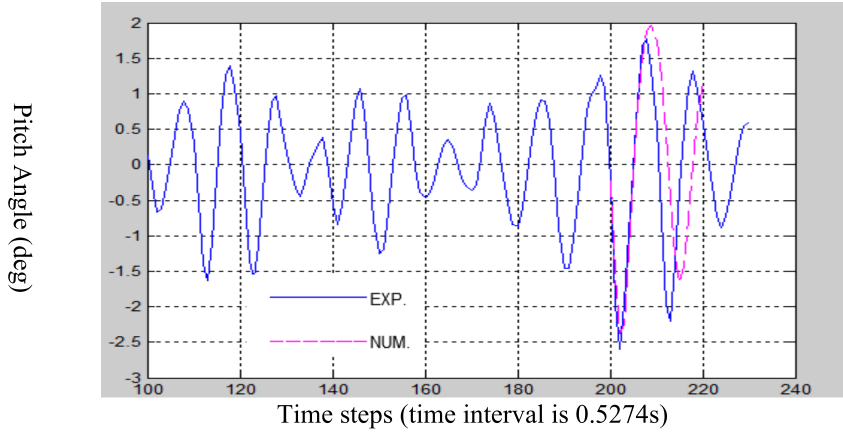


Fig. 10. Comparison of extreme-short-term predicted results of *VK* with experimental values

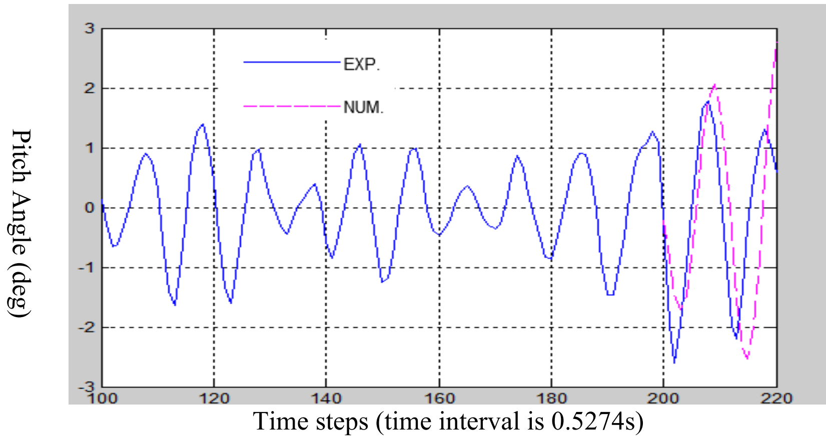


Fig. 11. Comparison of extreme-short-term predicted results of *VNLMS* with experimental values

Table 4

Relative error comparison of 3 different algorithms and ship model tests (pitch angle)

Time (s)	Experi-mental value (deg)	Predicted value by VR (deg)	error	Predicted value by VK (deg)	error	Predicted value by VNLMS/(deg)	error
200.91	-1.94	-1.85	4.60%	-1.55	20.09%	-0.84	18.82%
201.96	-2.60	-2.73	4.88%	-2.38	8.59%	-1.45	15.32%
204.60	-1.95	-2.24	14.90%	-2.24	14.87%	-1.70	6.81%

207.24	1.67	1.75	5.28%	1.50	9.88%	1.09	13.92%
209.87	1.78	1.86	4.94%	1.88	6.06%	1.80	20.21%
215.15	-1.97	-2.25	14.08%	0.43	122.19%	-0.36	14.09%
216.73	-2.21	-1.76	20.25%	-0.49	77.64%	-1.55	35.45%

It can be seen that, as ship speed increases, the error of the forecast results increases. The maximum error of the *VR* result calculation is 20.25%, the maximum error of *VK* is 122.19%, and the maximum error of *VNLMS* is 35.45%. On the whole， both the maximum error and the error variance show that the calculation results of *VR* are better than the other two algorithms.

4. Conclusions

It can be seen from the prediction results that,

(1) In terms of trends, whether it is rolling or pitching motions, the 3 algorithms are consistent with the ship model motion trends for extremely-short-term simulations. And the *Least square* algorithm shows better consistency than other algorithms which have certain delay.

(2) Neither the heading wave state of ship nor the ship's speed are the main factors that affect the accuracy of the prediction. The algorithm is the main reason that affects the accuracy of the forecast.

(3) The error analysis shows that under the same sea conditions and the same speed, the maximum error predicted by The *Least square* algorithm can be controlled within 30%, while the errors of the other two algorithms exceed 100% under extreme conditions, and the *NLMS* algorithm even exceeds 200 %.

Extreme-short-term ship motion prediction has always been the focus and difficulty of ship motion prediction. For a long time, researchers have used the nonlinear characterization ability of *Volterra* series to simulate the nonlinear and chaotic characteristics of ship motion under random ocean waves. The *RLS* algorithm, Kalman filtering algorithm and *NLMS* algorithm are main algorithms to estimate the vortex core of *Volterra* series model. In this paper, all the three algorithms are compared, and the *RLS* algorithm has good stability and high accuracy in prediction. In the later stage, this method will be used to predict the extreme-short-term motion response of large-scale ship model.

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