ANALYSIS OF PLATE FIN SURFACE TYPES PERFORMANCE BASED ON THE SECOND LAW OF THERMODYNAMICS. A CASE STUDY

Adina GHEORGHIAN[1], Alexandru DOBROVICESCU[2]

Any heat transfer augmentation technique modifies the irreversibility of the process to the extent to which the positive effect can be cut back by exergy loss. The paper is based on an analysis technique developed by Bejan [1], which quantifies the entropy generation rate induced by an augmentation technique applied to a passage compared to the entropy generation rate in the original passage. The method proposed in this paper can be regarded as an extension of that described by Bejan [1], being applied to different types of heat transfer surfaces, as opposed to Bejan's approach, which compares the augmented and un-augmented cases.

Keywords: Entropy generation; Second Law of Thermodynamics; Plate fin heat transfer surfaces

1. Introduction

Heat transfer augmentation techniques aim to enhance the thermal performance of a heat transfer device by increasing the overall heat transfer coefficient compared to the initial value for the original heat transfer surface. Usually, any heat transfer augmentation technique modifies the flow characteristics increasing rather than decreasing the power required for pumping. It is therefore essential to consider both effects (enhancement of heat transfer coefficient – desirable effect and supplementary head loss – undesirable effect) when assessing a certain heat transfer augmentation technique. Various comparison criteria were proposed [2, 3], but it is commonly agreed that it is difficult to develop a universally valid comparison methodology.

Design of heat transfer devices must consider both heat exchange between fluids and mechanical work required for pumping fluids. Heat exchange device analysis based on the Second Law of Thermodynamics allows integration of these two factors. Second Law analysis was proposed by Bejan [4] as an assessment method for heat transfer augmentation techniques. Sahiti et al [5] optimized a double-pipe pin fin heat exchanger by entropy generation minimization method. It was found that an optimal region for Re number exists, which results in the lowest

[1] PhD Student, University POLITEHNICA of Bucharest, Romania, e-mail: adina_gheorghian@yahoo.com
[2] Prof., University POLITEHNICA of Bucharest, Romania, e-mail:adobrovivescu@yahoo.com

value of the entropy generation number. Taufiq et al [6] identified the geometry of a radial fin array rectangular profile by minimizing entropy generation rate. It was also found that increasing the cross flow fluid velocity increased the heat transfer rate and reduced heat transfer irreversibility. Naphon [7] investigated theoretically and experimentally entropy generation due to heat transfer and fluid flow in a concentric tube heat exchanger. A one-dimensional model based on First Law and heat transfer equation was developed and the effects of heat exchangers parameters on entropy generation and exergy loss were considered. Ylmaz et al [8] reviewed the heat exchanger performance evaluation criteria based on Second Law analysis. Petrescu et al [9] applied the entropy generation analysis to a Carnot refrigeration machine, accounting for all irreversibility sources. Dobrovicescu et al [10] applied exergy analysis for optimization of temperature difference in a cryogenic heat exchanger.

Irreversibility associated with the heat transfer augmentation technique is quantified by means of entropy generation rate. Volumetric rate of entropy generation is given by [1]:

$$\dot{S}'''_{gen} = \frac{k}{T}(\nabla T)^2 + \frac{\mu}{T}\Phi \quad (1)$$

with k – thermal conductivity; μ – dynamic viscosity; Φ – viscous dissipation;

For a flow passage of length dx, hydraulic diameter D, and the flow cross-sectional area A, heat transfer per unit length q' and mass flow rate $\dot{m}$, the entropy generation rate per unit length can be expressed as [1]:

$$\dot{S}'_{gen} = \frac{q'\Delta T}{T^2} + \frac{\dot{m}}{\rho T}\left(-\frac{dp}{dx}\right) \quad (2)$$

The first term $\dot{S}'_{\Delta T} = \frac{q'\Delta T}{T^2}$ represents contribution to total irreversibility due to heat transfer while the second term $\dot{S}'_{\Delta p} = \frac{\dot{m}}{\rho T}\left(-\frac{dp}{dx}\right)$ represents the irreversibility caused by friction. Defining irreversibility distribution ratio as $\phi = \frac{\dot{S}'_{\Delta p}}{\dot{S}'_{\Delta T}}$, Eq. (2) becomes:

$$\dot{S}'_{gen} = \dot{S}'_{\Delta T}(1 + \phi) \quad (3)$$

Bejan [1] defined the augmentation entropy generation number as the ratio between entropy generation rate in the augmented passage $\dot{S}'_{gen,a}$ and entropy generation rate in the original flow passage $\dot{S}'_{gen,0}$:

$$N_{S,a} = \frac{\dot{S}'_{gen,a}}{\dot{S}'_{gen,0}} \quad (4)$$

Keeping $\dot{m}$ and q' constant the entropy generation number can be recast as:

$$N_{S,a} = \frac{N_T + \phi_0 N_P}{1 + \phi_0} \quad (5)$$

where [1]:

$$N_T = \frac{(St)_0}{(St)_a}\frac{D_a}{D_0} \quad (6) \qquad \text{and} \qquad N_P = \frac{f_a}{f_0}\frac{D_0}{D_a}\left(\frac{A_0}{A_a}\right) \quad (7)$$

where f is the friction factor.

Irreversibility distribution ratio for the reference passage (subscript *0*) can be cast in the form [1]:

$$\phi_0 = \left(\frac{T}{\Delta T}\right)_0^2 \left(\frac{w^2}{c_p T}\right)_0 \frac{f_0/2}{(St)_0} \quad (8)$$

It is essential to note (see Eq. 5) that assessment of a heat transfer augmentation technique requires the value of the irreversibility distribution ratio ϕ_0, which can only be calculated if the flow passage (geometry and dimensions) and flow conditions (fluid nature - c_p and velocity w) are given. An entropy generation number value greater than one means that the heat transfer augmentation technique (for the given ϕ_0) will increase the entropy generation compared to the un-augmented case. Analyzing N_T and N_P will provide further information concerning the causes of irreversibility.

The analysis technique described above will be applied to a number of various plate fin heat transfer surfaces types described in detail in [11]. Five groups of heat transfer surfaces will be analyzed. The comparison will be carried out taking as reference one surface type in each group, all other types in the same group being compared to the reference type, thereby allowing identification of the surface type with the lowest value of the entropy generation rate.

2. Analysis method

Heat transfer surfaces analyzed in this paper are described in detail in Kays and London [11]. Kays and London [11] compiled data for a significant number of various types of heat transfer surfaces. Geometrical characteristic as well as heat transfer data and friction factors were included in [11]. The analysis presented in this paper will be focused on plate fin surfaces and will aim to identify the heat transfer surface type with lowest entropy generation rate. The following plate fin surfaces types will be analyzed: plain-fin, louvered-fin, strip-fin, pin-fin and wavy fin types. Kays and London [11] used a semi-descriptive method to designate the surfaces analyzed. The following codding convention will be employed:

- *Louvered-fin*: surface code will consist of two figures as follows: the first represents the length of the louvered fin in the flow direction; the second represents the number of fins per inch transverse to the flow;

- *Plain-fin* surfaces: surface code will consist of the number of fins per inch transverse to the flow direction (Fig. 1);

- *Strip-fin*: Same as for louvered fins;

- *Pin-fin*: Non-descriptive designation;
- *Wavy-fin*: surface code will consist of two figures as follows: the first represents the number of fins per inch; the second represents the wavelength followed by the letter *W*.

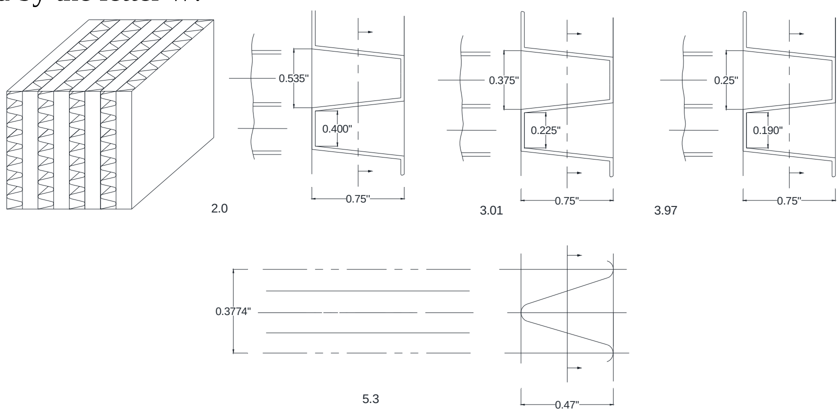

Fig. 1. Plain fins. Geometric elements.

Heat transfer and friction factor data is presented in [11] as data sets $StPr^{2/3} = f(Re)$ and $f = f(Re)$ respectively. The data sets were correlated by means of nonlinear curve fitting, searching functional dependencies of the form:

$$StPr^{2/3} = a_1 Re^{b_1} \tag{9}$$

$$f = a_2 Re^{b_2} \tag{10}$$

Heat transfer and friction data was processed using Origin© in order to calculate the parameters in Equations (9) and (10). Origin© [12] uses chi-square minimization method employing an iterative method based on Levenber-Marquardt algorithm to estimate the non-linear model parameters.

In order to account for the transition to turbulent flow the interval for Re values listed in [11] was divided into two subintervals. The Re break value (denoted Re_{cr}) was determined based on the minimum value of the mean deviation of the whole data set. Generally, it was observed that the Re value that minimizes the mean deviation for heat transfer data minimizes also the mean deviation for friction data. The parameters a_1, b_1, a_2, b_2 for each surface type are listed in Table 1.

Table 1

Heat transfer correlation and friction data for the heat transfer surfaces analyzed

No	**Surface code**	**Heat transfer correlation $StPr^{2/3} = a_1Re^{b_1}$**				**Friction factor correlation $f = a_2Re^{b_2}$**				Re_{cr}	**Surface type**
		$Re < Re_x$		$Re \geq Re_x$		$Re < Re_x$		$Re \geq Re_x$			
		a_1	b_1	a_1	b_1	a_2	b_2	a_2	b_2		
1	3/4-11.1	0.2	-0.44	0.08	-0.31	2.75	-0.62	0.2	-0.28	2500	uv ere d fin
2	1/2-11.1	0.27	-0.47	0.07	-0.28	2.47	-0.59	0.16	-0.22	2000	

3	3/8-6.06	0.21	-0.42	0.1	-0.31	0.93	-0.41	0.13	-0.15	2500	
4	3/8a-6.06	0.07	-0.25	0.14	-0.34	0.83	-0.34	0.23	-0.17	2000	
5	1/2-6.06	0.13	-0.35	0.07	-0.27	1.08	-0.44	0.16	-0.18	2000	
6	1/2(a)-6.06	0.11	-0.32	0.05	-0.2	0.22	-0.19	1.26	-0.42	2000	
7	3/8-8.7	0.38	-0.5	0.09	-0.3	2.25	-0.54	0.15	-0.18	2000	
8	3/8(a)-8.7	0.12	-0.34	0.11	-0.32	1.27	-0.43	0.23	-0.2	2000	
9	3/16-11.1	0.21	-0.4	0.17	-0.37	1.66	-0.47	0.51	-0.31	2000	
10	1/4-11.1	0.16	-0.37	0.11	-0.31	1.4	-0.45	0.44	-0.3	2000	
11	1/4(b)-11.1	0.17	-0.37	0.09	-0.28	1.78	-0.49	0.2	-0.19	2000	
12	3/8-11.1	0.29	-0.46	0.11	-0.33	2.24	-0.54	0.3	-0.28	2000	
13	3/8(b)-11.1	0.14	-0.35	0.17	-0.38	0.24	-0.25	2.15	-0.54	2000	
14	3/4(b)-11.1	0.06	-0.29	0.15	-0.4	2.25	-0.59	0.19	-0.28	2500	
1	3.97	0.02	-0.18	0.02	-0.19	0.11	-0.3	0.03	-0.15	12000	Plain fins
2	3.01	0.01	-0.15	0.02	-0.21	0.18	-0.34	0.03	-0.14	15000	
3	5.3	0.15	-0.46	0.03	-0.23	5.53	-0.82	0.18	-0.35	2000	
4	11.1	0.25	-0.55	0.02	-0.2	4.38	-0.78	0.08	-0.24	2000	
5	11.11(a)	1.36	-0.79	0.01	-0.17	10.5	-0.91	0.1	-0.28	2000	
6	14.77	0.23	-0.53	0.03	-0.24	4.56	-0.76	0.14	-0.3	2000	
7	2.0	0.02	-0.16	0.02	-0.21	0.14	-0.32	0.03	-0.14	15000	
8	9.03	0.6	-0.67	0.01	-0.13	5.1	-0.79	0.05	-0.2	3000	
9	15.08	0.8	-0.72	0.01	-0.1	10.6	-0.9	0.09	-0.28	2000	
10	19.86	0.02	-0.18	0.75	-0.7	0.09	-0.27	9.06	-0.88	2000	
11	10.27T	0.03	-0.25	0.72	-0.68	0.3	-0.41	8.67	-0.86	2000	
12	11.94T	0.75	-0.72	0.01	-0.05	7.66	-0.87	0.1	-0.28	2000	
13	12.00T	0.77	-0.73	0.01	-0.12	8.5	-0.88	0.15	-0.32	1500	
14	16.96T	1.02	-0.81	0	-0.02	8.03	-0.91	0.06	-0.25	1500	
15	25.79T	1.11	-0.8	0.04	-0.33	7.75	-0.9	0.25	-0.42	1500	
16	30.33T	1.06	-0.76	0.11	-0.44	12.9	-0.95	0.43	-0.47	1500	
1	1/4(s)-11.1	0.24	-0.45	0.12	-0.35	2.55	-0.59	0.3	-0.31	2000	Strip fins
2	1/8-15.2	0.06	-0.22	0.11	-0.3	2.2	-0.49	0.2	-0.17	2000	
3	1/8-13.95	0.14	-0.29	0.15	-0.3	3	-0.5	0.27	-0.16	1500	
4	1/8-15.61	0.52	-0.51	0.41	-0.46	6.04	-0.68	0.36	-0.28	1500	
5	1/8-19.86	0.41	-0.51	0.16	-0.37	5.56	-0.69	0.74	-0.4	1000	
6	1/9-22.68	1.44	-0.69	0.24	-0.43	8.48	-0.74	0.76	-0.39	1000	
7	1/9-25.01	0.53	-0.52	0.19	-0.38	4.39	-0.63	0.48	-0.31	1200	
8	1/9-24.12	0.26	-0.45	0.04	-0.17	2.89	-0.59	0.34	-0.28	1200	
9	1/10-27.03	0.67	-0.54	0.16	-0.34	4.52	-0.63	0.33	-0.26	1500	
10	1/10-19.35	0.3	-0.46	0.15	-0.36	4.32	-0.67	0.69	-0.41	1200	
11	1/10-19.74	0.27	-0.47	0.06	-0.27	3.07	-0.61	0.27	-0.26	1200	
1	17.8-3/8W	0.2	-0.4	0.22	-0.41	1.49	-0.47	0.85	-0.4	2000	Wavy fins
2	11.44-3/8W	0.11	-0.29	0.2	-0.37	1.11	-0.38	1.18	-0.39	2000	
3	11.5-3/8W	0.09	-0.26	0.22	-0.38	1.28	-0.39	1.31	-0.4	2000	
1	AP-1	0.18	-0.34	0.2	-0.36	0.2	-0.14	0.03	0.1	2000	Pin fins
2	AP-2	0.07	-0.17	0.18	-0.31	0.7	-0.22	0.08	0.1	1200	

The analysis will be carried out by comparing all heat transfer surfaces in each group to a reference surface. The entropy generation number given by Eq. (5) will be calculated based on N_T and N_P (Eqs. 6, 7) given by:

$$N_T = \frac{(St)_{ref}}{(St)}\frac{D}{D_{ref}} \quad \text{and} \quad N_P = \frac{f}{f_{ref}}\frac{D_{ref}}{D}\left(\frac{A_{ref}}{A}\right) \tag{11}$$

where subscript *ref* denotes the reference surface in the group to which all other surfaces will be compared. To ensure consistency, the reference surface for each group listed in Table 1 will be the first surface (e.g. surface 3/4-1.11 will be the reference for group Louvered fins).

Bejan's designation [1] augmentation entropy generation number was modified to characteristic entropy generation number since in this case we are not talking about heat transfer augmentation.

The analysis will be carried out at two levels:

1. The influence of ϕ_0 will be assessed. ϕ_0 is a key factor that influences the entropy generation for a certain surface type. However, ϕ_0 includes also the influence of the passage, i.e. it depends on the heat transfer device configuration. Therefore, the dependence $N = f(Re, \phi_0)$ will offer valuable information at the design phase allowing optimum selection of a heat transfer surface during the design phase.

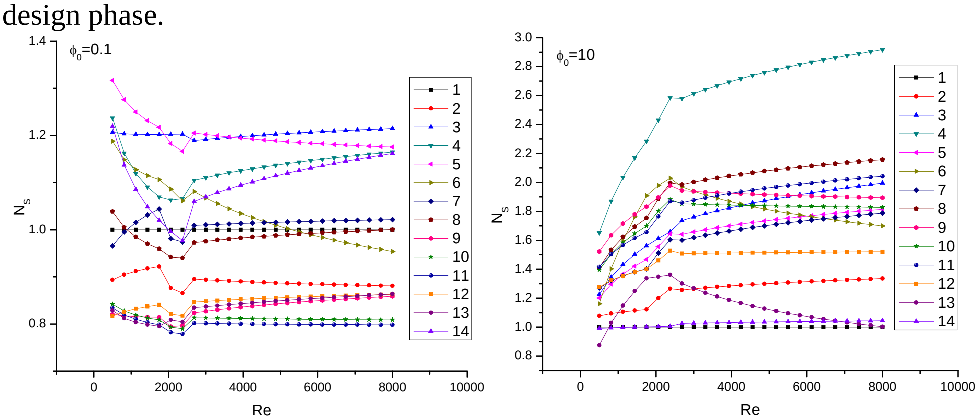

Fig. 2. Influence of ϕ_0 for surface group *Louvered fins* (curve index corresponds to surface number in Table 1 for group *Louvered fins*).

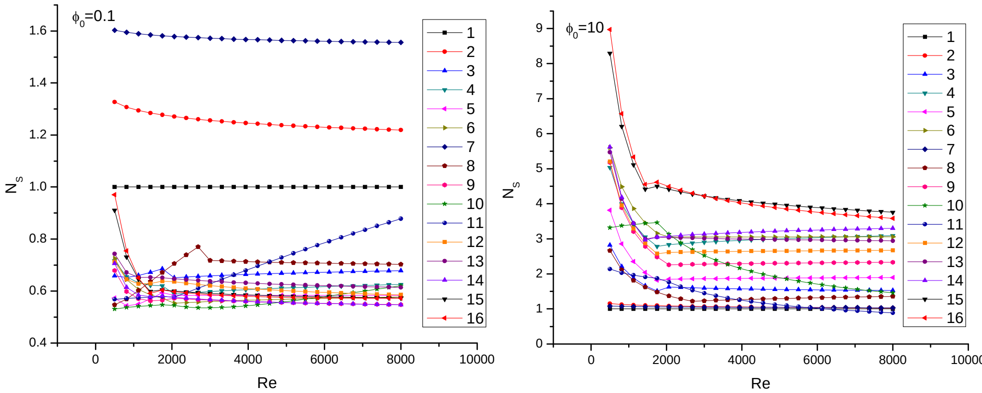

Fig. 3. Influence of ϕ_0 for surface group *Plain fins* (curve index corresponds to surface number in Table 1 for group *Plain fins*).

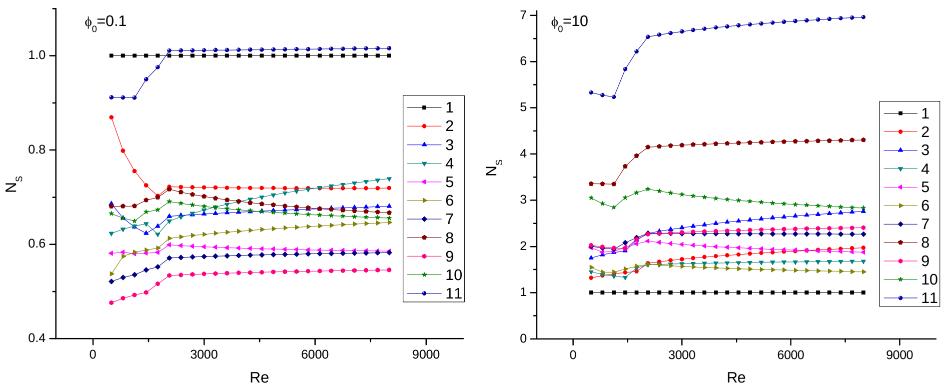

Fig. 4. Influence of ϕ_0 for surface group *Strip fins* (curve index corresponds to surface number in Table 1 for group *Strip fins*).

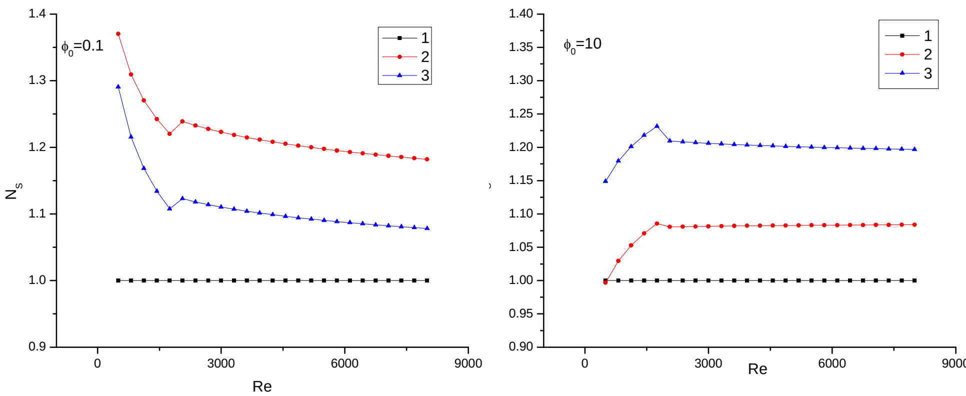

Fig. 5. Influence of ϕ_0 for surface group *Wavy fins* (curve index corresponds to surface number in Table 1 for group *Wavy fin*).

2. Level 2 analysis consists of the following: For each surface analyzed, the characteristic entropy generation number was plotted against *Re* for five values of ϕ_0 that cover the usual values that can occur for most heat transfer devices. By means of this approach, the heat transfer surface with lowest value of the characteristic entropy generation number corresponding to a certain value of ϕ_0 will be identified. Given the number of heat transfer surface types analyzed in this paper only a limited number considered representative will be presented. A similar convention as in the case of level 1 analysis will be employed: reference heat transfer surface type in each group will be always the first type (see Table 1).

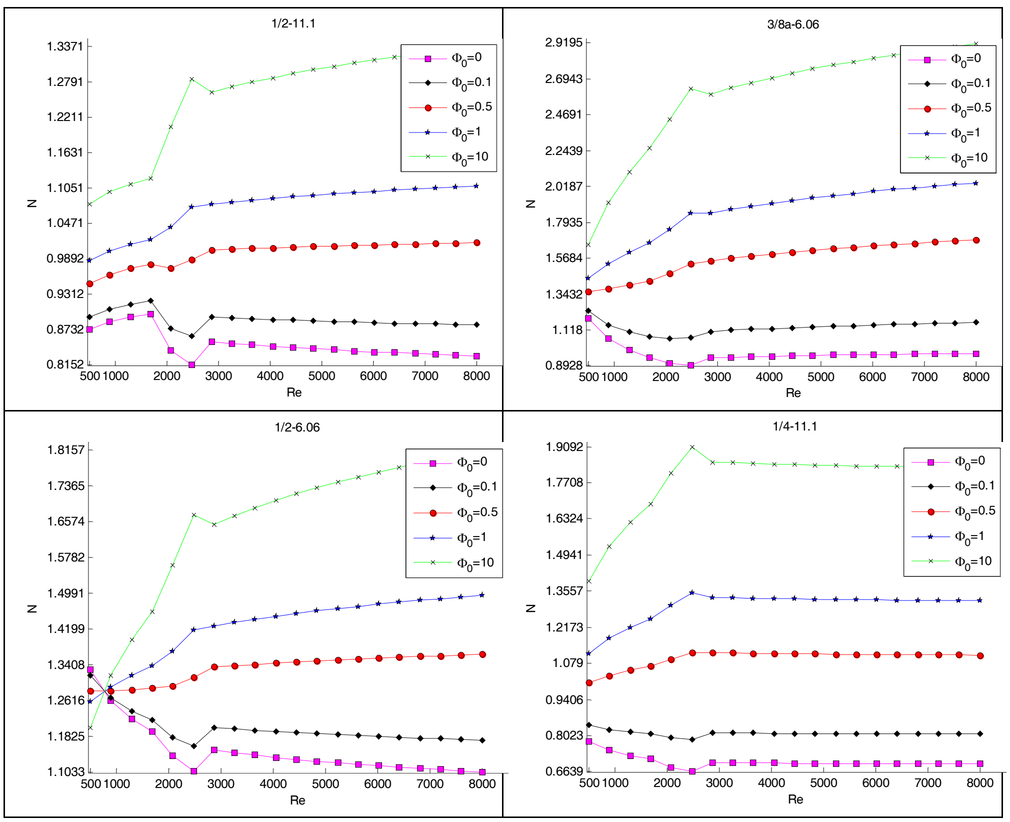

Fig. 6. Influence of irreversibility distribution ratio - *Louvered fins* surfaces.

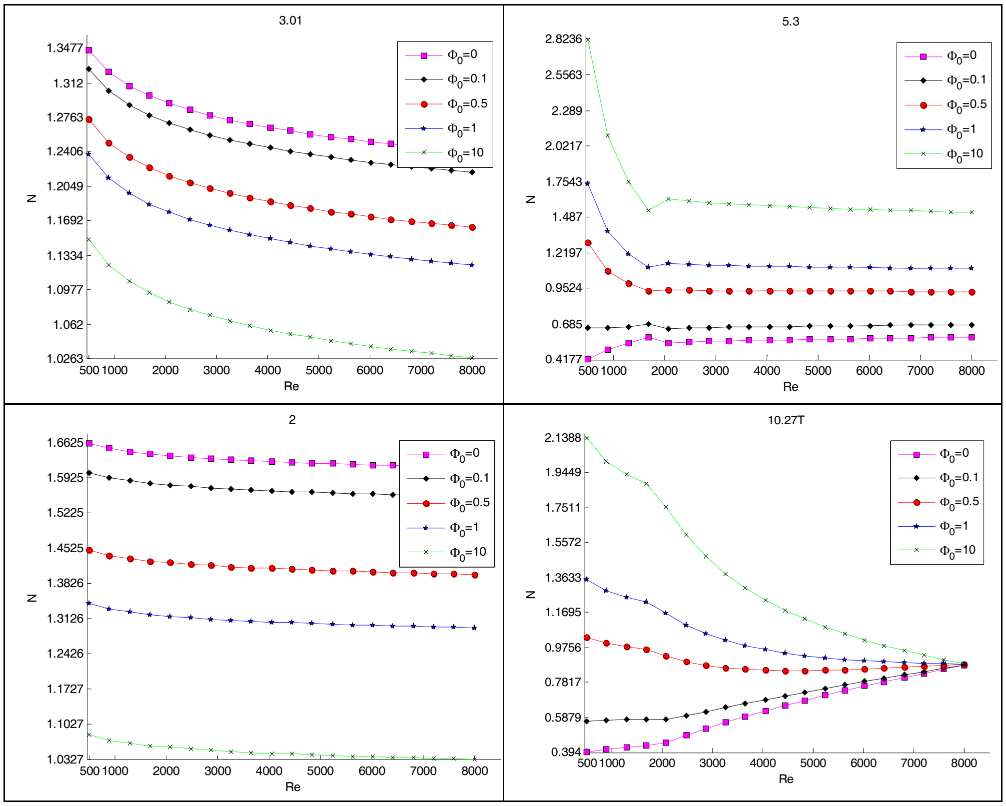

Fig. 7. Influence of irreversibility distribution ratio - *Plain fins* surfaces.

Level 2 analysis allows identification the surface type in each group that exhibits the lowest value of the characteristic entropy generation number. These surface types are presented in Table 2. It is essential to note that the surface type with the lowest value of the characteristic entropy generation number in each group depends on the irreversibility distribution ratio ϕ_0 for the reference passage.

Table 2

Surface types with lowest value of characteristic entropy generation number

ϕ_0	**Surface type with lowest value of characteristic entropy generation number**				
	Louvered fin	**Plain fin**	**Strip fin**	**Wavy fin**	**Pin fin**
0.1	1/4(b)-11.1	19.86	1/10-27.03	17.8-3/8W	AP-2
1.0	3/8(b)-11.1	10.27T	1/4(s)-11.1	17.8-3/8W	AP-1

It is possible to go one step further by applying the analysis method to the surfaces listed in Table 2. Reference type will be 1/4(b)-11.1 and the analysis will be carried out for irreversibility distribution ratio $\phi_0 = 0.1$. The results are presented in Fig. 9. It can be noticed that surface type with lowest characteristic entropy generation number among all types analyzed is 1/10-27.03 (*Strip fins* type). However, this conclusion is only valid for $\phi_0 = 0.1$.

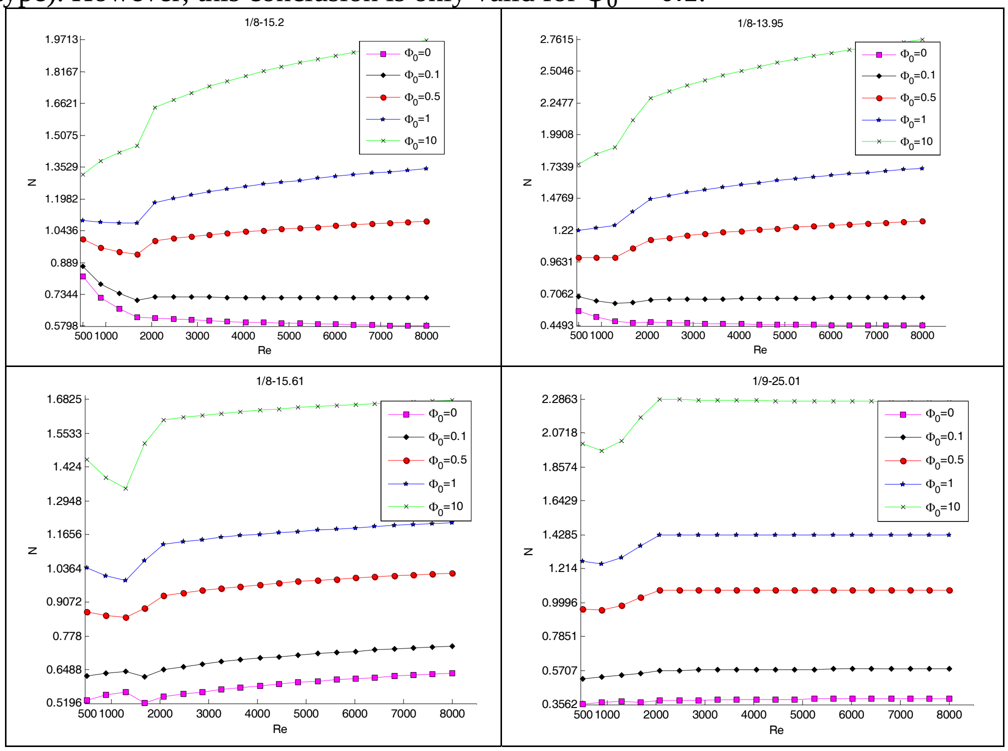

Fig. 8. Influence of irreversibility distribution ratio - *Strip fins* surfaces.

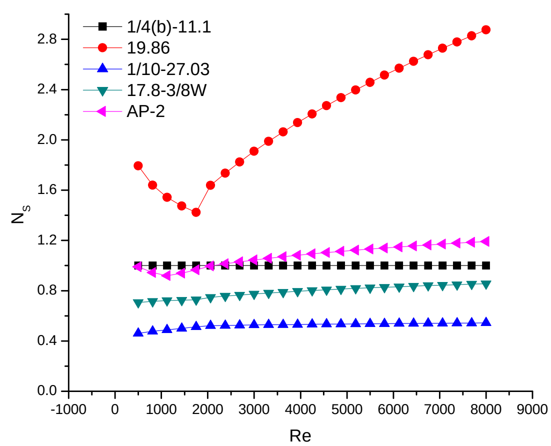

Fig. 9. Characteristic entropy generation number for surface types in Table 2 ($\phi_0 = 0.1$).

3. Discussion and results interpretation

Level 1 analysis allows selection of the heat transfer surface type with lowest value of the characteristic entropy generation number if irreversibility distribution ratio is given. It can be noticed from Figs. 6 and 7 that ϕ_0 influences significantly the choice of the optimum surface. Surface type 1/4(b)-11.1 exhibits the lowest value of the characteristic entropy generation number for $\phi_0 = 0.1$, while for $\phi_0 = 10$, the surface type with lowest N_S is 3/4-11.1. Influence of Re number can be synthesized as follows: While irreversibility distribution ratio is constant, Re number influences to a small extent the entropy generation (see Figs. 2-4) and generally the influence is very similar for all surface types. If irreversibility distribution ration varies, a significant change in the influence of Re number can be observed (see Figs. 6-7). Level 2 analysis provides further insight into the influence of irreversibility distribution ratio. It will focus on irreversibility distribution ration on characteristic entropy generation number for each surface type analyzed. By means of this approach the following conclusions can be drawn:

- For *Louvered fins* surfaces, the characteristic entropy generation number for each surface type decreases as ϕ_0 decreases. All surface types in the group *Louvered fin* follow the same trend (see Fig. 6);
- For *Plain fins* surfaces the influence of ϕ_0 is contradictory: while in the case of surface type 3.01 and 2 the characteristic entropy generation number increases as ϕ_0 decreases, in other cases such as 5.3 and 10.27T, the trend is reversed (see Fig. 7);
- For *Strip fins* surfaces all types follow the same trend: the characteristic entropy generation number decreases as ϕ_0 decreases (see Fig. 8);
- A mixed trend occurs also in the case of *Wavy fins* surfaces: for surface type 11.5-3/8W the characteristic entropy generation number decreases as ϕ_0 decreases, while for 11.44-3/8W the trend is reversed;
- Only two surface types were available for *Pin fins* surface group; characteristic entropy generation number decreases for AP-2 type. It is not possible to establish if this trend is consistent or mixed.

4. Conclusions

An analysis based on the Second Law of Thermodynamics was applied to a range of plate fin heat transfer surfaces taking into account geometric characteristics, heat transfer and friction data. The objectives of the study were the following:

- Identify the surface type with lowest entropy generation;
- Assess the influence of Re number on entropy generation;
- Assess the flow passage characteristics on entropy generation;

The conclusions of the study can be resumed as follows:

- Surface type is the main factor that influences the entropy generation; significant differences in terms of entropy generation can be observed even for surface types belonging to the same group;
- The effect of Re number is not identical for all surface type and it depends on the irreversibility distribution ratio;
- Irreversibility distribution number is the key factor that governs entropy generation; even for the same surface type, ϕ_0 value can radically modify the entropy generation.

The analysis technique presented in this paper can be applied for any heat transfer surface types allowing classification from the entropy generation viewpoint.

Optimal design of heat transfer devices involves a range of constraints. If minimization of entropy generation is included among the optimization criteria, selection of the heat transfer surface with lowest entropy generation requires an estimation of irreversibility distribution ratio.

R E F E R E N C E S

[1] *A. Bejan*, Entropy Generation Through Heat and Fluid Flow, John Wiley and Sons Publishing, New York, 1982

[2] *A. Bergles*, Recent developments in convective heat transfer augmentation, Appl. Mech. Rev. **26(6)**, pp. 675-682, 1973

[3] *A. Bergles*, *A. Blumenkrantz*, *J. Taborek*, Performance evaluation criteria for enhanced heat transfer surfaces, Heat Transfer **2**, pp. 239-243, Tokyo, 1974

[4] *A. Bejan*, *P. A. Pfister*, Evaluation of heat transfer augmentation techniques based on their impact on entropy generation, Letters in Heat anad Mass Transfer **7**, pp. 97-106, 1980

[5] *N. Sahiti*, *F. Krasniqi*, *Xh. Fejzullahu*, *J. Bunjaku*, *A Muriqi*, Entropy generation minimization of a double pipe pin fin heat exchanger, Applied Thermal Engineering **28**, pp. 2337-2344, 2008

[6] *B.N. Taufiq*, *H.H. Masjuki*, *T.M.I. Mahlia*, *R. Saidur*, *M.S. Faizul*, *E. Niza Mohamad*, Second law analysis for optimal thermal design of radial fin geometry by convection, Applied Thermal Engineering **27**, pp. 1363-1370, 2007

[7] *P. Naphon*, Second law analysis on the heat transfer of the horizontal concentric tube heat ecchanger, International Communication in Heat and Mass Transfer **33**, pp. 1029-1041, 2006

[8] *M. Ylmaz*, *O. N. Sara*, *S. Karsli*, Performance evaluation criteria for heat exchangers based on second law analysis, Exergy Int. J. **1(4)**, pp. 278-294, 2001

[9] *S. Petrescu, C. Dobre, M. Costea, A. Dobrovicescu, G. Tirca-Dragomirescu*, Entropy generation analysis for a reversed quasi-Carnot cycle (Refrigeration machine) by using the direct method from finite speed thermodynamics, UPB Sci. Bull. Series D, **74**. Iss. 4, 2012

[10] *A. Dobrovicescu, D. Stanciu, C. Padet, E. C. Mladin, J. Padet, M. Prisecaru*, Analyse exergetique pour la choix de la difference de temperature optimale dans les echangeurs de chaleur recuperateurs d’un systeme cryogenique, U.P.B. Sci. Bull., Series D, **71**, Iss.4, 2009

[11] *W. M. Kays*, *A. L. London*, Compact Heat Exchangers, Third edition, McGraw Hill, 1984

[12] www.originlab.com