U.P.B. Sci. Bull., Series A, Vol. 86, Iss. 4, 2024 ISSN 1223-7027

CONSTITUTIVE MATERIAL LAWS FROM MULTIFRACTAL PERSPECTIVE OF MOTION

Cristina-Marcela RUSU[1]*, Luminița BIBIRE[2], Monica MOLCĂLUȚ[3], Anișoara CORĂBIERU[4], Raluca GIURMĂ-HANDLEY[5]*, Ștefana AGOP[6]

Multifractal constitutive relations for ideal isotropic material structures in the framework of Multifractal Theory of Motion are build assuming that any material structures are assimilated to the multifractals and that multifractal scalar potentials, specified by means of Madelung type scenario motion functions as a multifractal elastic potential. Finaly, standard results from the classical deformation theories as Lode's parameter and Hook's law for an ideal, isotropic material structure are obtained from the present model considering only dynamics on Peano type curves (i.e mono-fractal case in the fractal dimension D_F≡2.

Keywords: multifractal, isotropic material structures, multifractal stress tensor, multifractal scalar potential, continuous and non-differentiable curves.

1. Introduction

Fractal/multifractal theory is a new method of approaching the dynamics of complex systems both at small scale resolutions and at large scale resolutions.

The multifractal theory of motion in the form of the theory of scale relativity allows multiple approaches in describing the dynamics of complex systems [1-3].

From such a perspective, two scenarios are possible: Schrodinger's multifractal theory and Madelung's multifractal theory.

[1] Dept.of Physics, "Gheorghe Asachi" Technical University of Iași, Romania, cristina-marcela.rusu@academic.tuiasi.ro
[2] Dept. of Environment Engineering, Mechanical Engineering and Agritourism; Faculty of Engineering, "Vasile Alecsandri" University of Bacau, lbibire@ub.ro
[3] Faculty of Physics "A.I.Cuza " University of Iasi, Romania, monicamolcalut@yahoo.com
[4] Faculty of Material Engineering and Science, "Gh. Asachi" Technical University of Iasi, Romania, anisoara.corabieru@academic.tuiasi.ro
[5] Faculty of Hydrotechnics, Geodesy and Environmental Engineering "Gheorghe Asachi" Technical University of Iasi, Romania, catrinel-raluca.giurma-handley@academic.tuiasi.ro
[6] Faculty of Material Engineering and Science, "Gh. Asachi" Technical University of Iasi, Romania, stefanaagop@yahoo.com
*Correspondence: cristina-marcela.rusu@academic.tuiasi.ro; catrinel-raluca.giurma-handley@academic.tuiasi.ro

2. Theory

In the Multifractal Theory of Motion [1,2] two scenarios have been outlined in describing the dynamics of any material structures:

a. The multifractal Schrödinger type scenario explained by the differential equation:

$$\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}-2\right]}\partial^l\partial_e\Psi + i\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial_t\Psi - U\Psi = 0, i = \sqrt{-1} \quad (1)$$

b. The multifractal Madelung type scenario explained by the system of differential equations:

$$\partial_e V_D^i + (V_D^e\partial_e)V_D^i = -\rho^{-1}\partial_e\hat{\sigma}^{ie} + \partial^i U \quad (2)$$

$$\partial_t\rho + \partial_i\left(\rho V_D^i\right) = 0 \quad (3)$$

where

$$\partial_l = \frac{\partial}{\partial x_l}, \partial_e\partial^e = \frac{\partial^2}{\partial x_e^2}, \ \partial_t = \frac{\partial}{\partial t}, \ i, e = 1,2,3 \quad (4)$$

In relations (1)-(3) the quantities have the following physical meanings: Ψ is the multifractal state function, ρ=ΨΨ¯with Ψ¯the complex conjugate of Ψ is the density of multifractal states, U is the external scalar potential, x^e is the multifractal spatial coordinate, t is the non-multifractal temporal coordinate,

$$V_D^i = 2\lambda(dt)^{\left[\frac{2}{f(\alpha)}-1\right]}\partial^i ln\Psi \quad (5)$$

is the non-multifractal speed,

$$\sigma^{ie} = -2\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}-2\right]}\rho\partial_i\partial_e \ln\rho \quad (6)$$

is the multifractal stress tensor, λ is the parameter associated with the multifractal-non-multifractal scale transition, dt is the scale resolution and f(α) is the singularity spectrum of order $\alpha = \alpha(D_F)$, where D_F is the fractal dimension of the motion curves. Recall that in Multifractal Theory of Motion the dynamics of any material structure are described by continuous and non-differentiable curves (fractal curves/multifractal curves). For more details see references [1-3]. Note that the two scenarios for describing the dynamics of material structures are not disjoint, but complementary. They allow, by an appropriate choice of scale resolution, either local or global descriptions, or specific local-global transition descriptions. In such a context in references [4,5] we have shown that the presence of a multifractal stress tensor can be correlated by means of the relation

$$\partial^i\sigma_{ie} + \rho\partial_e Q = 0 \quad (7)$$

with the multifractal scalar potential:

$$Q = -2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}-2\right]} \frac{\partial^e \partial_e \sqrt{\rho}}{\sqrt{\rho}} = V_F^e V_F^e - \lambda (dt)^{\left[\frac{2}{f(\alpha)}-1\right]} \partial_i V_F^i \quad (8)$$

where

$$V_F^i = \lambda (dt)^{\left[\frac{2}{f(\alpha)}-1\right]} \partial^i \ln \rho \quad (9)$$

is the multifractal velocity.

Therefore, taking into account our previous results, a correlation between the multifractal stress tensor, σ_{ie} and the multifractal scalar potential, Q, becomes functional. In a such perspective, assuming that any material structure can be assimilated to multifractal (we note that a such hypothesis confers holographic behaviors on the dynamics of the material structures [2,3]) and considering the multifractal scalar potential, Q functions as a multifractal elastic potential, in the present paper multifractal constitutive relations for any ideal isotropic material structures are obtained.

3. Multifractal cubic equations associated to the multifractal stress and deformation tensors

Both the multifractal stress tensor, σ^{ie}, and the multifractal deformation tensor,ε^{ie}, can be associated to the multifractal matrix 3x3. In such a framework let us consider the multifractal generic matrix (a_{ie}). Then the eigen values of the multifractal matrix (a_{il}) is the multifractal cubic:

$$x^3 - I_1 x^2 + I_2 x - I_3 = 0 \quad (10)$$

where we used the following notations:

$$\begin{aligned} I_1 &= TR(a) = a_{11} + a_{22} + a_{33} \\ I_2 &= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \\ I_3 &= det\,(a_{ie}) \end{aligned} \quad (11\text{ a-c})$$

Admitting that the multifractal cubic (10) has distinct solutions, the Tartaglia- Cartan procedure allows to make them explicit in the form

$$\begin{aligned} x_1 &= \frac{2}{\sqrt{3}} \sqrt{a_2}\, sin\, 3\theta + \frac{1}{3} a_1 \\ x_2 &= \frac{2}{\sqrt{3}} \sqrt{a_2}\, sin(3\theta + \frac{2\pi}{3}) + \frac{1}{3} a_1 \\ x_3 &= \frac{2}{\sqrt{3}} \sqrt{a_2}\, sin(3\theta + \frac{4\pi}{3}) + \frac{1}{3} a_1 \end{aligned} \quad (12\text{ a-c})$$

where

$$sin\, 3\theta = \frac{\sqrt{3}}{2} \frac{a_3}{a_2^{3/2}} \quad (13)$$

and

$$a_1 = I_1$$

$$a_2 = \frac{1}{6}\left\{(a_{22}-a_{33})^2 + (a_{33}-a_{11})^2 + (a_{11}-a_{22})^2 + \frac{3}{2}(a_{12}a_{21} + a_{13}a_{31} + a_{12}a_{32})\right\}$$

(14 a-c)

$$a_3 = 3 \cdot det\,(\frac{1}{3}a_1\delta_{ie} - a_{ie})$$

where θ is the representation angle of matrix (a) [6,7].

We note that in the monofractal case $f((\alpha) \equiv D_F = 2$, for dynamics on Peano type curves [2,3], i.e. in the standard deformation theory, θ it is connected to the ratio of radii of Mohr's circles, expressing either deformations or stress, being characterized by the Lode's parameter μ_α[6,7]:

$$\mu_\alpha = \frac{2x_1-x_2-x_3}{x_2-x_3} = \sqrt{3}\,tan\,3\theta \tag{15}$$

4. Multifractal constitutive relations for any ideal isotropic material structures

For any of the multifractal matrices ε^{ie} and σ^{ie} an analogous theory to that developed in the previous paragraph one can construct.

Thus, let us consider e_1, e_2, e_3 the invariants of the multifractal matrix ε^{ie} and Ψ the representation angle of this multifractal matrix. Also, let us consider s_1, s_2, s_3 the invariants of the multifractal matrix σ^{ie} and ξ *t*he representation angle of this multifractal matrix.

The fundament of multifractal constitutive relation for an ideal isotropic material structure can be related to the multifractal elastic potential (8), considered as a function of the multifractal deformation invariants E_1, E_2, E_3 i.e

$$Q \equiv Q(E_1, E_2, E_3), (E_i = E_i(\rho), i = 1,2,3 \tag{16}$$

Then we will have:

$$\sigma_{ij} = \frac{\partial Q}{\partial \varepsilon_{ij}} \tag{17}$$

or:

$$\sigma_{ij} = \sum_{k=1}^{3} \frac{\partial Q}{\partial E_k}\frac{\partial E_k}{\partial \varepsilon_{ij}} \tag{18}$$

The derivatives $\frac{\partial Q}{\partial E_k}$ can play the role of some multifractal elastic moduli: for example, the multifractal volume modulus *K*, the multifractal shear modulus *G*, etc. But first let us explain (18). It results:

$$\sigma_{11} = \frac{\partial Q}{\partial E_1} + \frac{\partial Q}{\partial E_2}(\varepsilon_{22} + \varepsilon_{33}) + \frac{\partial Q}{\partial E_3}\left(\varepsilon_{22}\varepsilon_{33} - \frac{1}{4}\varepsilon_{23}^2\right)$$

$$\sigma_{22} = \frac{\partial Q}{\partial E_1} + \frac{\partial Q}{\partial E_2}(\varepsilon_{11} + \varepsilon_{22}) + \frac{\partial Q}{\partial E_3}\left(\varepsilon_{11}\varepsilon_{33} - \frac{1}{4}\varepsilon_{13}^2\right)$$

$$\sigma_{33} = \frac{\partial Q}{\partial E_1} + \frac{\partial Q}{\partial E_2}(\varepsilon_{11} + \varepsilon_{22}) + \frac{\partial Q}{\partial E_3}\left(\varepsilon_{11}\varepsilon_{22} - \frac{1}{4}\varepsilon_{12}^2\right)$$

$$\sigma_{12} = -\frac{1}{2}\left\{\frac{\partial Q}{\partial E_2}\varepsilon_{12} - \frac{\partial Q}{\partial E_3}\left(\frac{1}{2}\varepsilon_{13}\varepsilon_{23} - \varepsilon_{33}\varepsilon_{12}\right)\right\}$$

$$\sigma_{13} = -\frac{1}{2}\left\{\frac{\partial Q}{\partial E_2}\varepsilon_{13} - \frac{\partial Q}{\partial E_3}\left(\frac{1}{2}\varepsilon_{23}\varepsilon_{12} - \varepsilon_{22}\varepsilon_{13}\right)\right\}$$

$$\sigma_{23} = -\frac{1}{2}\left\{\frac{\partial Q}{\partial E_2}\varepsilon_{23} - \frac{\partial Q}{\partial E_3}\left(\frac{1}{2}\varepsilon_{12}\varepsilon_{13} - \varepsilon_{11}\varepsilon_{23}\right)\right\} \qquad \text{(19 a-f)}$$

Let us rewrite s_1, s_2, s_3 in terms of e_1, e_2, e_3. For this, it is necessary to express s_1, s_2, s_3 by means of E_1, E_2, E_3. It results:

$$\begin{aligned}
s_1 &= 3\frac{\partial Q}{\partial E_1} + 2\frac{\partial Q}{\partial E_2}E_1 + \frac{\partial Q}{\partial E_3}E_2,\\
s_2 &= \frac{1}{3}\left\{(E_3^2 - 2E_2)\left(\frac{\partial Q}{\partial E_2}\right)^2 + (E_1E_2 - 9E_3)\frac{\partial Q}{\partial E_2}\frac{\partial Q}{\partial E_3} + \right.\\
s_3 &= \frac{1}{9}\left\{(2E_1^3 - 9E_1E_2 + 27E_3)\left(\frac{\partial Q}{\partial E_2}\right)^2 + \right.\\
&\quad +(2E_1^2E_2 - 18E_2^2 + 27E_1E_3)\left(\frac{\partial Q}{\partial E_2}\right)^2\frac{\partial Q}{\partial E_3} + \\
&\quad +(18E_1^2E_3 - 3E_1E_2^2 - 27E_2E_3)\left(\frac{\partial Q}{\partial E_3}\right)^2\frac{\partial Q}{\partial E_2} + \\
&\quad \left. +(-2E_2^3 + 9E_1E_2E_3 - 27E_3^2)\left(\frac{\partial Q}{\partial E_3}\right)^2\right\}
\end{aligned} \qquad \text{(20 a-c)}$$

If we replace the derivatives with respect to E_1, E_2, E_3 with the derivatives with respect to e_1, e_2, e_3 we obtain:

$$\frac{\partial Q}{\partial E_1} = \frac{\partial Q}{\partial e_1} + \frac{2}{3}\frac{\partial Q}{\partial e_2}E_1 + \frac{\partial Q}{\partial e_3}\left(E_2 - \frac{2}{3}E_1^2\right)$$

$$\frac{\partial Q}{\partial E_2} = -\frac{\partial Q}{\partial e_2} + \frac{\partial Q}{\partial e_3} E_1 \qquad (21\ \text{a-c})$$

$$\frac{\partial Q}{\partial E_3} = -3\frac{\partial Q}{\partial e_3}$$

Substituting (20 a-c) into (19 a-f) the result is:

$$s_1 = 3\frac{\partial Q}{\partial e_1}$$

$$s_2 = e_2\left(\frac{\partial Q}{\partial e_2}\right)^2 + 3e_3\left(\frac{\partial Q}{\partial e_2}\right)\cdot\left(\frac{\partial Q}{\partial e_3}\right) + 3e_2^2\left(\frac{\partial Q}{\partial e_3}\right)^2 \qquad (22\ \text{a-c})$$

$$s_3 = e_3\left(\frac{\partial Q}{\partial e_2}\right)^2 + 6e_2^2\left(\frac{\partial Q}{\partial e_2}\right)^2\left(\frac{\partial Q}{\partial e_3}\right) + 9e_2e_3\left(\frac{\partial Q}{\partial e_2}\right)\left(\frac{\partial Q}{\partial e_3}\right)^2 + 3(3e_3^2 - 2e_2^3)\left(\frac{\partial Q}{\partial e_3}\right)^2$$

The relations (22 a-c) can be expressed in terms of e_1, e_2, ψ, taking into account that:

$$sin\ 3\psi = \frac{\sqrt{3}}{2}\frac{e_3}{e_2^{3/2}} \equiv \beta$$

We obtain:

$$s_1 = 3\frac{\partial Q}{\partial e_1},$$
$$s_2 = e_2\left(\frac{\partial Q}{\partial e_2}\right)^2 + \frac{9}{4}\frac{1-\beta^2}{e_2}\left(\frac{\partial Q}{\partial \beta}\right)^2,$$
$$s_3 = \frac{2}{\sqrt{3}}e_2^{3/2}\left\{\beta\left(\frac{\partial Q}{\partial e_2}\right)^2 + \frac{9}{2}\frac{(1-\beta^2)}{e_2}\left(\frac{\partial Q}{\partial e_2}\right)^2\left(\frac{\partial Q}{\partial \beta}\right) - \right.$$
$$\left. - \frac{27}{4}\cdot\frac{\beta(1-\beta^2)}{e_2^2}\cdot\left(\frac{\partial Q}{\partial e_2}\right)\left(\frac{\partial Q}{\partial \beta}\right)^2 - \frac{27}{8}\cdot\frac{1-\alpha^2}{e_2^3}\left(\frac{\partial Q}{\partial \beta}\right)^3\right\} \qquad (23\ \text{a-c})$$

or, in terms of ψ,

$$s_1 = 3\frac{\partial Q}{\partial e_1}$$

$$s_2 = e_2\left\{\left(\frac{\partial Q}{\partial e_2}\right)^2 + \frac{1}{4e_2^2}\left(\frac{\partial Q}{\partial \psi}\right)^2\right\} \quad (24\text{ a-c})$$

$$s_3 = \frac{2}{\sqrt{3}}e_2^{3/2}\left\{\left(\frac{\partial Q}{\partial e_2}\right)\sin 3\psi + \frac{3}{2}\frac{1}{e_2}\left(\frac{\partial Q}{\partial e_2}\right)\left(\frac{\partial Q}{\partial \psi}\right)\cos 3\psi - \frac{3}{4}\frac{1}{e_2^2}\left(\frac{\partial Q}{\partial e_2}\right)\left(\frac{\partial Q}{\partial \psi}\right)^2 \sin 3\psi_0 - \frac{1}{8}\frac{1}{e_2^3}\left(\frac{\partial Q}{\partial \psi}\right)^3 \cos 3\psi\right\}$$

Denoting:

$$\chi = \frac{1}{2e_2}\frac{\partial Q}{\partial \psi}/\frac{\partial Q}{\partial e_2} \quad (25)$$

the relations of (24 b,c) allow to construct $sin\ 3\xi$

$$sin\ 3\xi = \frac{(1-3\chi^2)sin\ 3\psi + (-\chi+3)\chi cos\ 3\psi}{(1+\chi^2)^{3/2}} \quad (26)$$

For $\chi = tan\ \omega$ in (26), it results:

$$sin\ 3\xi = sin\ 3(\psi + \omega) \quad (27)$$

with the solution:

$$\omega = \xi - 2\psi + \frac{2n\pi}{3}$$

where $n \in N$. Taking $n = 0$, we have:

$$\omega = \xi - 2\psi \quad (28)$$

and relationship between ω, ξ and ψ becomes univocal.
Introducing $\chi = tan\ \omega$ into (24 a-c), these relations become:

$$s_1 = 3\frac{\partial Q}{\partial e_1}$$

$$s_2 = \frac{e_2}{cos^2\omega}\left(\frac{\partial Q}{\partial e_2}\right)^2$$

$$s_3 = \frac{2}{\sqrt{3}}e_2^{3/2}\frac{sin\ 3(\psi+\omega)}{cos^2\ \omega}\left(\frac{\partial Q}{\partial e_2}\right)^2 \quad (29\text{ a-c})$$

Now, it is useful to define the multifractal quantities:

$$K=\frac{1}{3}\frac{s_1}{e_1},G=\frac{1}{2}\sqrt{\frac{s_2}{e_2}} \tag{30}$$

and so, (22 a-c) becomes:

$$\frac{\partial Q}{\partial e_1}=Ke_1,$$
$$\frac{\partial Q}{\partial e_2}=2G\cdot cos\,\omega$$
$$\frac{\partial Q}{\partial e_3}=4Ge_2\cdot sin\,\omega \tag{31 a-c}$$

With these observations, (22 a-c) write:

$$\frac{\partial Q}{\partial E_1}=Ke_1+\frac{4}{3}Ge_1\frac{cos(3\psi+\omega)}{cos\,3\psi}-2G\frac{e_2+\frac{1}{3}e_1^2}{\sqrt{3e_2}}\frac{sin\,\omega}{cos\,3\psi}$$
$$\frac{\partial Q}{\partial E_2}=-2G\left\{\frac{cos(3\psi+\omega)}{cos\,3\psi}-\frac{e_1}{\sqrt{3e_2}}\frac{sin\,\omega}{cos\,3\omega}\right\}$$
$$\frac{\partial Q}{\partial E_3}=-2G\sqrt{\frac{3}{e_2}}\frac{sin\,\omega}{cos\,3\psi} \tag{32 a-c}$$

In view of (32 a-c), (20 a-c) receive the following multifractal tensor form:

$$\widehat{D}_\sigma=2G\left\{\frac{cos\,(3\psi+\omega)}{cos\,3\psi}\widehat{D}_\varepsilon-\sqrt{\frac{3}{e_2}}\frac{sin\,\omega}{cos\,3\psi}\left(\widehat{D}_\varepsilon^2-\frac{2}{3}e_2\hat{I}\right)\right\} \tag{33}$$

where $\widehat{D}$ denotes the multifractal deviator of the respective multifractal matrix, and $\hat{I}$ the unit multifractal matrix.

The multifractal matrix (33) stands for the most general multifractal constitutive relation for the multifractal ideal isotropic elastic material structure, but only for this one.

Since the multifractal potential Q also functions for multifractal reversible transformations, $\widehat{D}_\varepsilon$ can be expressed in terms of $\widehat{D}_\sigma$. Indeed, taking into account the operational procedures from [2] it results:

$$\widehat{D}_\varepsilon=\frac{1}{2G}\left\{\frac{cos\,(3\xi-\omega)}{cos\,3\xi}\widehat{D}_\sigma+\sqrt{\frac{3}{s_2}}\frac{sin\,\omega}{cos\,3\xi}\left(\widehat{D}_\sigma^2-\frac{2}{3}s_2\hat{I}\right)\right\} \tag{34}$$

We note that in the mono-fractal case $f(\alpha)\rightarrow D_F\equiv 2$ i.e for dynamics on Peano type curves [2,3,8,9] and for $\omega\equiv 0$, (33) yields the Hook's law for a perfectly isotropic material,

$$\sigma_{ij} = E\left[\frac{1}{1+\mu}\varepsilon_{ij} + \frac{\mu}{(1+\mu)(1-2\mu)}(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33})\right]$$

by means of the usual identifications:

$$2G = \frac{E}{1+\mu}, K = \frac{E}{1-2\mu},$$

where μ is Poisson's coefficient.

5. Conclusions

The main conclusions of the present paper are the following:
1. In the Multifractal Theory of Motion two scenarios have obtained in describing the dynamics of material structures: the multifractal Schrödinger type scenario and the multifractal Madelung type scenario;
2. In the multifractal Madelung type scenario correlations between the multifractal stress tensor and the multifractal scalar potential are obtained;
3. Assuming that any material structure can be assimilated to a multifractal and that, the multifractal scalar potential functions as a multifractal elastic potential, the constitutive material laws for the multifractal ideal isotropic elastic material structures are obtained;
4. Standards results from classical deformation theories as Lode's parameter, Hook's law for a ideal isotropic material structure are obtained using the present model considering only dynamics on Peano type curves (i.e the monofractal case $f(\alpha) \rightarrow D_F \equiv 2$);
5. The present model uses fractal/multifractal curves to describe the dynamics of material structures. Since fractal/multifractal curves have the property of self-similarity (the part reflects the whole and vice versa), such dynamics mimic holographic behavior. Moreover, as any hologram works "as a deep learning [10]", such a model could be useful in the analysis of some "dynamics" in the medical field [11-13].

R E F E R E N C E S

[1]. *Merches,I; Agop, M.* Differentiability and fractality in dynamics of physical systems, World Scientific, New Jersey, USA, 2016;
[2]. *Agop, M.; Paun V.P.* (2017), On the new perspectives of the fractal theory. Fundaments and applications, Romanian Academy publishing house, Bucharest, Romania, 2017;
[3]. *Gavrilut, A; Merches, I and Agop, M.*, 2019. Atomicity through Fractal Measure Theory. Springer International Publishing;

[4]. *Ana Rotundu, Stefana Agop, Maria-Alexandra Paun, C. Bejinariu, T.-C. Petrescu, Cristina Marcela Rusu, A. M. Cazac, Vl.-Al. Paun, M. Agop, V.P. Paun*. Constitutive Materials laws in the multifractal theory of motion, UPB seria A**, 4**/2023,2;

[5]. *S. Agop, M.-Al. Paun, C. Bejinariu, T-C. Petrescu, C. M. Rusu, A. M. Cazac, C-G. Dumitras, A. C. Hanganu, Vl.Al. Paun, M. Agop, V.P. Paun*. Constitutive Materials laws in the multifractal theory of motion, UPB seria A, **4**/2024,1;

[6**].** *Marco Maurizi, Chao Gao & Filippo Berto*, Predicting stress, strain and deformation fields in materials and structures with graph neural networks, Scientific Reports volume **12**, (2022)

[7]. *Mincai Jia, Wensen Luo, Yunhong Zhou, Shun Zhao, Zhen Zhang*, A Model of Undrained Stress–Strain Curves Considering Stress Path and Strain Softening International Journal of Geomechanics, **21** (11), 2023;

[8]. *Mandelbrot B.B.,* The Fractal Geometry of Nature, W.H. Freeman and Co., San Francisco, USA,1982;

[9]. *Nottale, L.* Scale Relativity and Fractal Space-Time, A New Approach to Unifying Relativity and Quantum Mechanics, Imperial College Press, London, UK,2011;

[10]. *SHENG-CHI LIU and DAPING CHU*, Deep learning for hologram generation, Vol. **29**, No. 17 / 16 Aug 2021 / Optics Express 27373;

[11]. *Tudor Florin Ursuleanu, Andreea Roxana Luca, Liliana Gheorghe, Roxana Grigorovici, Stefan Iancu, Maria Hlusneac, Cristina Preda, and Alexandru Grigorovici,* Deep Learning Application for Analyzing of Constituents and Their Correlations in the Interpretations of Medical Images, Diagnostics (Basel). 2021 Aug; 11(8): 1373;

[12]. *Oana Roxana Ciobotaru, Gabriela Stoleriu, Octavian Catalin Ciobotaru, Alexandru Grigorovici, Doina Carina Voinescu, Madalina Nicoleta Matei, Roxana Gabriela Cobzaru, Nicuta Manolache, and Mary-Nicoleta Lupu*, Postanesthetic skin erythema due to succinylcholine versus atracurium, Exp Ther Med; **20**(3): pp.2368–2372, 2020;

[13]. *Constantin Mihai, Dragos Octavian Palade, Cristian Budacu, Iuliu Fulga, Mihaela Gabriela Luca, Ana Gabriela Seni, Alexandru Grigorovici, Laurian Lucian Francu, Ioan Sarbu*, A Review of Operational Surgery in Ent and Cervical-Facial Tumours - Oral Cavity and Pharynx, Revista de Chimie (Rev. Chim.), Volume **70**, Issue 5, pp.1879-1883, 2019.