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THEORETICAL AND EXPERIMENTAL ANALYSIS FOR DETERMINING THE EIGENFREQUENCIES OF MECHANICAL SYSTEM WITH THREE MASSES

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This paper presents an analytical model for a mechanical system with three masses and a disruptive force applied to one of them. It presents an analysis of the free vibration, the vibration eigenmodes and forced vibration. Finally, there are several experimental determinations on a stand with three masses set with two elastic metal blades, made by authors and present measured values

Finally there is a good agreement between the values calculated using the theoretical model and the values resulting from experimental tests

Keywords: free vibration, eigenmodes, forced vibration, mechanical model.

1. Introduction

As general rule to data- processing equipment used for sorting/cleaning of the cereals are various mechanical systems that helps the process of sifting and sorting of cereals. The sieves for the sieving/sort are fixed to the engine frame in groups of two or three of them, a machine of this type, used in the process of sorting of cereals seed, because of the size and holes dimensions of sieves are obtained different aprons.Most systems of grading [1,2,3] uses mechanical drives (electric or thermal engines)and eccentric elements (or elements with eccentricity) produce a perturbing force in the system [2].

In this paper, the authors have identified the eigenfrequencies in a system modeled mechanically with three masses, where each mass is composed of an engine frame and a sieve by screening.

For the analytical study of the vibration transmitted to this system, we have introduced a perturbing force [4, 5, 10, 11] form $F_p = F_0 \cos \Omega t$ acting masses m_3 as shown in Fig.1.This simplified model has been built and some preliminary experimental tests were performed the most significant displacements produced in the horizontal direction. For this reason, in the model of the mechanical system with three masses only the displacements x_1, x_2, x_3 should be

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taken, as well as the active perturbing force F_p are in the horizontal direction, i.e., a motion from left to right, due to the perturbation introduced in the system.

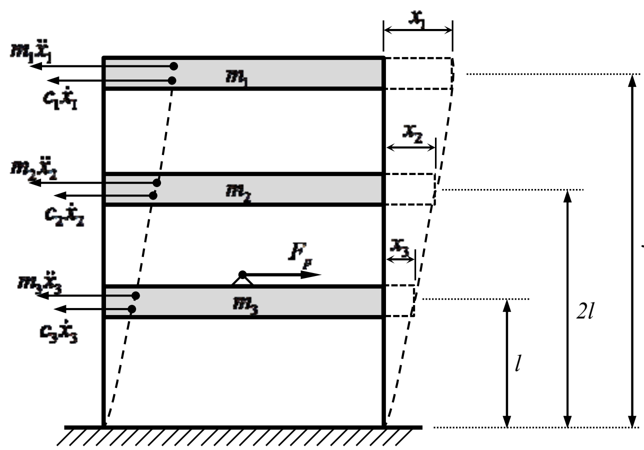

Fig. 1. Mechanical model with three masses

If the elastic elements are bending or systems of bars the expressions of the constraint forces are more difficult to determine, that referred expression entered to isolate masses, depending on movements. In these cases, for the analysis of vibration, the influence coefficient method [4, 6, 7, 12] is more convenient.

In the following, for this mechanical system with three masses, regarding the undamped free vibrations are analyzed, followed by the study of the forced vibrations, finally, some experimental results are available.

2. Analysis of free vibrations

Considering the system shown in figure 1 consisting of the masses m_1, m_2, m_3, fixed on two elastic vertical blades, rigid fixed. As mentioned above, the displacements of the masses x_1, x_2, x_3 on the horizontal direction are considered, appearing due to the bending of the elastic blades.

Using the method of coefficients of influence [5,12] the differential equations of motion of the vibrating system are:

$$\begin{aligned} x_1 &= \left(-m_1\ddot{x}_1 - c_1\dot{x}_1\right)\delta_{11} + \left(-m_2\ddot{x}_2 - c_2\dot{x}_2\right)\delta_{12} + \left(-m_3\ddot{x}_3 - c_3\dot{x}_3\right)\delta_{13} \\ x_2 &= \left(-m_1\ddot{x}_1 - c_1\dot{x}_1\right)\delta_{21} + \left(-m_2\ddot{x}_2 - c_2\dot{x}_2\right)\delta_{22} + \left(-m_3\ddot{x}_3 - c_3\dot{x}_3\right)\delta_{23} \\ x_3 &= \left(-m_1\ddot{x}_1 - c_1\dot{x}_1\right)\delta_{31} + \left(-m_2\ddot{x}_2 - c_2\dot{x}_2\right)\delta_{32} + \left(-m_3\ddot{x}_3 - c_3\dot{x}_3\right)\delta_{33} \end{aligned} \quad (1)$$

The system is written in matrix form:

$$\{x\} = -[\delta][M]\{\ddot{x}\} - [\delta][C]\{\dot{x}\} - [\delta][F]\cos\Omega t \quad (2)$$

or:

$$[\delta][M]\{\ddot{x}\} + [\delta][C]\{\dot{x}\} + \{x\} = [\delta]\{F\}\cos\Omega t\,, \quad (3)$$

where: $[M]$ - represent the mass matrix

$[C]$ - the damping matrix

By multiplying (3) to the left with $[\delta]^{-1}$ the following is obtained:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [\delta]^{-1}\{x\} = \{F\}\cos\Omega t\,. \quad (4)$$

Resulted the stiffness matrix:

$$[K] = [\delta]^{-1}. (5)$$

And the relation (5) can be written:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}\cos\Omega t\,. \quad (6)$$

A unit force is applied in the center of mass m_1, then in the center of mass m_2, then in the center of mass m_3 along the direction of the displacement.

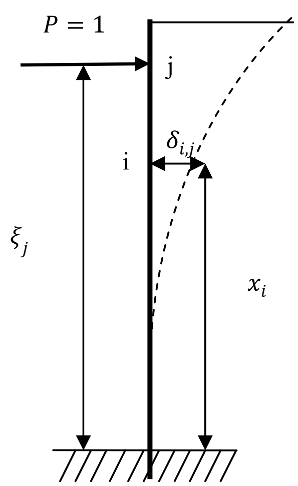

Fig. 2

Parameters δ_{ij} from Fig. 2 are the influence coefficients of the system and they represented the displacement in "*i*" section produced by a unit force in "*j*" section and the matrix $[\delta]$ is calculated using the relation [7]:

$$\delta_{ij} = \frac{x_i^2\left(3\xi_j - x_i\right)}{6EI}, \quad (7)$$

where: E is Young`s modulus (=2,1x 10^5N/mm^2);

I- geometrical moment of inertia of the cross section ($I=2I_0=40$mm^4).

and resulted:

$$\delta = \frac{l^3}{6EI}\begin{bmatrix} 54 & 28 & 8 \\ 18 & 16 & 5 \\ 8 & 5 & 2 \end{bmatrix}, \tag{8}$$

where: $l = \xi_i$ for mass m_3

that the stiffness matrix is:

$$[K] = [\delta]^{-1} = \frac{3EI}{13l^3}\begin{bmatrix} 7 & -16 & 12 \\ -16 & 44 & -46 \\ 12 & -46 & 80 \end{bmatrix}. \tag{9}$$

The different equations of the forced vibration with damping:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [\delta]^{-1}\{x\} = \{F\}\cos\Omega t \tag{10}$$

where: $[M]$ - represents the masses matrix $[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$,

$[C]$ - is the damping matrix $[C] = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$,

$[K]$ – is the stiffness matrixfrom eq.9.

The equation (10) considering force $F_0=0$ and damping coefficients $c_i = 0$, is the following:

$$[M]\{\ddot{x}\} + [K]\{x\} = 0. \tag{11}$$

It is considered that $m_1 = m_2 = m$ where m is the framework mass add sieve mass and $m_3 = \alpha m$ is sieve mass add eccentric mass, where α=1,308 it is constant.

The equations are the following:

$$\begin{cases} \ddot{x_1} + \frac{3EI}{13l^3m}(7x_1 - 16x_2 + 12x_3) = 0 \\ \ddot{x_2} + \frac{3EI}{13l^3m}(-16x_1 - 44x_2 - 46x_3) = 0, \\ \ddot{x_3} + \frac{3EI}{13l^3\alpha m}(12x_1 - 46x_2 + 80x_3) = 0 \end{cases} \tag{12}$$

with solutions:

$$\begin{cases} x_1 = a_1\cos\omega t \\ x_2 = a_2\cos\omega t \\ x_3 = a_3\cos\omega t \end{cases} \text{ and } \begin{cases} \ddot{x}_1 = -\omega^2 a_1\cos\omega t \\ \ddot{x}_2 = -\omega^2 a_2\cos\omega t \\ \ddot{x}_3 = -\omega^2 a_3\cos\omega t \end{cases}. \tag{13}$$

By substituting:

$$\omega_0^2 = \frac{3EI}{13l^3}, \tag{14}$$

algebraic equations are the following:

$$\begin{cases} a_1\left(7-\dfrac{\omega^2}{\omega_0^2}\right)-16a_2+12a_3=0 \\ -16a_1+\left(44-\dfrac{\omega^2}{\omega_0^2}\right)a_2-46a_3=0 \\ 12a_1-46a_2+\left(80-\dfrac{\omega^2}{\alpha\cdot\omega_0^2}\right)a_3=0 \end{cases}, \tag{15}$$

with solutions:

$\omega_1 = 10,47051\ rad/s$; $\omega_2 = 71,15817\ rad/s$; $\omega_3 = 201,6087\ rad/s$,

and eigenfrequencies are the following:

$$f_1 = 1,67\ Hz;\ f_2 = 11,32\ Hz;\ \ f_3 = 32,09\ Hz. \tag{16}$$

3. The analysis of the vibration eigenmodes.

Using the representation of the equations of motion of the system, the vibration mode shapes are determined. The vibration mode shapes are presented in the Fig. 3.

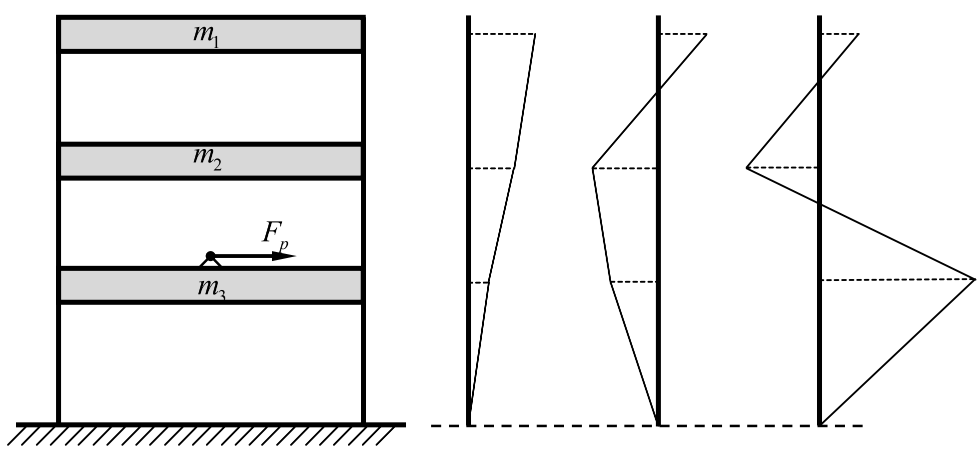

Fig. 3. Mode shapes (right)

The modal matrix $[\mu]$ is the following:

$$[\mu]=\begin{bmatrix} 1 & 1 & 1 \\ 0,5312 & -1,5955 & -3,3361 \\ 0,1560 & -1,2774 & 6,4742 \end{bmatrix}. \qquad (17)$$

In this case the solutions of free undamping vibration are the following:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}=\begin{bmatrix} 1 & 1 & 1 \\ 0,5312 & -1,5955 & -3,3361 \\ 0,1560 & -1,2774 & 6,4742 \end{bmatrix}\begin{Bmatrix} a_{11}\cos(\omega_1 t-\varphi_1) \\ a_{12}\cos(\omega_2 t-\varphi_2) \\ a_{13}\cos(\omega_3 t-\varphi_3) \end{Bmatrix}, \qquad (18)$$

where coefficients $a_{11}, a_{12}, a_{13}, \varphi_1, \varphi_2, \varphi_3$ are determined from the initial conditions regarding the position and the velocity.

4. The study of the forced vibration of the mechanical system with three masses

For the study of the forced vibration the normal coordinates are used.

The equation of the forced vibration of the system with viscous damping is considered:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F\}\cos\Omega t. \qquad (19)$$

The variable substitution:

$$\{x\}=[\mu]\{\xi\},\ \{\dot{x}\}=[\mu]\{\dot{\xi}\} \text{ and } \{\ddot{x}\}=[\mu]\{\ddot{\xi}\},$$

resulted:

$$[M][\mu]\{\ddot{\xi}\}+[c][\mu]\{\dot{\xi}\}+[K][\mu]\{\xi\}=\{F\}\cos\omega t \qquad (20).$$

Equation (38) is multiplied to the left with the transposed of matrix $[\mu]$, i.e. $[\mu]^t$. The matrix equation:

$$[\mu]^t[M][\mu]\{\ddot{\xi}\}+[\mu]^t[c][\mu]\{\dot{\xi}\}+[\mu]^t[K][\mu]\{\xi\}=[\mu]^t\{F\}\cos\omega t, \qquad (21)$$

is obtained.

In the equation above, the matrices multiplying $\{\ddot{\xi}\}$ and $\{\xi\}$ are diagonal. The damping matrix $[C]$, which multiplies $\{\dot{\xi}\}$, is considered proportional either with $[M]$ or with $[K]$. Thus, the matrix multiplying $\{\dot{\xi}\}$ is also diagonal.

In this case resulted:

$$[\mu]^t[M][\mu]=m\begin{bmatrix}1,3140 & 0 & 0\\ 0 & 5,6800 & 0\\ 0 & 0 & 66,9550\end{bmatrix} \tag{22}$$

and

$$[\mu]^t[K][\mu]=\frac{3EI}{13l^3}\begin{bmatrix}0,4843 & 0 & 0\\ 0 & 82,242 & 0\\ 0 & 0 & 6099,06\end{bmatrix}. \tag{23}$$

The non-diagonal elements are less than 0.001 and can be considered negligible, with an error less than 1‰.

The case of matrix $[C]$ proportional with $[M]$ is considered. The damping is due to the friction with the air, to the internal friction of the elements, to the friction in the connections of the parts and to the friction between the particles and the sieve. To achieve a mathematic model in a correlation so close with experimental construction, it was choose one of the considered model from vibrations study, proportional damping. These can be: proportional damping by inertia$[C]=c_0[M]$, by stiffness $[C]=c_1[K]$ or both $[C]=c_0[M]+c_1[K]$, where c_0, c_1 are constants [12, 14].

Proportional damping with inertia was choosing for the studied model because the material friction with sieve is significant. These models with proportional damping bring simplify the calculations.

It can be obtained:

$$[\mu]^t[C][\mu]=c_0\begin{bmatrix}1,3140 & 0 & 0\\ 0 & 5,6800 & 0\\ 0 & 0 & 66,9550\end{bmatrix}. \tag{24}$$

The product$[\mu]^t\{F\}$from the relation (28) is:

$$[\mu]^t\{F\}=\begin{bmatrix}1 & 0,5312 & 0,1560,\\ 1 & -1,5955 & -1,2774\\ 1 & -3,3361 & 6,4742\end{bmatrix}\begin{Bmatrix}0\\ 0\\ F_0\end{Bmatrix}=\begin{Bmatrix}0,1560\\ -1,2774\\ 6,4742\end{Bmatrix}F_0. \tag{25}$$

The coefficients of$\ddot{\xi}_k$, (*k=1,2,3)*are the modal masses; the coefficients of $\dot{\xi}_k$ are the modal damping; the coefficients of ξ_k are the modal stiffness..

From relation (21), after the division each relation with coefficient$\ddot{\xi}$ is obtained:

$$\begin{cases} \ddot{\xi}_1 + \dfrac{c_0}{m}\dot{\xi}_1 + 0,373\dfrac{3EI}{13ml^3}\xi_1 = 0,1187\dfrac{F_0}{m}\cos\Omega t \\ \ddot{\xi}_2 + \dfrac{c_0}{m}\dot{\xi}_2 + 17,2\dfrac{3EI}{13ml^3}\xi_2 = -0,2672\dfrac{F_0}{m}\cos\Omega t \\ \ddot{\xi}_3 + \dfrac{c_0}{m}\dot{\xi}_3 + 138,07\dfrac{3EI}{13ml^3}\xi_3 = 0,0739\dfrac{F_0}{m}\cos\Omega t \end{cases}. \quad (26)$$

The coefficients ξ_1, $\xi_2$and $\xi_3$in system (26) are the squares of the circular eigenfrequencies:

$$\omega_1^2 = 0,373\frac{3EI}{13ml^3};\ \omega_2^2 = 17,2\frac{3EI}{13ml^3};\ \omega_3^2 = 138,07\frac{3EI}{13ml^3}. \quad (27)$$

The system (26) for normal coordinate can be written:

$$\begin{cases} \ddot{\xi}_1 + \dfrac{c_0}{m}\dot{\xi}_1 + \omega_1^2\xi_1 = 0,1187\dfrac{F_0}{m}\cos\Omega t \\ \ddot{\xi}_2 + \dfrac{c_0}{m}\dot{\xi}_2 + \omega_2^2\xi_2 = -0,2672\dfrac{F_0}{m}\cos\Omega t \\ \ddot{\xi}_3 + \dfrac{c_0}{m}\dot{\xi}_3 + \omega_3^2\xi_3 = 0,0739\dfrac{F_0}{m}\cos\Omega t \end{cases}. \quad (28)$$

The damping factors

$$\zeta_1 = \frac{c_0}{2m\omega_1};\ \zeta_2 = \frac{c_0}{2m\omega_2};\ \zeta_3 = \frac{c_0}{2m\omega_3}. \quad (29)$$

Are introduced, as well as the amplitudes of the model of perturbation forces[5, 12].

$$q_1 = 0,1187\frac{F_0}{m};\ q_2 = -0,2672\frac{F_0}{m}; q_3 = 0,0739\frac{F_0}{m}. \quad (30)$$

By substituting above and relation(35) are obtained:

$$\begin{cases} \ddot{\xi}_1 + 2\cdot\omega_1\cdot\zeta_1\cdot\dot{\xi}_1 + \omega_1^2\cdot\xi_1 = q_1\cos\Omega t \\ \ddot{\xi}_2 + 2\cdot\omega_2\cdot\zeta_2\cdot\dot{\xi}_2 + \omega_2^2\cdot\xi_2 = q_2\cos\Omega t \\ \ddot{\xi}_3 + 2\cdot\omega_3\cdot\zeta_3\cdot\dot{\xi}_3 + \omega_3^2\cdot\xi_3 = q_3\cos\Omega t \end{cases}. \quad (31)$$

The general form of these equations is the following [5, 11, 13]:

$$\xi_i = \frac{q_i}{\omega_i^2 \sqrt{\left(1-\frac{\Omega^2}{\omega_i^2}\right)^2 + 4\zeta_i^2 \frac{\Omega^2}{\omega_i^2}}} \cos\left(\Omega t - \varphi_i\right);\ i = 1,2,3, \qquad (32)$$

where: $tg\varphi_i = \frac{2 \cdot \zeta_i \cdot \frac{\Omega}{\omega_i}}{1-\frac{\Omega^2}{\omega_i^2}};\ i = 1,2,3$.

The coefficients ζ_1, ζ_2 and ζ_3 represent the modal damping ratio, where it was considered the same values of the modal damping as against critical damping with an appropriate approximation. These were determinate separately and they have values close with 0.2.

The solution of forced vibrations with damping is the following:

$$\{x\} = [\mu]\{\xi\}, \qquad (33)$$

resulted:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0,513 & -1,5955 & -3,3361 \\ 0,1560 & -1,2774 & 6,4742 \end{bmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix}. \qquad (34)$$

That is:

$$\begin{cases} x_1 = A_1 \cos\left(\Omega t - \varphi_1\right) + A_2 \cos\left(\Omega t - \varphi_2\right) + A_3 \cos\left(\Omega t - \varphi_3\right) \\ x_2 = 0,513 A_1 \cos\left(\Omega t - \varphi_1\right) - 1,595 A_2 \cos\left(\Omega t - \varphi_2\right) - 3,336 A_3 \cos\left(\Omega t - \varphi_3\right), \\ x_3 = 0,156 A_1 \cos\left(\Omega t - \varphi_1\right) - 1,227 A_2 \cos\left(\Omega t - \varphi_2\right) + 6,474 A_3 \cos\left(\Omega t - \varphi_3\right) \end{cases} \qquad (35)$$

by substitute:

$$A_i = \frac{q_i}{\omega_i^2 \sqrt{\left(1-\frac{\Omega^2}{\omega_i^2}\right)^2 + 4\zeta_i^2 \frac{\Omega^2}{\omega_i^2}}};\ i = 1,2,3. \qquad (36)$$

5. Experimental results

For the experimental tests has been performed a stand similar to the previously proposed model shown in Fig. 1. It was conducted from three frames rectangular metal angle of known mass (equivalent masses m_1, m_2 and m_3) and were fixed on the side with two sofas rigidly fixed to the lower base. Adjustable vibration source was used a disc mounted eccentrically on an axle with two bearings mounted on the frame.

Experimental tests were performed with the eccentric shaft mounted in turn on three frames (masses m_1, m_2 and m_3) with vibration measurement, but for this paper was chosen only one situation, namely that it is excited mass m_3 which were previously presented analytical calculations.

To better understand the Fig. 4 shows some photos of the experimental tests. It can be seen in the figure, on the left side (Fig. 4a) signal acquisition module for USB and laptop used for recording data, in the middle Fig. 4b is observed masses and stand with three eccentric disc and shaft position in on the right side figure (Fig. 4c) accelerometer position on the metal frame. Actuation shaft was performed using electric motor with variable speed and a transmission cable which allows the transmission of motion from the engine to the axis of the eccentric disc.

a) *b)* *c)*

Fig. 4

For data acquisition was used an USB acquisition board, type NI 9233 from National Instruments with four-channel acquisition and a triaxial accelerometer type B 003 4306 Brüel&Kjaer. Data processing and identification of significant recorded parameters was performed with LabView 2011software.

The experimental results are summarized in Table 1 and Table 2, as compared with the previously analytical results.Among the identified values during experimental tests, three tests were chosen and then the average was calculated. After an analysis of the registered signals, the parameters of tables are separately indicated for every mass.

In table 2 the masses displacements were obtained by double integration of the measured acceleration signal values for each mass, of the horizontal direction (the direction of the perturbation force).

Table 1

Free vibration analysis

The corresponding mass	The calculated frequency [Hz]	The frequencies obtained by testing[Hz]			
		Test 1	Test 2	Test 3	**Average**
m_1	1,67	1,88	2,02	1,84	**1,91**
m_2	11,32	10,91	10,52	10,77	**10,73**
m_3	32,09	31,81	30,95	31,33	**31,36**

Table 2

Forced vibration analysis fora speedof 1200 rev/ min

The corresponding mass	The calculated displacements [mm]	The displacements obtained by testing[mm]			
		Test 1	Test 2	Test 3	**Average**
m_1	0,821	0,663	0,584	0,693	**0,647**
m_2	0,442	0,311	0,395	0,464	**0,390**
m_3	1,031	0,832	0,895	0,913	**0,880**

6. Conclusions

Based on the influence coefficients analytical methods and vibration modes, the classical sorting complex systems can be modeled. This can be useful in the equipment design calculations used in industrial processes for materials sifting and sorting.

The experimental results presented in Table 1 and Table 2 are quite close to the calculated results using the analytical calculation method. Therefore, we conclude that the analytical model is well defined and can be used for the other two cases, when the eccentric drive shaft is mounted on the table mass m_1 or m_2. Using this analytical model, recommendations can be made for the adjustment of the operating parameters of the equipment, for instance when the sieves or the materials to be sorted are changed, taking into account the influences of the approximated masses of the new sieves and material to be sorted. Also, for these types of models, it can be estimated the productivity, depending on operating parameters of the sorting equipment.

The assessment of the right resonance frequency for the sorting equipment can provide the right damping solution for the vibrations transmitted in the building (damping base).

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