U.P.B. Sci. Bull., Series D, Vol. 70, Iss. 4, 2008 ISSN 1454-2358

ON NITROGEN COMPOUNDS TRANSPORT IN UNSATURATED SOIL: ANALYTICAL AND NUMERICAL SOLUTIONS

Anca Marina MARINOV[1], Tudorel PETROVICI[2]

Lucrarea îşi propune să analizeze senzitivitatea unui model matematic folosit pentru descrierea comportării compuşilor azotului, într-un sol agricol. Modelul descrie matematic transformările biochimice suferite de diferitele forme ale azotului, într-un sistem sol-apă-plante. Modelul biochimic este cuplat cu un model hidrodinamic care descrie mişcarea apei în sol. Modelul cuplat, astfel obținut calculează concentrația nitraților care ajung la pânza freatică, ca urmare a folosirii îngrăşămintelor pe bază de azot. Sunt comparate, pentru un caz simplificat, soluția analitică cu cea numerică, pentru a aprecia comportarea modelului numeric propus, folosit apoi în cazuri mai complexe. Modelul permite aprecierea surselor difuze de poluare a apei subterane.

The goal of this paper is to analyze the sensitivity of a mathematical model used to describe nitrogen behavior in an unsaturated agricultural soil. The model describes mathematically the biochemical reactions among different compounds of nitrogen in a soil-water-plant system. This biochemical model is conjugated with a water transfer model, to compute the concentration of nitrates that reach groundwater. Hydraulic model provides the input data for the biochemical model. This paper will compare an analytical solution for a simplified case with the proposed numerical solution. The biochemical model represents an important link in the coupled model. The model provides data about the diffusive groundwater pollution sources.

Keywords: groundwater, water pollution, nitrogen, nitrate, coupled model.

1. Introduction

One of the most important groundwater pollution sources in the world is the intensive use of agricultural chemicals, especially nitrogenous fertilizers. Protecting the environment means first and foremost minimizing the harmful effects of man's activities. The problem of groundwater pollution by nitrogen compounds is of growing concern to many countries.

[1] Professor, Dept. of Hydraulics and Hydraulic Machinery, University POLITEHNICA of Bucharest, Romania
[2] Researcher, Dept. of Hydraulics and Hydraulic Machinery, University POLITEHNICA of Bucharest, Romania

Comprehensive field monitoring and research efforts are currently being carried out to establish the environmental management practices [1], [2], [3], [4].

Fig. 1 shows a general plants-soil-groundwater system. The groundwater (GW), (3), lies between two surface water bodies (5) and (6), beneath an unsaturated layer. The plants develop theirs roots in the first unsaturated (1), agricultural soil (RL- roots layer). The depth (z_1) of the roots layer depends on the crops type. Between RL ($z=z_1$) and the water table (4), the unsaturated (UL) layer, (2), doesn't contain roots.

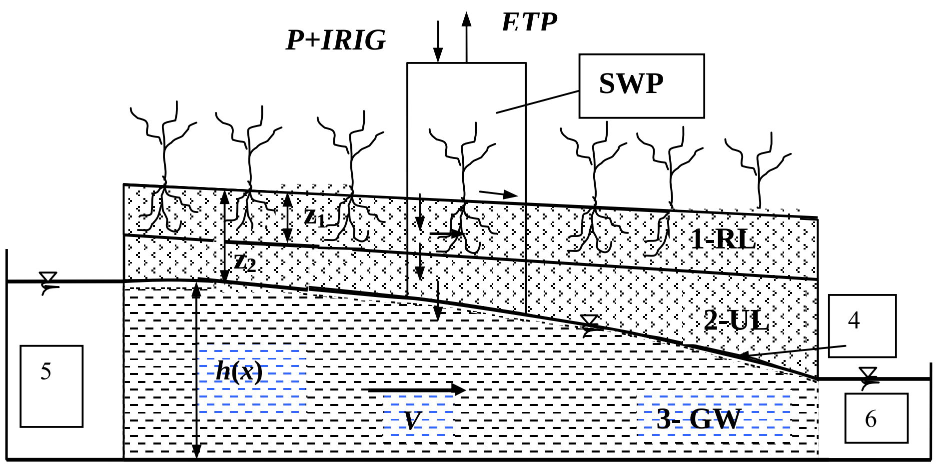


Fig. 1. A general groundwater-soil-plant system.

We will study the behavior of a local soil-water-plant system (SWP) considering 1ha ($10^4 m^2$) surface of the general system (fig.1). The water transfer through the (SWP) system influences the nitrogen cycle and the plant's life.

In this paper we consider a simple model connecting the water transfer with the nitrogen cycle model to predict the nitrogen –nitrate concentration ($N\text{-}NO_{3n}^-$) at the boundary between unsaturated layer (UL) and groundwater (GW).

The nitrogen concentration in the agricultural soil depends on the inorganic and organic fertilizers, organic mater, meteorological and irrigation conditions, soil properties and crops type.

2. Mathematical modeling of a simplified nitrogen cycle for a soil – water - plant system

The mechanism of natural water pollution by nitrogen compounds is governed by three distinct processes types: hydrodynamic mass transport, biological transformations and chemical reactions, and mass transfer at interface surfaces. The transformation study of the nitrogenous compounds is very complex. In the nitrogen cycle the chemical, hydro chemical and physical interaction, convert and reconvert the organic nitrogen into mineral nitrogen with an influence on the life cycle.

The model considers the nitrogen compounds transformations in SWP system (Fig.2). We refer to Geng [2] studies of these biochemical transformations.

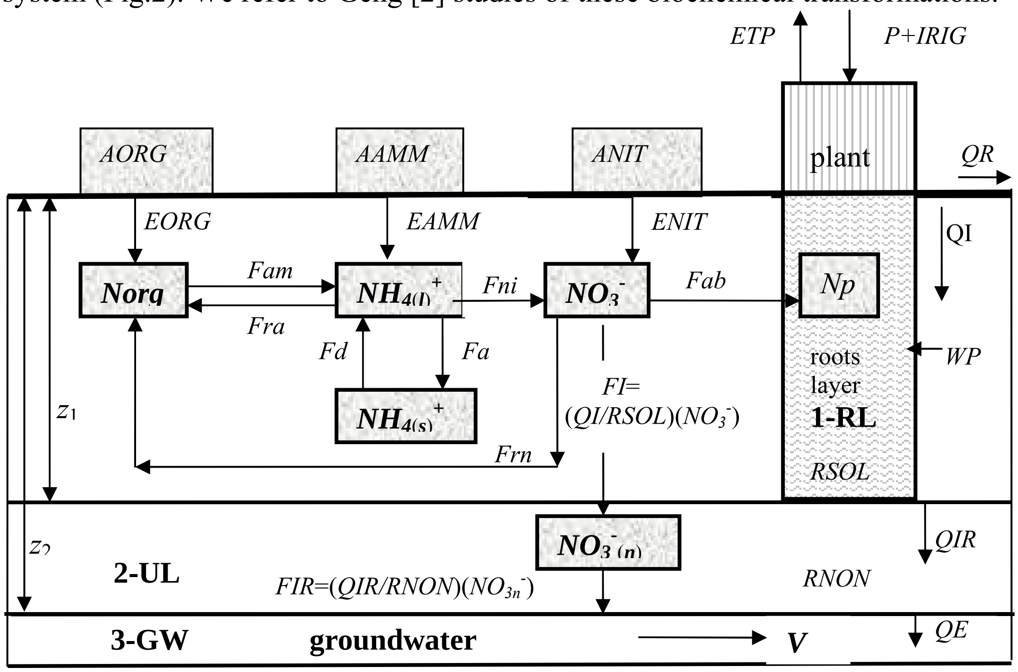


Fig. 2. The simplified nitrogen cycle and the water transfer elements for a local soil-water-plant system (SWP).

The biochemical model assumptions are:

1. Sources of nitrogen in the soil results from the plant residues decay, microbial and animal material decay, and fertilization.
2. The nitrogen forms, considered in the model are nitrate (NO_3^-), ammonium (NH_4^+), and organic nitrogen Norg.
3. Nitrogen transformations occur by means of mineralization (ammonification), immobilization (reorganization), nitrification and denitrification.
4. We will consider the nitrification one step reaction (neglecting the unstable nitrites form).
5. The effects of precipitation reaction and biologic fixation are compensated by denitrification and volatilization.
6. A quasi-deterministic modelling approach was adopted to simulate plant nitrogen uptake (NO_3^-). The nitrate uptake sub-model calculates the cumulative amount of nitrogen, taken by the crops since germination using a phenomenological equation proposed by Watts and Hanks [3].

The nitrification-denitrification processes, usually presented in literature as successive reactions of first order kinetics, are affected by: dissolved oxygen, micro-organisms, temperature, *pH* and carbon content.

The nitrogen concentrations in each component are related with the chemical entrances (fertilizers concentrations: *AORG, AAMM, ANIT*), and with the water transfers in the soil. The system describing the nitrogen transfers represents the changes, between the different nitrogen compounds in a simplified cycle (fig.2):

$$\frac{\mathrm{d}\left(NO_3^-\right)}{\mathrm{d}t} = ENIT - FI + Fni - Frn - Fab\,, \tag{1}$$

$$\frac{\mathrm{d}\left(NH_{4l}^+\right)}{\mathrm{d}t} = EAMM + Fam - Fni - Fra + Fd - Fa\,, \tag{2}$$

$$\frac{\mathrm{d}\left(NH_{4s}^+\right)}{\mathrm{d}t} = Fa - Fd\,, \tag{3}$$

$$\frac{\mathrm{d}\left(NH_4^+\right)}{\mathrm{d}t} = EAMM + Fam - Fni - Fra\,, \tag{4}$$

$$\frac{\mathrm{d}\left(Norg\right)}{\mathrm{d}t} = EORG + Frn + Fra - Fam\,, \tag{5}$$

$$\frac{\mathrm{d}\left(NO_{3n}^-\right)}{\mathrm{d}t} = FI - FIR\,. \tag{6}$$

The (4)-th equation has been obtained by summation of eqns. (2) and (3). For the biochemical intern transformations (ammonification, nitrification, reorganization) have been considered first order kinetics reactions:

$$\frac{\mathrm{d}C}{\mathrm{d}t} = -kC\,, \tag{7}$$

where C (kg ha^{-1}) is the nitrogen concentration in each compound, and k is the reaction rate coefficient. The transfer functions used in the eqns. system (1-6) are:

$$Frn = k_{rn}(NO_3^-) \tag{8}$$

$$Fam = k_{am}(Norg) \tag{9}$$

$$Fni = k_{ni}\,(NH_{4l}^+) = [k_{ni}/(1+k_d)](NH_4^+) \tag{10}$$

$$Fra = k_{ra}\,(NH_{4l}^+) = [k_{ra}/(1+k_d)](NH_4^+) \tag{11}$$

$$FI = (QI/RSOL)(NO_3^-) \tag{12}$$

$$FIR = (QIR/RNON)(NO_3^-{}_{(n)}) \tag{13}$$

The transfer functions *Fni* and *Fra* are calculated considering the total (NH_4^+) in the soil: $(NH_4^+) = (NH_4^+{}_{(l)}) + (NH_4^+{}_{(s)})$, (14)
where $(NH_4^+{}_{(l)})$ is the dissolved form, $(NH_4^+{}_{(s)})$ the absorbed one, and they are connected by a distribution coefficient k_d**:** $(NH_4^+{}_{(s)}) = k_d\,(NH_4^+{}_{(l)})$ (15)
FI (kg ha^{-1}/month) is the nitrate flux through roots layer RL (fig.2), and depends

on the nitrogen-nitrate concentration in this layer, (NO_3^-) (kg ha^{-1}), multiplied by the ratio between the water flux *QI* (m^{3}ha^{-1}/month) carrying the nitrates, and RL water content *RSOL* (m^{3}ha^{-1}). *FIR* (kg ha^{-1}/month) is the nitrate flux through unsaturated layer UL, and depends on the water flux *QIR* (m^{3}ha^{-1}/month), UL water content *RNON* (m^3 ha^{-1}) and on the nitrogen nitrate concentration in this layer, ($NO_3^-{}_{(n)}$) (kg ha^{-1}). k_{am}, k_{ra}, k_{ni}, k_{rn} are the rate values of chemical reactions, obtained by model calibration [2]. *ANIT, AAMM, AORG* are nitrogenous fertilizers (kg ha^{-1}), on the soil surface, and *ENIT, EAMM, EORG* are nitrogenous fertilizers (kg ha^{-1}), entering through the soil surface.

$$ENIT = (1-QR/P)ANIT \tag{16}$$

is the nitrate fertilizer quantity entering in SWP system considering *ANIT* solubility, the rain intensity, *P*, (m^{3}ha^{-1}/month), and the runoff discharge *QR* (m^{3}ha^{-1}/month). For *QR* = 0, (no runoff occurs) all the nitrate fertilizer quantity enters the soil. The ammoniacal and organic fertilizers are not soluble so all the quantities remain in the first layer RL and contribute to the nitrogen cycle:

$$EAMM = AAMM \text{ and } EORG = AORG \tag{17}$$

Plant nitrogen uptake can be considered by transfer function *Fab* (a Michaelis-Menten approach [1]):

$$Fab = B\, f_{pp}^{'}(t/\mathrm{T})\, [(NO_3^-)/((NO_3^-)+k_{ab})]. \tag{18}$$

B is the potential (maximum) nitrogen-nitrate uptake by the plant during a life cycle, *t* is the time since germination started, *T* is the total crop's life, k_{ab} is an equilibrium coefficient.

$$f_{pp}^{'}(t/T) = \frac{dF_{pp}^{'}(t^{'})}{dt^{'}}\frac{1}{T}, \quad t^{'} = \frac{t}{T}. \tag{19}$$

F'_{pp} (t/T) is the reduced function of potential cumulated uptake (fraction of total nitrogen uptake), and $t' = t/T$ is a fraction of growing season.

$$F_{pp}^{'}\left(\frac{t}{T}\right) = \begin{cases} 8.878\left(\frac{t}{T}\right)^{3.87}, & 0 \le \frac{t}{T} \le 0.3 \\ 0.66\frac{t}{T} + 3.485\left(\frac{t}{T}\right)^2 - 0.93\left(\frac{t}{T}\right)^3 - 0.899\left(\frac{t}{T}\right)^4, & 0.3 \le \frac{t}{T} \le 1 \end{cases} \tag{20}$$

3. Analysis of the analytical and numerical solutions for the equations system that describes the biochemical model

Let's consider the system of differential equations (1), (4), (5), (6) describing the distribution of nitrogen compounds in the local soil-water-plant system (fig. 2). Let us denote:

$$\begin{gathered} X_1 = NO_3^-,\ X_2 = NH_4^+,\ X_3 = N_{org},\ X_4 = NO_{3(n)}^-, \\ QI/RSOL = \alpha,\ QIR/RNON = \beta,\ QR/P = \gamma. \end{gathered} \tag{21}$$

By considering the transfer functions in the chemical reactions, *Fam, Fni, Fab, Fra, Frn, Fd, Fa, FI, FIR*, see fig. 2, and (8) – (13), the equations (1), (4), (5), (6) become:

$$\begin{cases} \dfrac{dX_1}{dt} = (-\alpha - B\dfrac{1}{X_1 - k_{ab}} f_{pp}'(\dfrac{t}{T}) - k_{rn})X_1 + (\dfrac{k_{ni}}{1+k_d})X_2 + (1-\gamma)ANIT \\ \dfrac{dX_2}{dt} = -(\dfrac{k_{ni}+k_{ra}}{1+k_d})X_2 + k_{am}X_3 + AAMM \\ \dfrac{dX_3}{dt} = k_{rn}X_1 + \dfrac{k_{ra}}{1+k_d}X_2 - k_{am}X_3 + AORG \\ \dfrac{dX_4}{dt} = \alpha X_1 - \beta X_4 \end{cases} \qquad (22)$$

We have studied three scenarios (different fertilizers schedule) for wheat crops with T = 9 months vegetation period with the plant nitrate consumption computed with (18), (19), (20).

The model parameters used for analytical and numerical analyses are:
k_d=0.35, k_{am} =0.06 month^{-1}, k_{ra}=1.1 month^{-1}, k_{ni}=2.2 month^{-1}, k_{rn}=0.8 month^{-1}, k_{ab}=3 kg ha^{-1}, B=218 kg ha^{-1}/year. (23)

The total fertilizers amount is 240 kg ha^{-1}/year. (*ANIT*=140 kg ha^{-1}/year, *AAMM*=60 kg ha^{-1}/year, *AORG*=40 kg ha^{-1}/year). (24)

α, β, γ are the hydrological parameters taken as monthly average values from the water transfer model. The fertilizers spreading schedules, (F.S.), for each scenario are presented in Table 2. For all three scenarios, the initial conditions are:

$$X_1(0) = X_2(0) = X_3(0) = X_4(0) = 10 \text{ kg ha}^{-1} \qquad (25)$$

The combination of the hydraulic model with the biochemical model (with input hydrological properties α, β, γ and output properties X_1, X_2, X_3, X_4) imposes the analysis of the system (22) solutions validity limits. The analysis is performed for a linearized model. The system (22) can be written:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = AX + F\,, \qquad (26)$$

$$A = \begin{bmatrix} -(\alpha+0.8) & 1.667 & 0 & 0 \\ 0 & -2.481 & 0.06 & 0 \\ 0.8 & 0.8185 & -0.06 & 0 \\ \alpha & 0 & 0 & -\beta \end{bmatrix},\ X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}, F = \begin{bmatrix} 140 - Bf_{pp}'(\frac{t}{T}) \\ 60 \\ 40 \\ 0 \end{bmatrix} \qquad (27)$$

and its initial conditions are described by (25).

As the hydraulic output parameters of the water transfer model lie between the margins $\alpha \in [0; 0.6161]$, $\beta \in [0;\ 0.0983]$ and $\gamma = 0$ we can consider:

$0 \le \alpha, \beta, \gamma \le 1$. The matrix associated with the linearized system of differential equations (26) admits the characteristic polynomial of A:

$$P_A(\lambda) = a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 \tag{28}$$

with the polynomial coefficients: $a_0 = 1$; $a_1 = \alpha + \beta + 3.341$,

$a_2 = 2.541\alpha + 3.341\beta + \alpha\beta + 2.1326$,

$a_3 = 0.0998\alpha + 2.1362\beta + 2.541\alpha\beta - 2.016/10^4$, $a_4 = -2.016/10^4\beta + 0.0998\alpha\beta$.

For $0 \le \alpha, \beta \le 1$ the problem admits real negative eigenvalues for all the studied cases. This way we compute the four eigenvalues in the ranges:
$\lambda_1 = -\beta$, $\lambda_2 \in [-0.0041, -0.0217]$, $\lambda_3 \in [-2.4816, -2.4501]$, $\lambda_4 \in [-0.8595, -1.8692]$.

We also check that the solutions is stable asymptotically and completely controllable.

For all scenarios we have studied the analytical solutions having the form:

$$X(t) = W(t)C + W(t)C(t), \text{ with } W(t) = e^{tU}V \text{ and } U = VDV^{-1} \tag{29}$$

where: V (modal matrix of A) and D (canonical form of A) are the eigenvectors and eigenvalues matrices for matrix A, (27), and C is the integration constants matrix. For the limit case $\alpha = \beta = B = 0$, system (22) is asymptotically unstable, but even in this case the theoretical solution is monthly robust and very closed with the numerical one. In fig. 3 we compare the analytical and numerical solutions for the 10-th month (first year), with the initial conditions:

$$X_1(9) = 0.65152, \; X_2(9) = 3.64397 \; X_3(9) = 148.44953, \; X_4(9) = 35.34 \tag{30}$$

The analytical solution, obtained for these conditions is:

$$\begin{cases} X_1(t) = 8.6103 - 7.9120e^{0.0015t} + 110.8115\,e^{-0.8617t} + 103.3223e^{-2.41797t} \\ X_2(t) = 7.4803 - 3.7918e^{0.0015t} - 10.6073e^{-0.8617t} - 100.3018e^{-2.41797t} \\ X_3(t) = 303.8024 - 153.2630e^{0.0015t} - 106.4424e^{-0.8617t} - 0.3958\,e^{-2.41797t} \\ X_4(t) = 35.34 \end{cases} \tag{31}$$

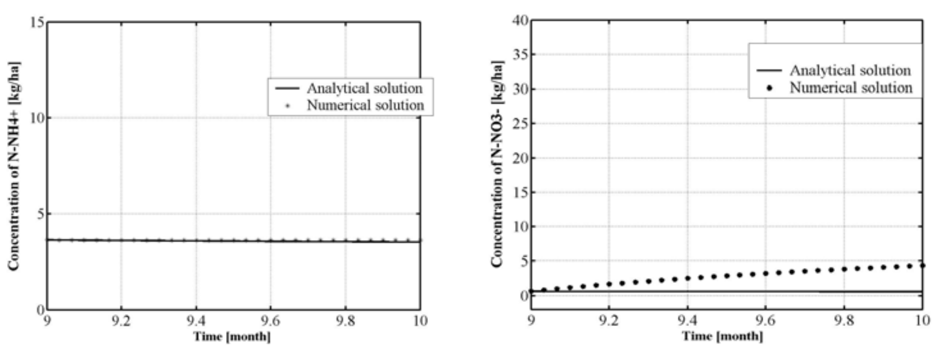


Fig. 3. Concentration of $N\text{-}NH_4^+$ and $N\text{-}NO_3^-$ (kg/ha) for the 10-th month (* analytical solution, - numerical solution).

The slightly complicated form of the analytical solutions and the difficulties related to the weak conditioning of certain operations during certain phases of the algorithm imposes to use them carefully in prediction models and only after stability Liapunov checks are performed.

Starting from the numerical analysis referring to the asymptotical stability of the biochemical model we looked at three scenarios (table 2) for the system (22) with biochemical input data (23), (24) and initial conditions (25) using numerical simulation. The numerical results demonstrate that the proposed method provides a robust and efficient alternative to analytical modeling.

Solution algorithm for water transport model (§ 4) and biochemical model were used together, simultaneously in standard software to solve system (22) (Octave-using the lsode procedure- and Matlab - using the ode15s procedure).

For each scenario and each month, we coupled the two models (the biochemical model takes over the monthly hydraulic parameters α, β, γ, provided by water transfer model).

The system's (22) numerical solutions, obtained using different integration methods (lsode and ode15s procedure) are identical. Such kind of numerical solutions are represented in fig. (5).

4. Water transfer model in a local soil-water-plant system

The volumetric water content of unsaturated layers (*RSOL* and *RNON*) and the water flux (*QI*) through RL or (*QIR*) through UL (fig.2), are very important in the nitrogen cycle behavior. We propose here a simple model to describe monthly, the water flow in a local SWP system.

Major assumptions of the water simulation model are:

1. The hydraulic balance has been done for two consequently years.
2. The monthly meteorological data are the same for two years.
3. For the wheat crops the effective rooting depth is z_1=0.5 m.
4. The water table is beneath the level z_2=1.5 m (fig.1).
5. The initial soil water content corresponds to field capacity.
6. The total watering quantity during irrigation scheduling is calculated imposing to meet the optimum plants requirement.
7. The runoff doesn't occurs (QR=0).

For each month, j=1 to j=12, the water content in the RL layer has to assure the field capacity: $RSOL \geq Vc$, (32)

$$Vc = 100\,(z_1)\,C(\%)\,\rho_b/\rho_w, \tag{33}$$

where Vc(m^3/ha) is the soil volumetric water content corresponding to field capacity $C(\%)$, ρ_b (kg/ m^3) is the bulk density (dry soil mass/soil total volume), ρ_w(kg/ m^3) is water density. The volumetric water content corresponding to field capacity is: $\theta(\%)$= C(%) ρ_b/ρ_w

The volumetric water content $\theta(\%)$ couldn't be less than a residual θ_r.

Another important coefficient is the wilting point of the soil, $Cw(\%)$ in percents from the dry soil mass. The Vw (m^3/ha) is the soil volumetric water content corresponding to the wilting point of the soil, $Cw(\%)$:

$$Vw = 100\,(z_1)\,Cw(\%)\,\rho_b/\rho_w. \quad (34)$$

In our model we will use the assumption (32). If at the beginning of the month the RL water content is less than Vc, $RSOL(j\text{-}1) < Vc$, (35)
the irrigation quantity $IRIG(j)$ (m^{3}ha^{-1}/month) used in the month 'j' will be:

$$IRIG(j) = Vc - RSOL(j\text{-}1). \quad (36)$$

The total watering requirement (TWR) during irrigation scheduling is calculated so that it satisfies the water plant necessity, WP (m^{3}ha^{-1}/month) [5], and the fertilization program.

$$TWR\ (\text{m}^3\text{ha}^{-1}/\text{year}) = \sum_{j=1}^{j=12} IRIG(j). \quad (37)$$

Considering the runoff QR=0, and

$$QI(j) = IRIG(j) + P(j) - ETP(j), \quad (38)$$

the water balance during each month gives the volumetric water content at the end of the month:

$$RSOL(j) = RSOL(j\text{-}1) + [IRIG(j) + P(j) - ETP(j) - WP(j) - QIR(j)]\,\mathrm{d}t. \quad (39)$$

$P(j)$ (m^{3}ha^{-1}/month) is the monthly rain volume (on a 1ha land surface). The evapotranspiration $ETP(j)$ (m^{3}ha^{-1}/month) and $WP(j)$ (m^{3}ha^{-1}/month) are input data for the hydraulic model. $QIR(j)$ is the discharge percolating through UL layer (fig.2). If $RSOL(j\text{-}1)$ is greater then saturated soil volumetric water content, $RSAT$

$$RSOL(j\text{-}1) > RSAT, \quad (40)$$

the water volume entering in UL layer is:

$$VIR(j) = RSOL(j\text{-}1) - RSAT = QIR(j)\,\mathrm{d}t. \quad (41)$$

Water balance for the layer UL:

$$RNON(j) = RNON(j\text{-}1) + [QIR(j) - QE(j)]\,\mathrm{d}t, \quad (42)$$

$QE(j)$ is the discharge percolating toward the groundwater.

$$RNON(j\text{-}1) > RNONSAT, \quad (43)$$

the water volume entering in GW layer is:

$$VIN(j) = RNON(j\text{-}1) - RNONSAT, = QE(j)\,\mathrm{d}t \quad (44)$$

The hydraulic model gives the values $\alpha(j) = QI(j)/RSOL(j)$ and $\beta(j) = QIR(j)/RNON(j)$ used in the biochemical model (Table 1).

5. Soil-water-plant coupled model application

The results for two years modeling of a local SWP system are presented in the figures 4 and 5. For a wheat crop (15 X-15VII) (z_1=0.5m, z_2=1.5m), the soil (loamy send) properties are: ρ_b/ρ_w =1.27**;** C =17.3 (%)**;** Cw=7.5(%)**;** $\theta r(\%)$=0.09

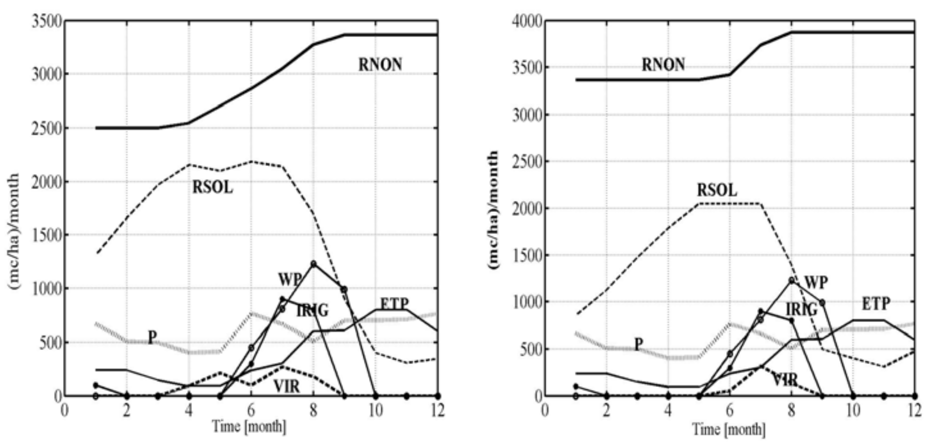
(%); *θsat*(%)=41 (%)**.** The input data, *P*, *ETP* and *WP* (Table 1) are total monthly values, in (mm), and *IRIG*(*j*) (m^3ha^{-1}/month) is the total irrigation water volume used during the *j*–th month.

Figures (4.a) and (4.b) show the water transfer model results for two years.

Table 1

Input and output data in the water transfer model (for the first year)

j	1	2	3	4	5	6	7	8	9	10	11	12
month	X	XI	XII	I	II	III	IV	V	VI	VII	VIII	IX
P	66.7	50.8	50.1	40.4	41.1	76.4	66.7	50.8	70.1	70.4	71.1	76.4
ETP	24.0	24.0	15.0	9.3	9.3	24.0	30.4	60.0	60.8	80.0	80.0	60.0
WP	-	-	-	-	-	45	81	123	99	-	-	-
IRIG	100	-	-	-	-	-	-	-	300	900	800	-
α	0.40	0.16	0.18	0.15	0.15	0.40	0.62	0.42	0.10	0	0	0.47
β	0	0	0	0.037	0.080	0.036	0.089	0.055	0	0	0	0



a. First year b. Second year

Fig. 4. Water transfer model results for two years.

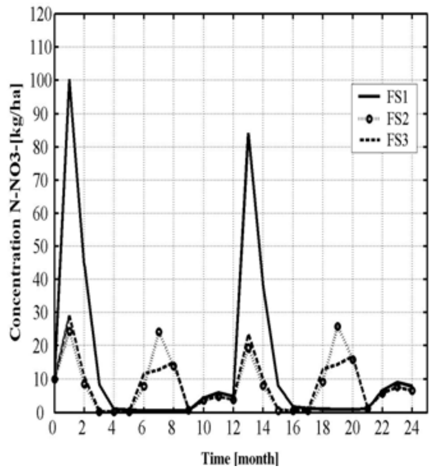
The input data for the biochemical model, have been done in § 3 by (23) and (24). Three schedules for fertilizers spreading (F.S.) are considered (Table 2). These values are percents from the total quantity. Each component has this percent from its annual value.

The model results are plotted in fig. 5. The (N-NO_3^-) values decrease rapidly during the first four months because the soil is almost saturated and the water is drained through the roots layer RL to the UL (where the (N-$NO_{3\ (n)}^-$) values increase). Also, the plants roots are growing during these months (Norg increases). In FS1 the plants do not have nitrate enough (in the 5-8 th months). The bests schedule for fertilizers spreading is the third because it achieves the minimal concentration of ($N\text{-}NO_3^{-}{}_{(n)}$) (fig.5.d) and enough $N\text{-}NO_3^-$ in RL layer (fig.5.a)

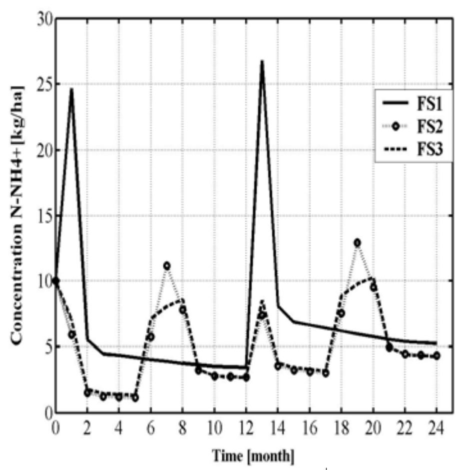
Table 2

Fertilization schedule (percents from the total fertilizers quantity)

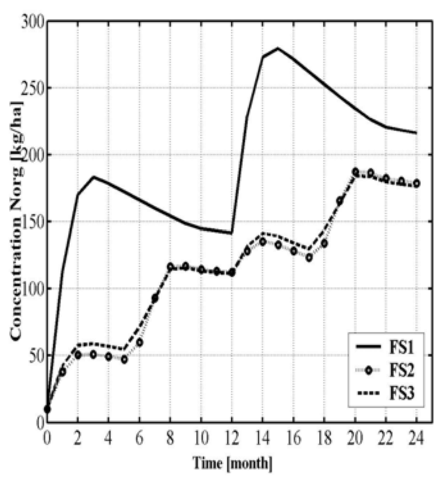
j	1	2	3	4	5	6	7	8	9	10	11	12
month	X	XI	XII	I	II	III	IV	V	VI	VII	VIII	IX
F.S. 1	20	-	-	-	-	20	40	20	-	-	-	-
F.S. 2	100	-	-	-	-	-	-	-	-	-	-	-
F.S. 3	25	-	-	-	-	25	25	25	-	-	-	-



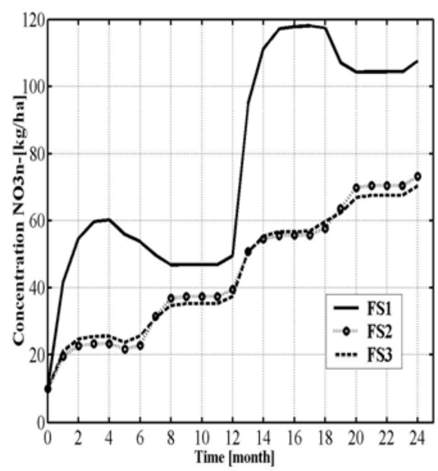
a) Concentration of $N\text{-}NO_3^-$ (kg/ha)



b) Concentration of $N\text{-}NH_4^+$ (kg/ha)



c) Concentration of Norg (kg/ha)



d) Concentration of $(N\text{-}NO_3^{-}{}_{(n)})^-$ (kg/ha)

Fig.5. Biochemical model results for two years.

6. Conclusions

Our work has studied mathematical models of various kinds of nitrogen compounds transfers in an agricultural soil.

The biochemical model requires the hydraulic parameters of the soil-water-plant system and provides the nitrogen compounds concentrations.

Our analysis shows that an analytical solution (closed form) of the model is available under restrictive constraints only. We show the solutions asymptotic stability.

A numerical scheme to integrate the differential equations has been proposed.

Another issue is to couple the above biochemical model with water transfer one. Thus, a quantitative link between precipitation, evapotranspiration, irrigation, fertilization schedule, and the nitrogen concentration can be studied.

By taking into account the water and nutrients demand, we can give an optimal fertilization schedule. It ensures the minimum nitrates reaching the groundwater and the necessary nitrates in the roots plants.

The soil characteristics and the reaction coefficients which appear in our model have to be found out by field measurements.

By comparison with the current models, the one proposed here is relatively simpler by its fewer parameters number.

For better results the coupled model can be improved by using a more detailed water transfer model [2], [4]. In such kind of hydraulic models the daily precipitation and evapotranspiration regimes are necessaries.

The nitrate concentration from unsaturated layer above the groundwater can be used further as monthly boundary condition for the nitrates dispersion equation in the phreatic aquifer.

R E F E R E N C E S

[1] *J. Feher, M.TH. Van Genutchen, G. Kienitz, T. Nemeth, Gy. Biczok, G.J. Kovacs*, DISNIT2, a Root Zone Water and Nitrogen Management model, Viena Symposium: Hydrological Interaction between Atmosphere, Soil and Vegetation, IAHS Publ. no.204, 1991, pp. 197-205

[2] *Q.Z. Geng*, Modélisation conjointe du cycle de l'eau et du transfert des nitrates dans un système hydrologique, PhD. Thesis E N S M de Paris, 1988.

[3] *D.G Watts, R.J. Hanks,* A soil-water-nitrogen model for irrigated corn on sandy soil, Soil Sci. Soc. Amer. J., no. 42, pp.492-499, 1978.

[4] *A.M. Marinov, M.A. Diminescu*, Experimental research and mathematical modelling of soil and groundwater contamination with nitrogen compounds, Water pollution 2008, WIT Conference, Alicante 9-11 June 2008, (accepted paper).

[5] *I. Plesa, GH. Florescu*, Irigarea Culturilor, Editura Ceres, Bucuresti, 1974.