U.P.B. Sci. Bull., Series D, Vol. 81, Iss. 4, 2019 ISSN 1454-2358

NUMERICAL ANALYSIS ON THE CORRELATION BETWEEN BOGIE DYNAMIC RESPONSE AND VERTICAL TRACK IRREGULARITIES

Mădălina DUMITRIU[1], Dorina FOLOGEA[2]

The paper herein submits an analysis regarding the correlation between the dynamic response of a two-axle bogie and the displacements of the axles during circulation on a track with vertical irregularities. To this purpose, the results from numerical simulations are used, in a steady-state harmonic behaviour. The correlation between the bogie dynamic response at different velocities and the track vertical irregularities is evaluated by means of the Pearson correlation coefficient calculated depending on the frequency response of the bogie chassis in two points against the axles. The results show that the correlation between the bogie dynamic response and the displacements induced by the vertical track irregularities depends on velocity, frequency and the suspension damping.

Keywords: railway bogie, vertical track irregularities, Pearson correlation coefficient, suspension, fault detection

1. Introduction

It is well known that the vibrations of the railway vehicle are mainly generated by the track irregularities - track geometric irregularities, irregularities in the rolling surfaces and discontinuities of the rails, which mostly come from the construction imperfections, track exploitation, change in the infrastructure due to the action of the environment factors or soil movements [1, 2]. Running on a track with irregularities will generate vibrations of the wheelset [3 - 5], which are transmitted to the bogie and the carbody, so that the dynamic response of the entire vehicle is affected by the track irregularities [6 - 8].

Establishing a correlation between the track irregularities and the dynamic response of the vehicle is essential from the perspective of monitoring the track quality, so as to facilitate an efficient track maintenance [9, 10], and of monitoring the condition in the vehicle suspension [11 - 14]. As for the latter, it provides a series of benefits to the rail operators. When faults of the suspension components are detected in an early stage, the deterioration in the vehicle performance is

[1] Assoc. Prof., Department of Railway Vehicles, University Politehnica of Bucharest, Romania, e-mail: madalinadumitriu@yahoo.com
[2] PhD student, Doctoral School of Transports, University Politehnica of Bucharest, Romania, e-mail: f_dorina@yahoo.com

prevented and safety increased. Repairs or timely replacement of the faulty components increases the reliability and availability of the railway vehicle. Last but not least, the costs associated to the vehicle maintenance can be greatly reduced by using the condition-based instead of the calendar-based maintenance [14, 15].

As a principle, there should be a close correlation between the geometrical quality of the track and the dynamic response of the vehicle. Nevertheless, as shown in paper, the correlation depends under certain conditions on frequency, velocity as well as on the condition of the vehicle suspension.

The paper examines the correlation between the dynamic response in a two-axle bogie and the vertical track irregularities, based on the Pearson correlation coefficient, calculated in function of the frequency response of the bogie chassis in two points against axles. The derived results confirm that premises are favorable for developing a method to detect the faulty dampers during operation, based on the coefficient of correlation between the bogie dynamic response and the displacements induced by the vertical track irregularities in a selected convenient frequency interval.

2. The bogie mechanical model and the motion equations

The mechanical model in figure 1 is used to study correlation between the dynamic response of a two-axle bogie and the vertical track irregularities [15]. The hypothesis of a perfectly rigid track is considered, which means that the axles closely follow the vertical irregularities of the track described against each axle by the functions $\eta_{1,2}$.

The bogie model includes 3 rigid bodies that help with modelling the bogie chassis and the two axles connected between them by Kelvin-Voigt type systems that model the suspension corresponding to each axle. The elastic element of the suspension has the constant $2k$ and the damping constant $2c_1$ and $2c_2$, respectively. In the suspension corresponding to each axle in a bogie is normally noticed that damping elements with identical characteristics are used ($2c_1 = 2c_2 = 2c$). To analyze the correlation between the bogie response and the vertical track irregularities when a damper ceases to function, the $2c_1 \neq 2c$ and $2c_2 \neq 2c$, respectively, have been considered.

The modes of vibration of the bogie in a vertical plan – bounce z_b and pitch θ_b are being taken into account. The bogie parameters are as such m_b – mass of the bogie, $2a_b$ - wheelbase, J_b – moment of inertia.

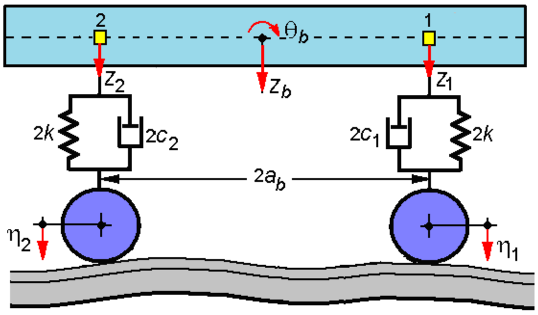

Fig. 1. The mechanical model of the bogie.

The bogie motion equations are as below:

$$m_b\ddot{z}_b = F_1 + F_2\,, \tag{1}$$

$$J_b\ddot{\theta}_b = a_b(F_1 + F_2)\,, \tag{2}$$

where F_1 and F_2 stand for the forces due to the suspension,

$$F_{1,2} = -2c_{1,2}(\dot{z}_b \pm a_b\dot{\theta}_b - \dot{\eta}_{1,2}) - 2k(z_b \pm a_b\theta_b - \eta_{1,2})\,. \tag{3}$$

If notations below are introduced,

$$z_{1,2} = z_b \pm a_b\theta_b\,, \tag{4}$$

where $z_{1,2}$ are the displacements of the bogie chassis against the two axles, the bogie motion equations become:

$$\frac{m_b}{2}(\ddot{z}_1 + \ddot{z}_2) + 2c_1\dot{z}_1 + 2c_2\dot{z}_2 + 2k(z_1 + z_2) = 2c_1\dot{\eta}_1 + 2c_2\dot{\eta}_2 + 2k(\eta_1 + \eta_2)\,, \tag{5}$$

$$\frac{J_b}{a_b^2}(\ddot{z}_1 - \ddot{z}_2) + 2c_1\dot{z}_1 - 2c_2\dot{z}_2 + 2k(z_1 - z_2) = 2c_1\dot{\eta}_1 - 2c_2\dot{\eta}_2 + 2k(\eta_1 - \eta_2)\,. \tag{6}$$

3. The calculation of the Pearson correlation coefficient

The Pearson correlation coefficient is defined as the covariance of the two variables divided by the product of their standard deviations. For instance, the Pearson correlation coefficient between the time periodic variables $z(t)$ and $\eta(t)$ is

$$\rho_{z,\eta} = \frac{\operatorname{cov}(z,\eta)}{\sigma_z \sigma_\eta} = \frac{\frac{1}{T}\int_0^T [z(t) - \tilde{z}][\eta(t) - \tilde{\eta}]\mathrm{d}t}{\sqrt{\frac{1}{\mathrm{T}}\int_0^T [z(t) - \tilde{z}]^2 \mathrm{d}t}\sqrt{\frac{1}{\mathrm{T}}\int_0^T [\eta(t) - \tilde{\eta}]^2 \mathrm{d}t}}, \tag{7}$$

where cov(z, η) denotes the covariance, $\sigma_{z,\eta.}$ – the standard deviation, T is the period and $\tilde{z}$ and $\tilde{\eta}$ are the mean values. When the two variables are harmonic functions, the Pearson correlation coefficient becomes

$$\rho_{z.\eta} = \cos\varphi, \tag{8}$$

where φ is the phase difference between $z(t)$ and $\eta(t)$.

To calculate the Pearson correlation coefficient, it is necessary to firstly find out the frequency response functions of the bogie. For this, the vertical track irregularities are considered to be in a harmonic shape with the wavelength Λ and amplitude η_0 that are outphased against the axles corresponding to the distance between them, $2a_b$, respectively. Therefore, the functions $\eta_{1,2}$ describing the track irregularities against the two axles are in the form of

$$\eta_{1,2}(x) = \eta_0 \cos\frac{2\pi}{\Lambda}(x \pm a_b), \tag{9}$$

where $x = Vt$ is the coordinate for the bogie centre and V is the velocity.

The functions $\eta_{1,2}$ can also be written as time harmonic functions, namely

$$\eta_{1,2}(t) = \eta_0 \cos\omega\left(t \pm \frac{a_b}{V}\right), \tag{10}$$

where $\omega = 2\pi V/\Lambda = 2\pi f$ represents the angular frequency induced by the track excitation, and $f = V/\Lambda$ is the correlative frequency.

As for the bogie response, this is assumed to be harmonic, with the same frequency as the track excitation induced frequency. The coordinates describing the motions of the bogie are written under the general form as

$$z_{1,2}(t) = Z_{1,2}\cos(\omega t - \varphi_{1,2}), \tag{11}$$

where $Z_{1,2}$ is the amplitude, and $\varphi_{1,2}$ represents the phase compared to the track vertical irregularities with respect to the bogie centre.

The complex numbers associated with real ones are introduced in the equations (5 - 6)

$$\overline{\eta}_{1,2}(t) = \overline{E}_{1,2} e^{\mathrm{i}\omega t}, \text{ with } \overline{E}_{1,2} = \eta_0 e^{\pm\frac{\mathrm{i}\omega a_b}{V}} \tag{12}$$

$$\bar{z}_{1,2}(t) = \overline{Z}_{1,2} e^{\mathrm{i}\omega t}, \text{ with } \overline{Z}_{1,2} = z_{1,2} e^{-\mathrm{i}\varphi_{1,2}}. \tag{13}$$

A linear system of non-homogeneous algebraic equations is obtained, whose solution allows the calculation of the frequency response functions of the bogie, against the two axles,

$$a_{11}\overline{Z}_1 + a_{12}\overline{Z}_2 = b_1\overline{\eta}_0, \tag{14}$$

$$a_{21}\overline{Z}_1 - a_{22}\overline{Z}_2 = b_2\overline{\eta}_0, \tag{15}$$

with: $a_{11} = -\frac{1}{2}\omega^2 m_b + 2\mathrm{i}\omega c_1 + 2k$, $a_{12} = -\frac{1}{2}\omega^2 m_b + 2\mathrm{i}\omega c_2 + 2k$,

$$a_{21} = -\frac{1}{2}\omega^2 \frac{J_b}{a_b^2} + 2\mathrm{i}\omega c_1 + 2k, \quad a_{22} = -\frac{1}{2}\omega^2 \frac{J_b}{a_b^2} + 2\mathrm{i}\omega c_2 + 2k,$$

$$b_1 = (2\mathrm{i}\omega c_1 + 2k)e^{\frac{\mathrm{i}\omega a_b}{V}} + (2\mathrm{i}\omega c_2 + 2k)e^{-\frac{\mathrm{i}\omega a_b}{V}},$$

$$b_2 = (2\mathrm{i}\omega c_1 + 2k)e^{\frac{\mathrm{i}\omega a_b}{V}} - (2\mathrm{i}\omega c_2 + 2k)e^{-\frac{\mathrm{i}\omega a_b}{V}}.$$

The response functions of the bogie chassis in the points 1 and 2 (see fig. 1) are as such

$$\overline{H}_1 = \frac{\overline{Z}_1}{\overline{E}_1} = e^{-\frac{\mathrm{i}\omega a_b}{V}} \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}, \quad \overline{H}_2 = \frac{\overline{Z}_2}{\overline{E}_2} = e^{\frac{\mathrm{i}\omega a_b}{V}} \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}}. \tag{16}$$

Corroborating the frequency response functions and Eq. (8), the Pearson correlation coefficient between the bogie displacement above the front/rear wheelset and the corresponding track irregularity can be calculated applying the following equations

$$\rho_1 = \frac{\mathrm{Re}\,\overline{H}_1}{\left|\overline{H}_1\right|},\ \rho_2 = \frac{\mathrm{Re}\,\overline{H}_2}{\left|\overline{H}_2\right|}. \tag{17}$$

The Pearson correlation coefficient is a measure of the correlation between the bogie response $z_{1,2}(t)$ and the vertical track irregularities $\eta_{1,2}(t)$, both of a constant amplitude $Z_{1,2}$, and η_0, and variable angular frequency ω. This can take values between 1 and -1, where 1 show total positive correlation – the bogie response above a wheelset and the corresponding irregularity are in phase, and −1 is total negative correlation – the two variables are out of phase. The value zero shows the lack of correlation – the bogie response and the track irregularity are in quadrature.

4. The results of the numerical simulations

This section features the results from numerical simulations about the bogie response to the vertical track irregularities and the correlation between them, depending on velocity and various cases of reduction in the damping constant of the suspension reported to its reference value. The reference parameters of the bogie used in the numerical simulations are included in Table 1 and the natural frequencies of the vibration bogie modes are: 5.9 Hz for bounce and 9.4 Hz for pitch [15].

Table 1

Reference parameters of the bogie

Bogie mass	m_b = 3200 kg
Bogie wheelbase	$2a_b$ = 2.56 m
Moment of inertia	$J_b = 2.05\cdot10^3$ kg·m^2
Elastic constants of suspension	k = 1.10 MN/m
Damping constants of suspension	c = 13.05 kNs/m

Figure 2 shows the frequency response functions of the bogie chassis against the two axles (in the points 1 and 2 – see figure 1) for a velocity range from 20 to 200 km/h. The presented result highlights a series of basic properties of the vibrations behaviour of the bogie.

One is the asymmetrical vibrations bogie behaviour – the level of vibrations above axle 2 is higher than above axle 1. The asymmetry of the dynamic bogie response against the two axles is due to the suspension damping. If $2c_1 = 2c_2 = 0$, then the bogie response against the two axles is symmetrical (see figure 3).

Next, the level of bogie vibrations is noticed to be increasing along with the velocity, but this growth is not uniform because of the geometric filtering effect given by the bogie wheelbase [6, 7, 16, 17].

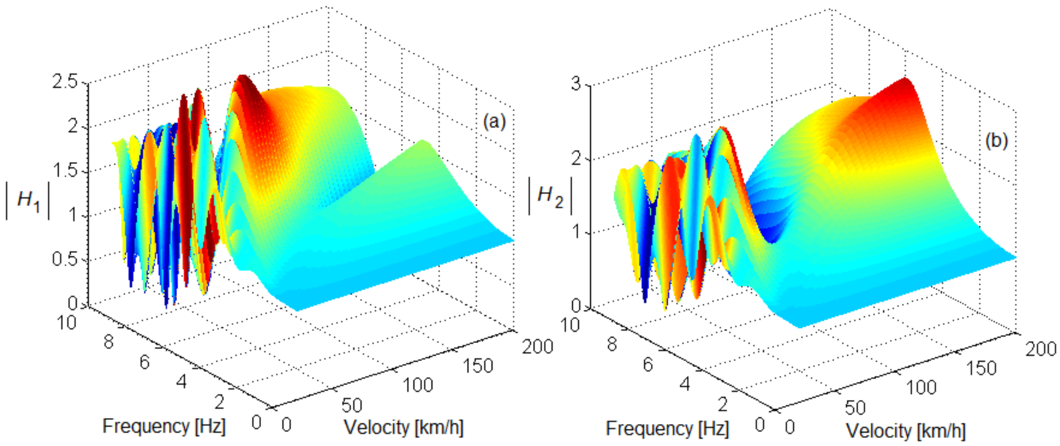

Fig. 2. Frequency response functions of the bogie: (a) in point 1; (b) in point 2.

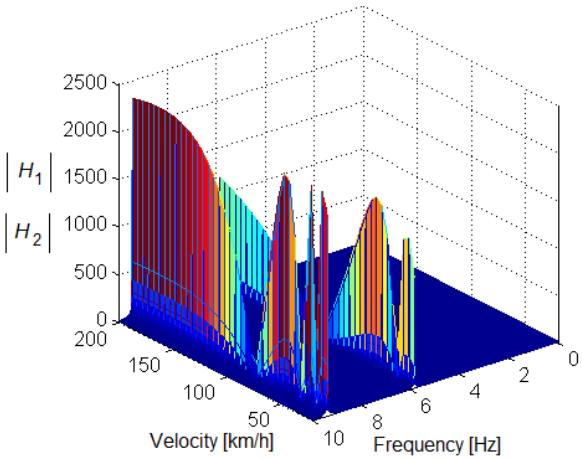

Fig. 3. Frequency response functions of the bogie for $2c_1 = 2c_2 = 0$.

This effect is mainly due to the manner in which the track excitations are conveyed to the bogie via the axles, irrespective of the suspension characteristics. Essentially, the geometric filtering effect is the result of the displacement between the vertical movements in the axles coming from running on a track with vertical irregularities. Due to the geometric filtering effect, the bogie response present a succession of maximum and minimum points, depending on the distance between axles and on velocity. The maximum points correspond to the situation where the geometric filtering does not operate, while the minimum points show themselves as anti-resonance frequencies that are consistent with the geometric filtering frequencies. Should the anti-resonance frequencies coincide with the natural frequency of one of the bogie vibration modes, then its influence is much diminished. This is how the change of the importance of the natural vibration modes in the bogie can be explained in dependence on the velocity and the fact that the vibration behaviour of the bogie does not continuously intensify when velocity increases [18]. The geometric filtering effect is more visible at low velocities, yet high frequencies.

Figure 4 exhibits the correlation coefficients between the dynamic response of the bogie chassis against the two axles and the vertical irregularities of the track. For the small frequencies, of up to 3 - 4 Hz, the system to behave like a rigid body, the bogie response being perfectly correlated with the displacements of the axles set by the vertical track irregularities, irrespective of velocity; the correlation is positive ($\rho_{1,2}$ = 1). At higher frequencies, the correlation coefficient changes in dependence on the velocity, taking values in the - 0.52 ... +1 interval.

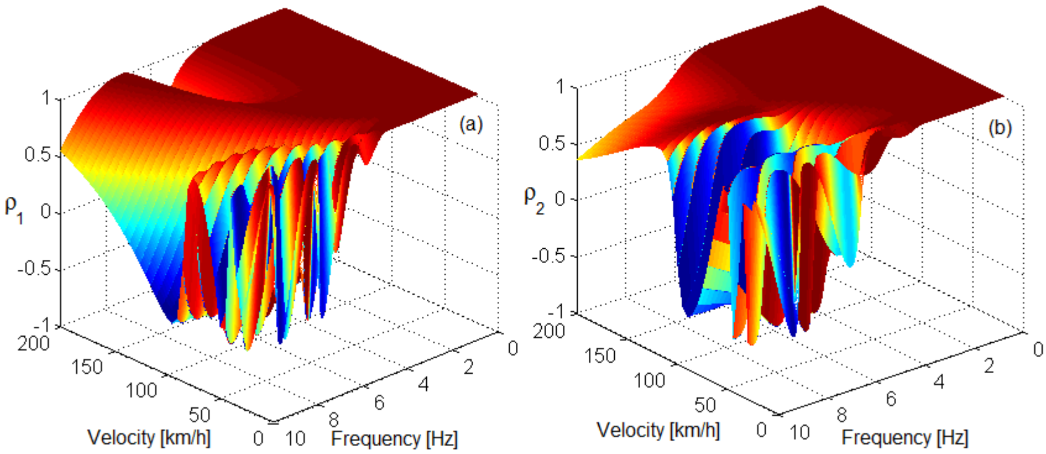

Fig. 4. Correlation coefficient: (a) in point 1; (b) in point 2.

The diagrams in figures 5 and 6 help with the analysis of the influence that the suspension damping has upon the correlation between the dynamic response of the bogie chassis against the two axles and the vertical track irregularities. The case of partial fault ($c_{1,2}$ = 0.5c) and total ($c_{1,2}$ = 0) of a damper in the suspension of the axle 1 (fig. 5) and axle 2 (fig. 6) have been taken into account and the correlation coefficient has been calculated at the resonance frequencies in the bogie bounce and pitch modes.

At the bounce resonance frequency (diagrams a), the correlation between the dynamic chassis response and the displacements of the axles set by the vertical track irregularities is positive for the entire velocity range. For the same velocity, the correlation coefficient generally varies within a limited interval upon the reduction of damping. The variation in the correlation coefficient with the suspension damping is noticed not to be visible in the same direction for all velocities; the correlation coefficient has contrary behaviours – it goes down or up with the reduction in damping, depending on velocity. There are large velocity intervals (for instance 43 – 115 km/h – fig. 5 or 103 – 200 km/h – fig. 6) for which the correlation coefficient is higher when damping is absent ($c_{1,2}$ = 0) than the correlation coefficient calculated for the reference value of damping ($c_{1,2} = c$) or velocity intervals where the situation is reversed - the correlation coefficient for the reference value of damping is higher than the coefficient calculated for c_1 = 0 or c_2 = 0 (eg. 120 – 195 km/h – fig. 5). All the above makes it harder to use a detection method for the damper faulty during operation, based on the calculation of the correlation coefficient.

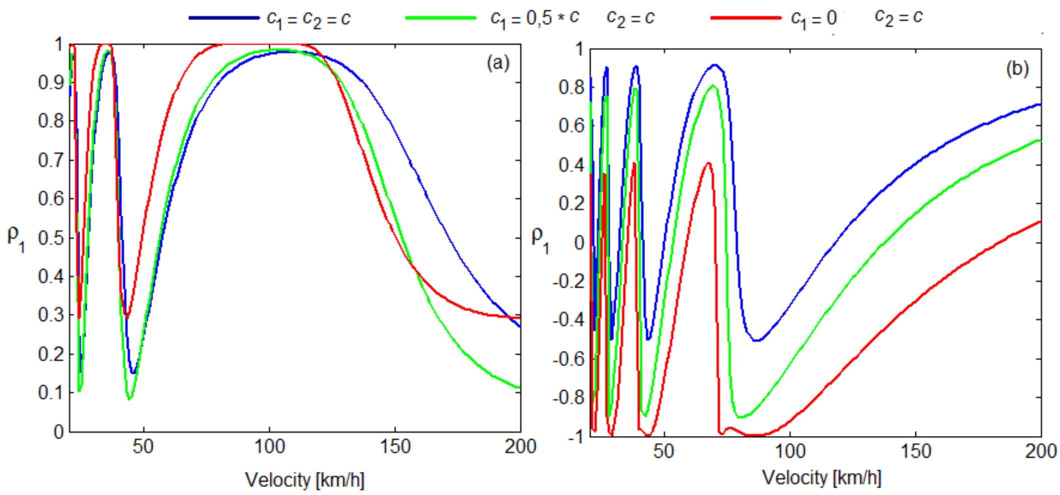

Fig. 5. Correlation coefficient during damper fault in the suspension of axle 1:
(a) at bogie bounce frequency (5.9 Hz); (b) at bogie pitch frequency (9.4 Hz).

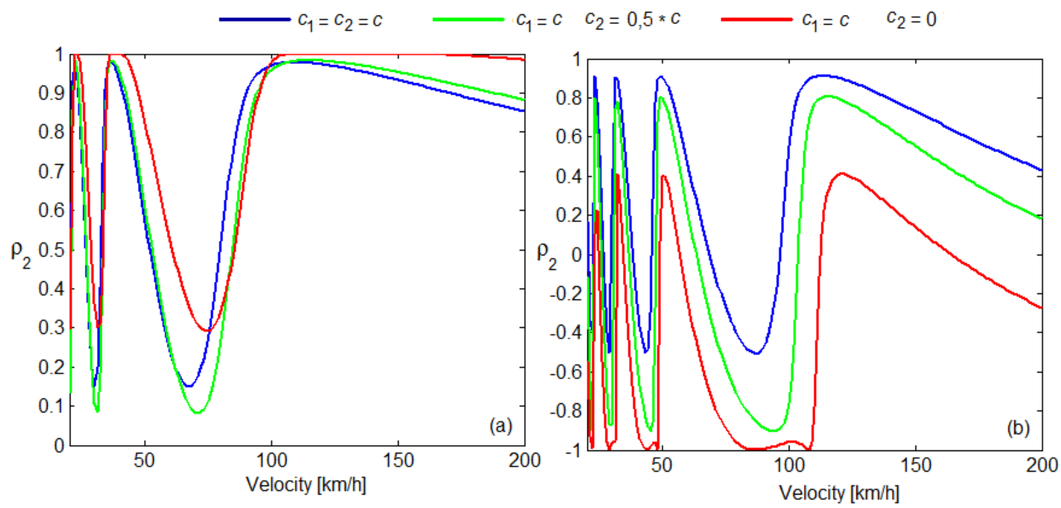

Fig. 6. Correlation coefficient during damper fault in the suspension of axle 2:
(a) at the bogie bounce frequency (5.9 Hz); (b) at the bogie pitch frequency (9.4 Hz).

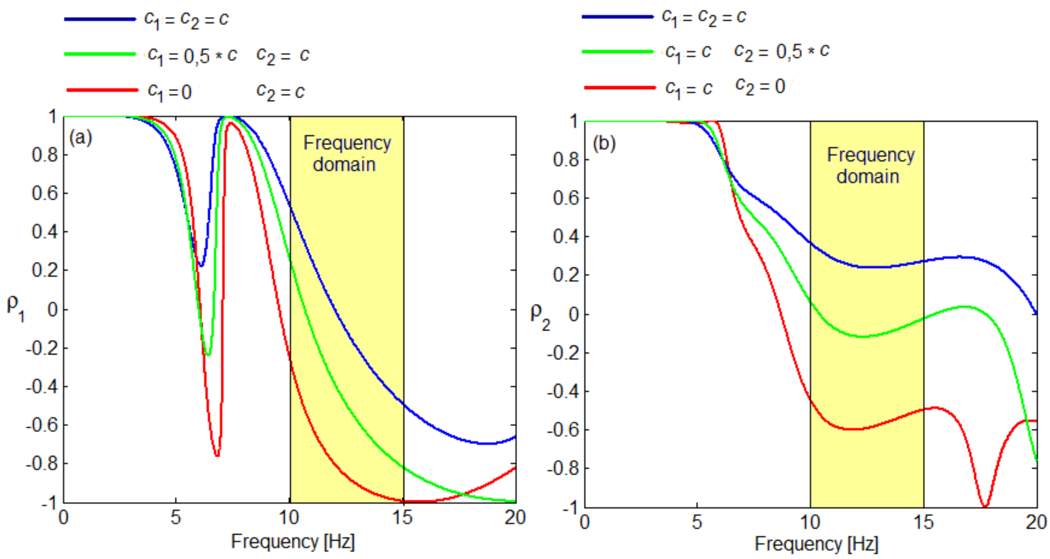

Fig. 7. Correlation coefficient at 200 km/h: (a) upon the damper fault in the suspension of axle 1; (b) upon the damper fault in the suspension of axle 2.

At the resonance frequency of the bogie pitch (diagrams b), the correlation coefficient varies from +1 to -1. Even though this coefficient changes with velocity, this variation only manifests in one direction during reduction of the suspension damping; irrespective of velocity, the correlation coefficient lowers when the suspension damping does the same. The above shows that there are favorable premises of developing a method to detect the fault in the damper during operation, based on the calculation of the coefficient of correlation between the dynamic response of the chassis and the displacements of the axles induced by the vertical track irregularities, in a selected frequency interval. It is about a frequency range in which the decrease of the correlation coefficient is correlative with the reduction degree in the suspension damping. Similarly, the method is preferred to be applied in a frequency domain, in which the difference between the correlation coefficients calculated for various fault degrees of the damper is higher. For instance, such domain is noted in the diagrams in figure 7, where the correlation coefficients are calculated at velocity of 200 km/h.

6. Conclusions

The paper herein submits a numerical analysis regarding the correlation between the dynamic response of a two-axle bogie and the displacements of the axles during circulation on a track with vertical irregularities, in a harmonic behaviour of vibration.

The correlation is evaluated by means of the Pearson correlation coefficient calculated depending on the frequency response of the bogie chassis in two points against the axles, in a velocity interval also including high values.

The results concerning the frequency response functions have highlighted a series of basic properties in the vibration bogie behaviour, such as the asymmetry of the response of the bogie chassis against the two axles due to the suspension damping or to the non-uniform increase in the level of vibrations of the bogie along with velocity, as a result of the geometric filtering effect introduced by the bogie wheelbase.

Based on the values in the correlation coefficient, the bogie response has been shown to be perfectly correlated at any velocity with the displacements of the axles induced by the vertical irregularities of the track in the range of low frequencies, of up to circa 4 Hz. The analysis regarding the influence that the suspension damping has upon the correlation between the dynamic response of the bogie chassis against the two axles and the vertical track irregularities has confirmed that there are favorable premises to develop a method to detect the faulty dampers during operation. This method underlies on the calculation of the coefficient of correlation between the dynamic response of the chassis and the displacements of the axles induced by the vertical track irregularities in a selected convenient frequency interval.

R E F E R E N C E S

[1]. *F. Cheli, R.Corradi*, On rail vehicle vibrations induced by track unevenness: Analysis of the excitation mechanism, Journal of Sound and Vibration, **vol. 330**, 2011, pp 3744–3765.

[2]. *J. Pombo, J. Ambrósio*, An alternative method to include track irregularities in railway vehicle dynamic analyses, Nonlinear Dynamics, **vol. 68**, issue 1-2, 2012, pp 161-176.

[3]. *T. Mazilu, Interaction between moving tandem wheels and an infinite rail with periodic supports – Green's matrices of the track method in stationary reference frame,* Journal of Sound and Vibration, **vol. 401**, 2017, pp. 233-254.

[4]. *T. Mazilu, Analysis of infinite structure response due to moving wheel in the presence of irregularities via Green's functions method*, Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, **vol. 10**, no.2, 2009, pp. 139-150.

[5]. *T. Mazilu, On the dynamic effects of wheel running on discretely supported rail***,** Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, **vol.10**, no.3, 2009, pp. 269-276

[6]. *M. Dumitriu*, Analysis of the dynamic response in the railway vehicles to the track vertical irregularities. Part I: The theoretical model and the vehicle response functions, Journal of Engineering Science and Technology Review, **vol. 8**, issue 4, 2015, pp. 24 – 31.

[7]. *M. Dumitriu*, Analysis of the dynamic response in the railway vehicles to the track vertical irregularities. Part II: The numerical analysis, Journal of Engineering Science and Technology Review, **vol. 8**, issue 4, 2015, pp. 32 – 39.

[8]. *M, Dumitriu, I. C. Cruceanu, Effect of vertical track irregularities on the vibration of railway bogie*, UPB Scientific Bulletin, Series D: Mechanical Engineering, Vol. 81, Issue 1, 2019, pp. 67-78
[9]. *T. Karis, M. Berg, S. Stichel, M. Li, D. Thomas, B. Dirks*, Correlation of track irregularities and vehicle responses based on measured data, Vehicle System Dynamics, **vol. 56**, issue 6, 2018, pp. 967-981.
[10]. *T. Karis, M. Berg, S. Stichel*, Analysing the correlation between vehicle responses and track irregularities using dynamic simulations and measurements, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail Rapid Transit, Article first published online: April 2, 2019, https://doi.org/10.1177/0954409719840450.
[11]. *R.W. Ngigi, C. Pîslaru, A. Ball, F. Gu*, Modern techniques for condition monitoring of railway vehicle dynamics, Journal of Physics: Conference Series, **vol. 364**, 2012, 012016.
[12]. *S. Bruni, R. Goodall, T.X. Mei, H. Tsunashima*, Control and monitoring for railway vehicle dynamics, Vehicle System Dynamics, **vol. 45**, 2007, pp. 743-779.
[13]. *P. Li, R. Goodall, P. Weston, C.S. Ling, C. Goodman*, Estimation of railway vehicle suspension parameters for condition monitoring, Control Engineering Practice, **vol. 15**, 2007, pp. 43-55.
[14]. *M. Dumitriu*, Fault detection of damper in railway vehicle suspension based on the cross-correlation analysis of bogie accelerations, Mechanics& Industry, **vol. 20**, number 1, 2019, Article number 102.
[15]. *M. Dumitriu, M. A. Gheţi*, MATEC Web of Conferences - 22nd International Conference on Innovative Manufacturing Engineering and Energy - IManE&E 2018, **vol. 178**, 2018, article number 06001.
[16]. *J. Zhou, S. Wenjing*, Analysis on geometric filtering phenomenon and flexible car body resonant, Vibration of Railway Vehicles, Journal of Tongji University, Natural Science, **vol. 37**, iss. 12, 2009, pp. 1653-1657.
[17]. *D. Gong, Y. Jia, G. Song, Y.J. Zhou*, Study on geometry filtering phenomenon and flexible car body resonant vibration of articulated trains, Advanced Materials Research, **vol. 787**, 2013, pp. 542-547.
[18]. *M. Dumitriu, C. Cruceanu*, Influences of carbody vertical flexibility on ride comfort of railway vehicles, Archive of Mechanical Engineering, **vol. LXIV**, iss. 2, 2017, pp. 119-238.