ON THE IMPLEMENTATION OF RESONANT CONTROLLERS IN CONTINUOUS AND DISCRETE TIME DOMAINS

Sergiu OPREA[1], Constantin RĂDOI[2], Dan-Alexandru STOICHESCU[3]

This paper analyses the implementation of resonant controllers in the continuous (analogue) and discrete (digital) domains. The authors present some topologies that can be used for implementing the resonant controllers in fundamental domains, s and z. The effects of various processes like the discretization and quantization are also discussed. The performances of proposed topologies are evaluated by simulations and by measurements performed on the developed hardware platform.

Keywords: resonant controllers, Proportional-Resonant, PR

1. Introduction

The resonant control was proposed as an alternative for the control of static power converters that operate with time-vary reference signals like inverters [1], controlled rectifiers [2], active power filters (APF) [3] and power factor correction circuits (PFC) [4]. The advantage of this kind of control over the classical method of using Proportional-Integral (PI) controllers in Synchronous Reference Frame (ScRF) is the simplicity. Fig. 1 presents a typical control system for a three-phase inverter using PIs in ScRF. The complexity of this approach is very high due to the presence of multiple reference frame transformations. These transformations are necessary in order to ensure the elimination of stationary error by PI controllers in case of time-varying signals. In this case the system is balanced; if the system is unbalanced the complexity of the control system is even higher because the symmetrical components must be extracted. The selective harmonic elimination in case of APFs requires also a complex implementation if the PI controllers in ScRF are used; each Park transform requires his own angular frequency that corresponds to the frequency of the harmonic that must be eliminated.

[1] Faculty of Electronics, Telecommunications and Information Technology, University POLITEHNICA of Bucharest, Romania, e-mail: sergiu.oprea44@gmail.com
[2] Faculty of Electronics, Telecommunications and Information Technology, University POLITEHNICA of Bucharest, Romania, e-mail: conrad_1944@yahoo.com
[3] Faculty of Electronics, Telecommunications and Information Technology, University POLITEHNICA of Bucharest, Romania, e-mail: dan_stoich@yahoo.com

The Proportional-Resonant (PR) controllers belong to the class of resonant controllers and operate directly in the Stationary Reference Frame (StRF). The PR controller provides the same performance as the PI controller in ScRF being able to eliminate the stationary error for time-varying signals. A control system based on PRs is presented in Fig. 2. The complexity is greatly reduced, the entire control system operating now in StRF. The PR controller acts on both sequences (positive and negative) for unbalanced systems; so, the complexity of the control system is greatly reduced, and it is not necessary to explicitly extract the symmetric sequences.

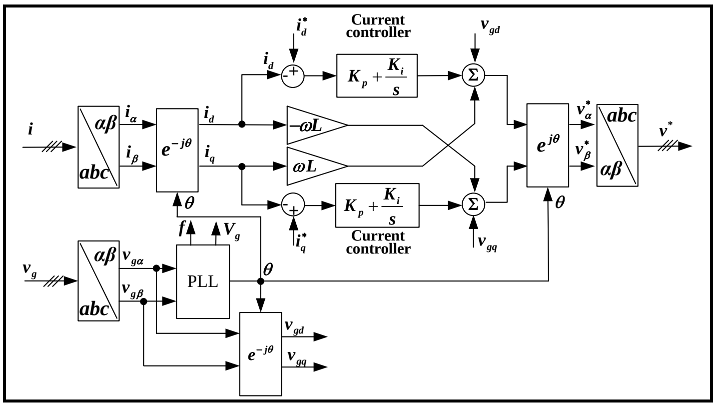


Fig. 1. A typical control system for a three-phase inverter using PI controllers in ScRF

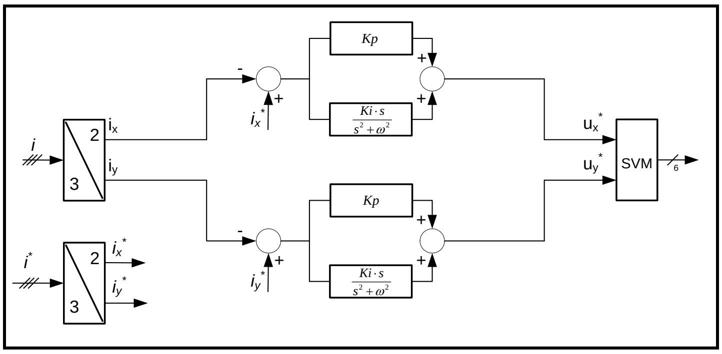


Fig. 2. A typical control system for a three-phase inverter using PR controllers

The Laplace domain form of a PR controller is:

$$G_C(s) = K_P + K_R \frac{s}{s^2 + \omega_R^2} \quad (1)$$

Where K_P - the gain of the proportional term, K_R - the gain of the resonant term, ω_R - the resonant frequency

The gain at the resonant frequency is infinite, similar to a second-order system where the damping factor is zero. Such controller cannot be implemented using analog or digital circuits because it is impossible to obtain an infinite gain in practice. The practical form of a PR controller is:

$$G_C(s) = K_P + K_R \frac{s}{s^2 + \xi s + \omega_R^2} \tag{2}$$

Where: ξ – damping ratio.

The second term of (1) is associated with a generalized integrator. Virtually, a sinusoidal signal will be integrated over time without introducing an additional phase error [5]. The existence of this term leads to a similarity between the PI type and PR type controllers, with the specification that in the case of the PR type the integral term acts only on the signals with frequencies very close to the resonance frequency.

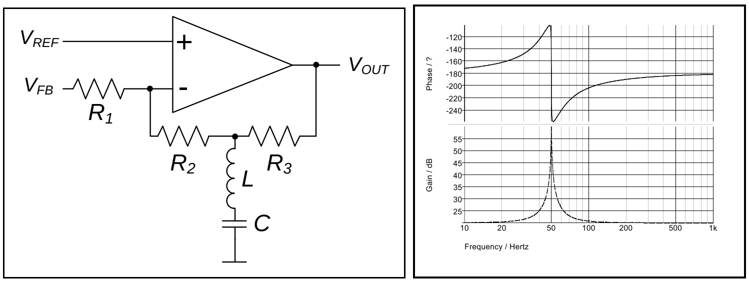
2. The Implementation of PR Controller in s-domain

In the literature, the presentation and analysis of the PR controller is in a high proportion in the s-domain. However, when it comes to the practical implementation of this controller it is, in most cases, done in the z-domain. The main reason is the flexibility offered by digital implementation and the rapid development of solutions dedicated to digital control. However, implementation in the s-domain has several advantages that can simplify the design process of this type of control. These can be listed:

- The maturity of the analysis methods in the s-domain. The entire design process can be done in the s-domain and then it can be converter in the z-domain using one of the well-known methods (Euler direct and inverse transformations, Tustin transformation, etc.)
- Implementation using active circuits such as operational amplifiers. This can result in substantial price reductions for some cost-sensitive applications.
- Lack of constraints associated with discrete systems regarding the frequency spectrum of the signals. For discrete systems, the maximum frequency must comply with the Nyquist criterion; in the case of analogue systems this is not necessary.
- A large number of simulators operate in the continuous time domain. Here are SPICE-based ones.

Fig. 3 shows a possible configuration of a PR controller in the s-domain. The implementation is based on an operational amplifier and passive components of type R, L and C. The presence of the LC series circuit in the amplifier's feedback loop modulates its gain according to the frequency. The impedance of LC series resonant circuit is minimal at the resonance frequency, the gain of amplifier being at maximum at this frequency. The resonance frequency is given by:

$$fr = \frac{1}{2\pi\sqrt{LC}} \tag{3}$$



(a) *(b)*

Fig. 3. PR controller implemented using LC circuit (a) and the Bode plots (b), (fr=50Hz)

The amplifier transfer function is:

$$\frac{V_{OUT}}{V_{FB}} = -\left[\frac{R_2}{R_1} + \frac{R_3}{R_1}\left(1 + \frac{sR_2C}{1+s^2LC}\right)\right] \tag{4}$$

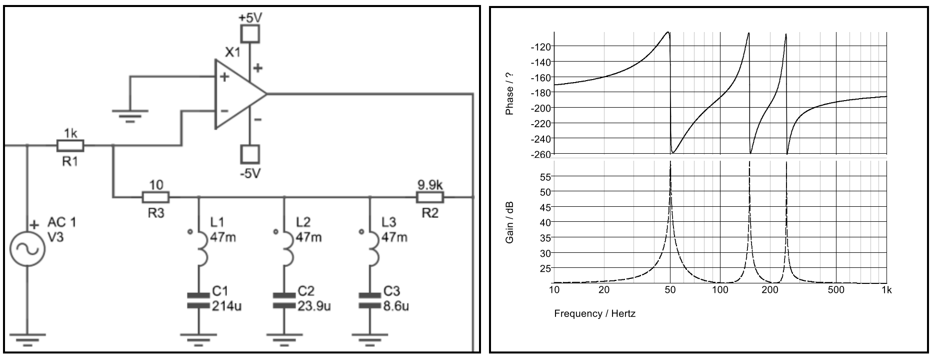
Suppose $R_2 << R_1$ and $R_2 << R_3$:

$$\frac{V_{OUT}}{V_{FB}} \cong -\left[\frac{R_3}{R_1}\left(1 + \frac{sR_2C}{1+s^2LC}\right)\right] \tag{5}$$

The proportional gain is set by R_3 and R_1 and is given by:

$$K_P \cong -\frac{R_3}{R_1} \tag{6}$$

The topology from Fig. 3 can be easily modified to support multiple resonant frequencies by adding additional resonant terms. Fig. 4 (a) presents a diagram of a PR controller with three resonant frequencies: 50 Hz (fundamental frequency), 150 Hz (third harmonic) and 250 Hz (fifth harmonic).

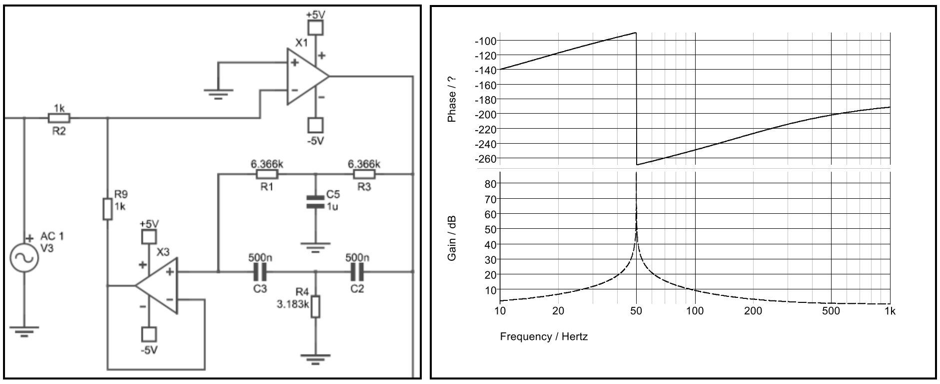


(a) *(b)*

Fig. 4. Multiple resonant frequencies PR controller (a) and the associated Bode plots (b)

The topologies presented are simple and can be used successfully in simulations of inverters, APFs and PFCs in the s-domain. There is, however, a major disadvantage associated with these circuits, namely the non-practical values for the LC circuit components. An inductor with a value of 47mH, good tolerance and high-quality factor is very difficult to be produced in practice. The cost and dimensions of such a component are very high. Also, the capacitor has a large value and is non-polarized. It must have a tight tolerance to avoid important errors that may affect the resonance frequency. Such a component has a very high price and occurs in series of values, finding the value required for the resonance frequency adjustment (generally different from the standard values) being virtually impossible. Under these circumstances, it is necessary to seek for solutions that can be easily implemented using common components produced in large series at a low cost. Large components with wide tolerances such as inductors are also to be avoided.

Using the same principle of introducing into the loop of networks whose impedance varies with frequency topologies that can be used for *PR* controllers can be obtained. The "*double T*" type network, commonly used in the synthesis of "*band-stop*" filters, presents a specific response of a second-order system with a sharp decrease in gain at resonant frequency. A typical topology of a PR controller implemented using a “*double T*” network is presented in Fig. 5 (a).



(a) *(b)*

Fig. 5. PR controller implemented using a “double T” network (a) and the Bode plots (b)

The resonant frequency for this topology is given by:

$$fr = \frac{1}{2\pi R_3 C_1} \tag{7}$$

The “*double T*” network’s transfer function is:

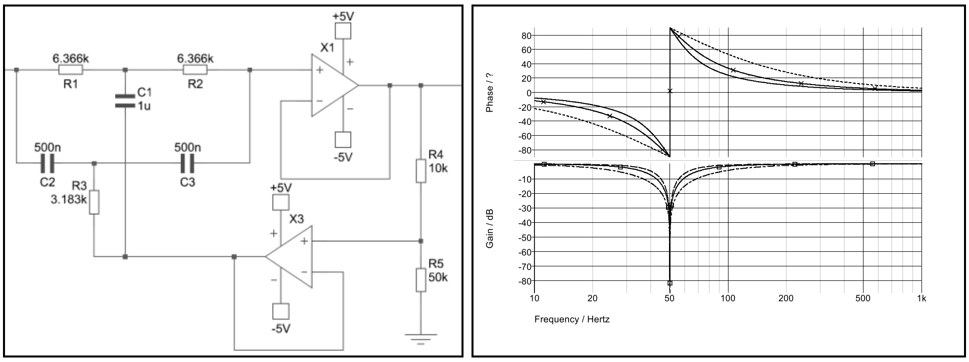
$$H_T(s) = \frac{{\omega_0}^2 - \omega^2}{{\omega_0}^2 - \omega^2 - s\Delta\omega} \tag{8}$$

Where $\omega_0 = 1/RC$. The quality factor is 0.25, relatively low. The proportional gain is well defined for frequencies different from the resonance frequency and is given by: $K_P = -R_9/R_1$.

In order to improve the selectivity, the "*double T*" network can be modified as is shown in Fig. 6 (a). In this case the "*double T*" network is used in a positive feedback loop with adjustable ratio; this ratio is adjusted by R_4 and R_5. The transfer function in this case is:

$$H_T(s) = \frac{\omega_0{}^2 - \omega^2}{\omega_0{}^2 - \omega^2 - \dfrac{4s\omega_0 R_4}{R_4 + R_5}} \tag{9}$$

By identification: $Q = (R_4 + R_5)/4R_4$; ω_0 is the resonant frequency.



(a) *(b)*

Fig. 6. The modified "double T" network (a) and the associated Bode plots (b)

Implementation of PR controllers in the s-domain can be done using several amplifier topologies in which the feedback loop contains elements with frequency-dependent impedance. The practical importance of these circuits is relatively limited due to their high sensitivity to component tolerances and very low flexibility. Basically, the resonance frequency cannot be dynamically adjusted, constituting a major disadvantage for applications requiring continuous control over this parameter such as inverters connected to the national power distribution network and APFs.

S-domain implementation is, however, of interest when targeting applications where final cost must be kept under control such as fixed frequency inverters that are part of "Uninterruptible Power Supply" (UPS) type systems. The topologies presented may also be useful for simulating power circuits using simulators in the continuous time domain like those based on SPICE. In this case, the influence of parasitic elements and noise on controller performance can be estimated using real components instead of the ideal ones commonly encountered in simulations.

3. Implementation of PR Controller in the z-domain

The implementation of this controller in the discrete domain is particularly important due to the fulminant development of the solutions dedicated to digital control. The digital implementation of controllers eliminates the disadvantages associated with analog implementations such as lack of flexibility, variation of parameters with component tolerance and undesirable effects from aging components [6, 7]. The specialized literature presents the PR controller in the s-domains, one of the reasons being the maturity of the analysis made in this domain. However, the vast majority of applications including PR controllers are implemented in the z-domain using microprocessors, digital controllers or *FPGAs*. For z-domain implementation, some discretization methods have been proposed, each with its own advantages and disadvantages. The effects of the discretization process cannot be neglected and the conclusions of the analysis in the field can be affected to a significant extent by these effects [8].

The design of resonant controllers is generally done in the s-domain. Stability of the control system is also evaluated in the s-domain, and then the entire controller is discretized using one of the dedicated methods and implemented in a DSP. The effects introduced by the quantization process as well as the implications for the actual implementation of the controller in the DSP should be carefully assessed to achieve the expected performance.

One of the adverse effects of the discretization process is the movement of poles and zeros [8]. Their original position in the complex s-plane is different from their position in the z-plane. The consequence of this frequency shift is the significant degradation of controller performance. Fig. 7 presents the Bode plots for the original resonant term in s-domain and for the resonant term discretized with the Tustin method. The original resonant term has a resonance frequency equal to 350 Hz. The resonance frequency after the discretization process is 348.6 Hz, an error of about -1.4 Hz. The gain is significantly affected, basically being very low for the original frequency of 350 Hz.

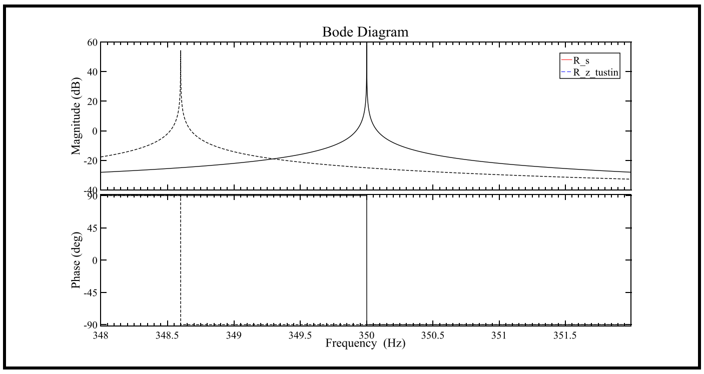


Fig. 7. Bode plots for Tustin discretization

The implementation of the controller in the *DSP* involves the representation of the coefficients and the signal from the output of the analog-to-digital converter in a format supported by the *DSP*. This process introduces additional adverse effects that must be considered for a good assessment of the performance of the controller. Most digital signal controllers use fixed-point representation on a number of bits that varies between 16 and 32 bits.

One of the side effects introduced by the coefficient quantization process is the shift of the frequency response of a digital filter. It is possible to determine the variation of the position of poles p_i to the coefficients variation a_k by introducing the sensitivity function [9]:

$$\frac{\partial p_i}{\partial a_i} = \left. \frac{z^{-k}}{-\left(z^{-1}\prod_{j\neq i,1}^{N}(1-p_j z^{-1})\right)} \right|_{z=p_i} \tag{10}$$

It can be noted that if the poles have close positions, the sensitivity to the disruption of the coefficients is high. In the case of the resonant term of the *PR* controller, the poles are complex conjugated having the same angular frequency. In this case, the associated sensitivity is extremely high. In [10] is proposed the following expression for moving the double pole frequency in the case of a system of the order M:

$$\Delta z_{pn} \approx \frac{\sum_{k=1}^{N} -\Delta a_k z_{pn}{}^{k}}{\prod_{l=1, z_{pl}\neq z_{pn}}^{N} (z_{pn}-z_{pl})} \tag{11}$$

For the resonant term, the effects of quantization can be extremely pronounced. For 16-bit quantization the double pole frequency error is about 0.04 Hz gain being strongly affected by this error (Fig. 8).

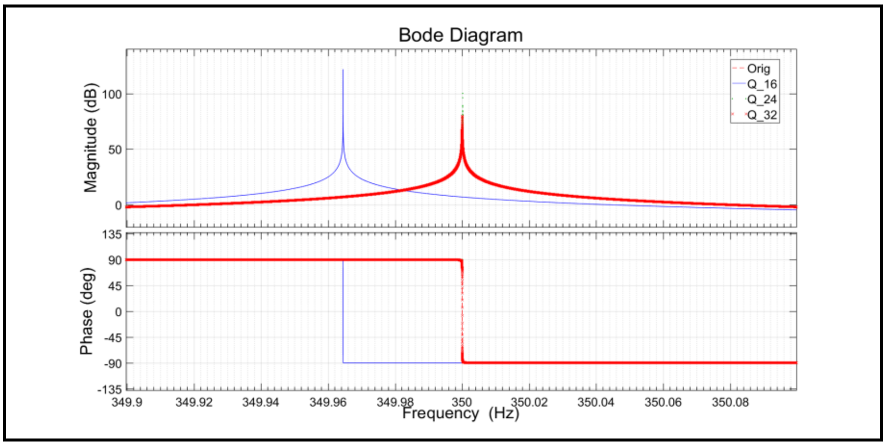


Fig. 8. Bode plots for the original resonant term and 16, 24, and 32-bit quantized terms

When using the 24-bit and 32-bit quantization the error is small and can be neglected.

The analysis of the effects introduced by the quantization process reveals virtually the impossibility of implementing the resonant term using a low-cost DSP with 16-bit resolution and fixed-point representation. This is a major disadvantage if a cost sensitive power converter is to be designed. To overcome this problem, the resonant term can be implemented based on the phase-sensitive amplifier principle known in the literature as "*Lock-in Amplifier*" [11]. The block diagram of such amplifier is shown in Fig. 9.

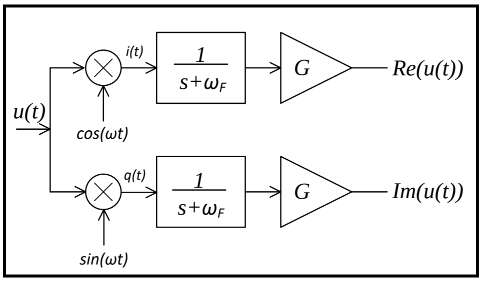


Fig. 9. Phase-sensitive Amplifier

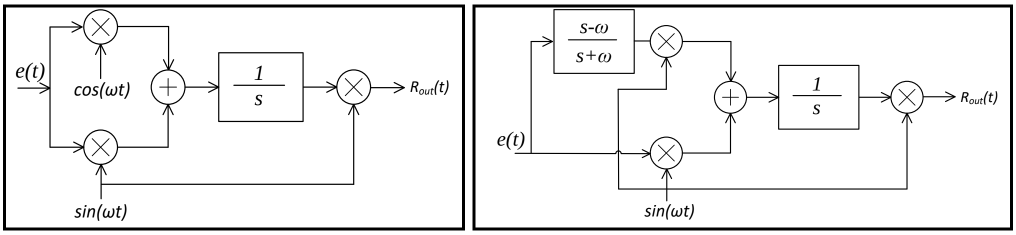
The two *i(t)* and *q(t)* signals at the output of the multipliers are also called "in phase" or "quadrature" respectively. The AC components are filtered with a low-pass filter and the resulting DC components are amplified by the G amplifiers. For different frequencies the DC component is null, virtually the output of the G amplifiers being null. The circuit behaves like an extremely narrow bandwidth filter with a central frequency equal to $\omega/2\pi$. If the low-pass filters are replaced by ideal integrators the resulting circuit behaves as a generalized integrator with the central frequency $\omega/2\pi$; the gain is infinite for the signals with the frequency $\omega/2\pi$ and null for the signals having another frequency.

The resonant term implemented with the aid of the phase-sensitive amplifier principle has the block diagram represented in Fig. 10 (a). Generally, the $sin(\omega t)$ and $cos(\omega t)$ reference signals are available inside the control system being generated locally using controlled oscillators or PLL circuits. For a PR controller the resonance frequency is an initial design parameter. Considering this aspect, the configuration can be simplified as shown in Fig. 10 (b). In this case, the error signal *e(t)* is further offset by 90° by means of an "*all-pass*" filter [12]. Thus it is necessary to provide only the reference signal $sin(\omega_r t)$ and ω. The "*all-pass*" filter can be easily implemented in the z-domain and is less sensitive to the effects of quantization having the order of one [12].

If the phase error introduced by the output filter of power converter is low the resonant term can be implemented using the simplified topology of Fig. 11 (a). To maximize the DC component the phase difference between the two signals must be zero. The compensation of phase error introduced by the controlled system can be done using "*all-pass*" phase-shift filters as shown in Fig.11 (b). The additional phase introduced by the "*all-pass*" filter is estimated by:

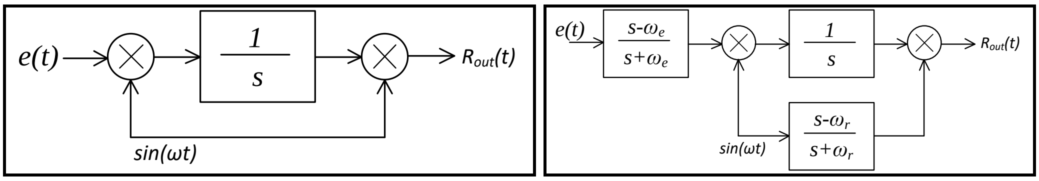
$$\angle H_{AP}(s) = 180 - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right) \tag{12}$$

Where ω_0 is the own frequency of the "*all-pass*" filter.



(a) *(b)*

Fig. 10. Resonant term implemented using a phase-sensitive amplifier (a) and his simplified form (b)



(a) *(b)*

Fig. 11. The simplified resonant term (a) and how the phase errors can be compensated using "all-pass" filters

The major advantage of implementation based on the phase-sensitive amplifier is the low sensitivity to quantization process. Practically, the performance depends only on the accuracy of the *sin(*ω_r*t)* reference signal. If this type of control is practically implemented using a low cost, fixed-point, 16-bit DSP it can be a significant advantage. There is also a disadvantage of this implementation: the calculation of the *sin(ωt)* reference in real time requires important resources. This may become a major problem if there are several resonant terms requiring different reference signals like in the case of APFs. For the case of fixed-point processors trigonometric functions are calculated using tables, the required calculation time being very low. For high-power applications in industry, the costs associated with using a powerful floating-point DSP are not a major issue.

Resonant control of inverters allows for excellent performance and can successfully replace classical control based on P and PI controllers in StRS or PIs in ScRS. The cost of implementing the resonant control is generally higher than classical solutions using dedicated integrated circuits because it requires the use of a DSP. A low-cost solution that can be used to control simple inverters that are part of UPSs is based on a hybrid control system. Within this system the resonant controller is implemented in the s-domain and output is controlled by a digital PWM modulator that is part of a digital system. The reference signal is generated by the digital system using a Digital-to-Analog Converter (DAC). Implementation of this topology can be done using 8-bit microcontrollers whose cost is very low compared with a cost of a dedicated DSP.

The block diagram of the proposed hybrid controller is shown in Fig.12. The PR controller is implemented in the s-domain using one of the topologies presented in the previous chapter. The reference signal is generated by the microcontroller using the internal DAC converter and a Numerically Controlled Oscillator (NCO). The amplitude of the output voltage of the inverter is controlled in the closed loop by adjusting the amplitude of the reference sinusoidal signal. The signal from the PR controller output is passed to the z-domain with the internal ADC converter and directly controls the digital PWM generator inside the microcontroller. Thus, a low-cost microcontroller can be used to implement a resonant control system for UPSs.

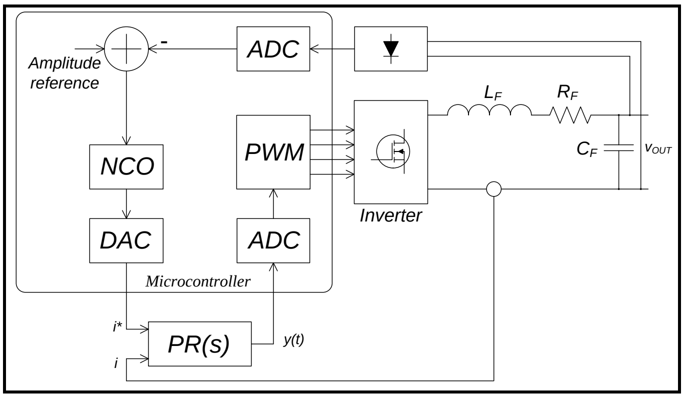


Fig. 12. The block diagram of the hybrid controller

4. Conclusions

A series of circuit topologies that can be used for implementation of resonant PR controllers in the continuous time domain was presented. These circuits can be used in simulators that operate in this time domain like those based on SPICE. Same topologies can be used to implement the hybrid control where the PR controller is implemented in the s-domain but the reference and PWM signals are generated by a low-cost microcontroller in the z-domain. The effects introduced by the discretization and quantization cannot be ignored as can significantly affect the performance of resonant controllers. A method to implement the resonant term using the phase-sensitive amplifier was presented. This method can be used to implement the PR controller using low-cost, fix-point, 16-bit digital signal controllers specially developed for the control of power converters.

REFERENCES

[1] *R. Teodorescu, F. Blaabjerg, U. Borup, M. Liserre*, "A new control structure for grid-connected LCL PV inverters with zero steady-state error and selective harmonic compensation," in Proceedings of IEEE Applied Power Electronics Conference, APEC, vol. 1, 2004, pp. 580–586.
[2] *D. Roiu, R. I. Bojoi, L. R. Limongi, A. Tenconi*, "New stationary frame control scheme for three-phase PWM rectifiers under unbalanced voltage dips conditions," IEEE Transactions on Industry Applications, vol. 46, no. 1, pp. 268–277, Jan./Feb. 2010.
[3] *M. Teodorescu*, "Metode și Sisteme Avansate de Comandă pentru Filtre Active de Putere", Teză de doctorat, 2012
[4] *Oprea S., Rădoi C., Florescu A.*, "Single-phase power factor correction circuit with Proportional-Resonant control", ECAI 2014, October 23 – October 25, 2014
[5] *R. Teodorescu, F. Blaabjerg, M. Liserre*, "Proportional-Resonant Controllers. A New Breed of Controllers Suitable for Grid-Connected Voltage-Source Converters,' Journal of Electrical Engineering, 2006
[6] S. *Buso, P. Mattavelli*, "Digital Control in Power Electronics", J. Hudgins, Ed. Morgan and Claypool Publishers, 2006
[7] *H. T. N. Charles L. Phillips*, "Digital Control System Analysis and Design", 3rd Edition. Prentice Hall, 1995
[8] *A. G. Yepes, F. D. Freijedo, J. Doval-Gandoy, O. Lopez, J. Malvar, P. Fernandez-Comesana*, "Effects of Discretization Methods on the Performance of Resonant Controllers," IEEE Transactions on Power Electronics, vol. 25, no. 7, July 2010
[9] *A. G. Place, G. H. Allen*, "Generalized Pole Sensitivity Analysis Due to Parameter Perturbation," IEEE Transactions on Circuits and Systems—II: Analog and Digital Signal Processing, vol. 44, no. 10, October 1997
[10] *A. G. Place, G. H. Allen*, "Perturbation Effects on Filters Having Localized Poles," International Conference on Acoustics, Speech, and Signal Processing, May 1995
[11] *M. Roșu-Hamzescu, S. Oprea, C. Polonschii, E. Gheorghiu, M. Gheorghiu*, „High Performance Low Cost Impedance Spectrometer for Biosensing", 21st International Conference on Control Systems and Computer Science, 2017, ISBN: 978-1-5386-1839-4
[12] *Phillip A. Regalia, Sanjit K. Mitra, P. P. Vaidyanathan* "The Digital All-Pass Filter: A Versatile Signal Processing Building Block," Proceedings of the IEEE, Vol. 76, No. 1, Jan. 1988