SENSITIVITY FUNCTION BASED ROBUSTNESS EVALUATION OF HYSTERETIC PIEZO ELECTRIC ACTUATOR USING INTERNAL MODEL PRINCIPLE

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Piezo actuated system is a promising solution for precision positioning applications provided the presence of complex nonlinear hysteresis is properly mitigated. This research is targeted towards reducing the effect of nonlinear hysteresis dynamics for enhanced precision positioning of piezoelectric actuated positioner. System modelling and parameter identification has been carried using Dahl based model for a second order system. Sensitivity function and complimentary sensitivity analysis has been carried out using transfer function where the design of Low Pass Filter (LPF) in internal model principle is formulated. It has been found that it is not possible to simultaneously reduce both sensitivity and complimentary sensitivity functions. Investigations have been carried out to observe the effect of disturbance attenuation at high and low frequency range. A considerable improvement in sensitivity function towards rejection of disturbance at low frequency is seen in Improved Model based Piezo Controller (IMPC) compared to Classical Model based Piezo controller (CMPC). Analysis of robustness and closed loop stability of IMPC has also been carried out based on conventional stability analysis techniques.

Keywords: Sensitivity, robust, piezoelectric, nonlinear, hysteresis

1. Introduction

There has been a breakthrough in the production of ultra-positioning systems mostly used for achieving up to the range of nano-meter level accuracy for manufacturing devices, precision metrology, positioning systems, and several industrial applications of different genres like atomic force microscope [1]. Piezoelectric actuator plays a prominent role in such applications because of several advantages it offers. However, hysteresis is a major non-linearity which bottlenecks the positioning accuracy due to such nonlinear dynamics [2-3]. Modelling and identification of hysteresis is a challenging research aspect towards efficient control of piezoelectric actuated systems. Asymptotic tracking of prescribed trajectories and/or asymptotic rejection of disturbances towards efficient control of the output of a system is a crucial problem in control theory. In

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essentiality, there are three different avenues to resort to these problems: a) tracking via dynamic inversion, b) adaptive tracking, and c) tracking through system internal models. Tracking by dynamic inversion comprise in figuring out an exact initial system state and a precise control input with the objective that provided the system is accordingly set up to initial conditions and run, the reference signal is exactly tracked by the output. This calls for a knowhow of "perfect knowledge" of the complete trajectory which is to be tracked along with "perfect knowledge" of the plant model that is to be controlled. Hence, for plants with large uncertainties either on parameters or the reference signal, this approach is not preferable. Dynamic inversion plays primary role in adaptive tracking where the control input parameters are tuned so as to achieve asymptotic convergence to zero of the tracking error. A reference trajectory with a very slow varying nature may be considered as a stabilizing issue with gradual variation of unknown parameter, however, mostly leading to a very conservative solution. On the contrary, simultaneous handling of uncertainties of plant parameters as well as in the trajectory that is to be tracked can be dealt by Internal model-based control approach. The principle of Internal Model Control (IMC) states that if the trajectory to be tracked belongs to the set of all trajectories generated by some fixed dynamical system, a controller which incorporates an internal model of such a system is able to secure asymptotic decay to zero of the tracking error for every possible trajectory in this set and does it robustly with respect to parameter uncertainties [4].This offers a sharp advantage and is a marked contrast over the two approaches discussed earlier where instead of the presumption that a trajectory belongs to a class of trajectories set up by an exogenous system, one rather requires to know a complete information of the trajectories` past, present and future time history. Due to this flexibility, internal model-based approach poses to be most suited for dealing with scenarios that involves rejecting unknown disturbances as well as tracking unknown references.

Reference [5] suggests a tuning method of IMC for controlling the overshoot of the speed-governing system, which is one of the most complex systems in the hydro-electric power plant including water hammer effect. Real-time control of a coupled tank liquid level system has been carried out in [6]. The proposed system has been compared with Ziegler-Nichols tuned PI controller, simulation as well as experimental results suggests controller efficiency with respect to traditional control. In the parlance of chemical process and chemical reactor plants, [7] recommends a new IMC controller based on frequency domain analysis which can decrease the over loop in Continuous Stirred Tank Reactor (CSTR) model.

Formulation of sensitivity function to reflect the effect of disturbance attenuation and also rigid mechanical system robustness in presence of uncertainties has been presented in [8-9]. Fan et al presents sensitivity optimization of nano manipulating systems based on piezo actuated positioners using Modified Disturbance Observer structure in [10]. Sensitivity analysis has also been beneficial towards hysteresis suppression and precision motion tracking of piezo-actuated micro positioning system in [11].

2. Piezoelectric Actuator System Description

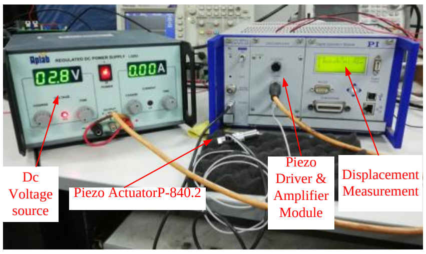
The piezoelectric actuator system has been considered as a second order system along with Dahl hysteresis model [12-13]. The experimental set-up has been shown in Fig. 1a) to identify the plant parameter of the piezo electric actuator. The hysteretic parameters have been found from experimental open loop step response of the piezo actuator shown in Fig.1 b) using the following equations:

$$a_1=\frac{2}{(t_1-t_2)}\ln\frac{H_1-H_s}{H_2-H_s}=\frac{2}{(0.021-0.0150)}\ln\left(\frac{28.6-21.1}{26.4-21.1}\right)=115.732 \quad (1)$$

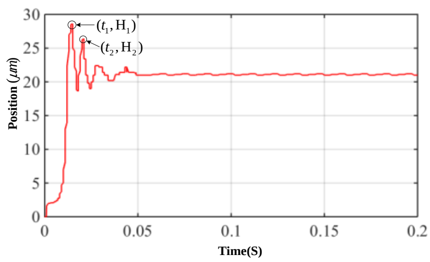
$$a_2=\frac{4\pi^2}{(t_1-t_2)^2}+\frac{a_1^2}{4}=\frac{4\pi^2}{(0.021-0.0150)^2}+\frac{115.732^2}{4}=1098859.58 \quad (2)$$

$$b_1=G_{dc}a_2=0.281*a_2=0.281*1098859.58=308779.54198 \quad (3)$$

$$b_0=S_0=0 \quad (4)$$



a)



b)

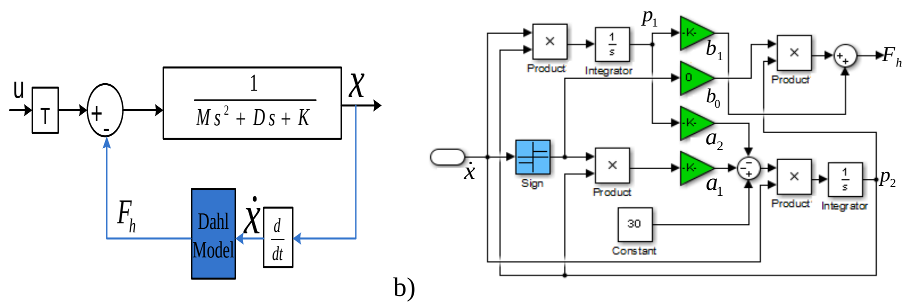
Fig.1. a) Experimental set-up for system Identification. b) Open Loop time domain response of the piezoelectric actuator for 75 V input

A State Space representation of the entire piezoelectric system using state vector $X=[x,\dot{x},p_1,p_2]^T$ and shown in Fig.2 based on following state matrices.

$$\dot{X}=AX+Bu,\ \ Y=CX \quad (5)$$

Where the system, input and output matrix are

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K}{M} & -\frac{D}{M} & -\frac{b_1}{M} & -\frac{b_0}{M}\mathrm{sgn}(\dot{x})\\ 0 & 0 & 0 & \dot{x}\\ 0 & u_p & -a_2\dot{x} & -a_1\dot{x}\mathrm{sgn}(\dot{x})\end{bmatrix},\ B=\begin{bmatrix}0\\ \frac{T}{M}\\ 0\\ 0\end{bmatrix}\ \text{And}\ C=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} \quad (6)$$



a) b)

Fig. 2 .a)Schematic Block Diagram of the entire piezoelectric system leading to a output of `x` for a input voltage `u`, b)Dahl model for calculating the hysteresis force, F_h

Piezoelectric actuator of make Physik Instrumente, P 841.20 has been used in the study. System transfer function has been experimentally determined using multi amplitude, multi frequency test signals on the piezo actuator through ARX model, shown in equation 7.

$$TF=\frac{T}{Ms^2+Ds+K}=\frac{0.0336}{0.1828S^2+190.154S+119206.85} \quad (7)$$

3. Sensitivity And Complimentary Sensitivity Functions

The main aim of a controller in a closed loop system is to bridge the gap between the output of controller and pre specified set point as far as possible when the system is subjected to extraneous disturbances. The control system parameters are normally decided in accordance with the plant properties and because the dynamics may change, it is crucial that the control parameters are not responsive to changes in the plant. Sensitivity functions – namely, sensitivity and complimentary, are helpful perceptible quantifiers to assess the fulfillment of a definite control algorithm depending upon the relationship amongst error, proposed controller and system. Sensitivity function correlates to capability of a system towards effective disturbance rejection and robustness while its complimentary form provides an estimate of tracking response which is related to system performance.

3.1 Basic feedback controller structure for a Piezoelectric plant

A hypothetical portrayal of the controller with piezoelectric plant is presented in Fig.3(a). The control signal subjected to the plant is referred to as `u` for an arbitrary input, `v` also termed as reference signal, `r` and `d` represents the disturbance. The system output is referred to as `y` while the internal state variables are represented by `n`. Another representation of the system input and outputs are as follows: the input signal to the plant i.e. the control voltage `u` that can be processed and signal `v` which collectively represents the independent disturbances as `r` and `d` which effects the closed loop performance. System outputs can also be classified to be of two types, tangible plant output, `y` and other intermediate outputs like `e` and `x`. Fig. 3 (a) possess a detailed illustration of the process flow while Fig. 3(b) contains a much compact form.

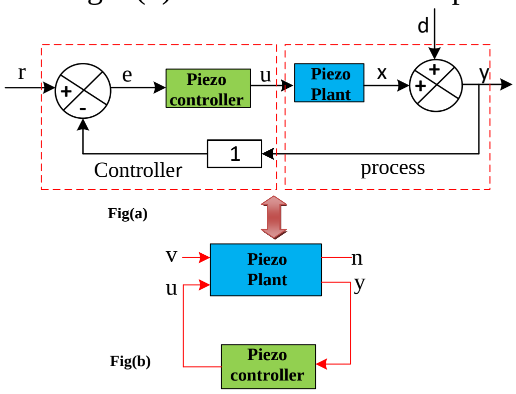


Fig. 3. a) Basic structure of the feedback loop of the piezo electric actuator system b) Equivalent conceptual representation.

The above illustration is also applicable for a system with a large number of input and outputs. An enhanced level of encapsulation is helpful for theoretical progress as it enables to emphasize on basics and take into consideration a broad ambit of utility.

The closed loop of Fig. 3 is effected by two exterior signals, the reference input denoted by `r` and the perturbation, `d`. However, from control point of view, the signals `x`, `y` and `u` are especially vital. Accordingly, six relations can be formed amongst the reference input and output signals, expressed below. Considering `X`, `Y`, `U`, `D`, R to be the Laplace equivalent of `x`, `y`, `u`, `d`, `r`, the subsequent relations are derived from Fig. 3(b)

$$X=-\frac{\text{Piezo Plant.Piezo Controller}}{1+\text{Piezo Plant.Piezo Controller}}D+\frac{\text{Piezo Plant.Piezo Controller}}{1+\text{Piezo Plant.Piezo Controller}}R$$

$$Y=\frac{1}{1+\text{Piezo Plant.Piezo Controller}}D+\frac{\text{Piezo Plant.Piezo Controller}}{1+\text{Piezo Plant.Piezo Controller}}R$$

$$U=-\frac{\text{Piezo Controller}}{1+\text{Piezo Plant.Piezo Controller}}D+\frac{\text{Piezo Controller}}{1+\text{Piezo Plant.Piezo Controller}}R$$

The sensitivity function of the controller can be presented as $\varepsilon(s)=\frac{1}{1+\text{Piezo Plant.Piezo Controller}}=\frac{1}{1+L(s)}$ where $L(s)=\text{Piezo Plant.Piezo Controller}$, is the loop transfer function of the piezo system. The complementary sensitivity function of the control structure can be represented as $\eta(s)=\frac{\text{Piezo Plant.Piezo Controller}}{1+\text{Piezo Plant.Piezo Controller}}=\frac{L(s)}{1+L(s)}$. It follows that

$$\varepsilon(s)+\eta(s)=1\ . \tag{8}$$

It is implied that it is not possible to reduce ε and η at the same time. The loop gain, L is generally high for minimal values of `s` and tends to zero with `s` tending to infinity. It implies that `$\varepsilon$` is intuitively low for a low `s` and nears unity for a higher `s`. Similarly, the complementary sensitivity function approaches unity for low `s` and it moves towards zero as `s` tends to infinity. A fundamental issue is to determine if `$\varepsilon$ ` can be requested for a large frequency range. Step response has been plotted for the proposed piezo electric actuator system with basic feed-back control loop topology, as shown in Fig.4. The step response of the sensitivity function settles at around 8.9 sec has no undershoot which is essential for piezoelectric actuators. Exact complimentary step response is also seen with similar settling time and nil overshoot for complimentary sensitivity function.

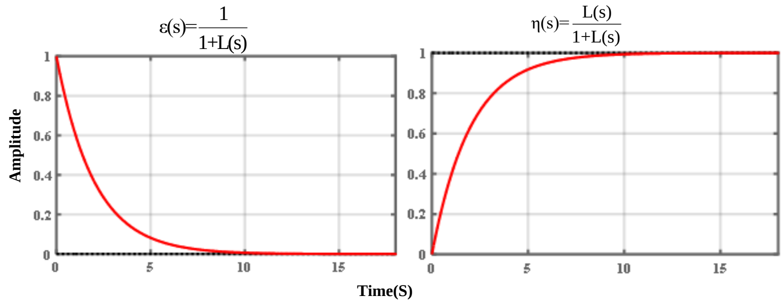


Fig. 4. Step response of the sensitivity and complementary sensitivity function

3.2 Classical Model Based Piezo Controller (CMPC)

The Model Based Design flow is a dual step approach which provides a reasonable trade-off between robustness and performance. An advantage of the Model Based Design approach is its capability to directly specify the nature of the closed-loop response through sensitivity and complementary sensitivity functions, `$\eta$`and `$\varepsilon$` respectively. The initial step is to ensure that the controller $G_{INV}(s)$ is stable and causal. The design procedure followed for the Model Based Design control of Piezo actuator system as represented in Fig. 5

- The PZA is chosen and the transfer function of the plant process $G_{PZA}(s)$ is determined as a second order system using Dahl model to encapsulate hysteretic non linearity.
- The process model $\tilde{G}_{PZA}(s)$ is estimated from the actual plant $G_{PZA}(s)$.

• The process model $\tilde{G}_{PZA}(s)$ is factorized into two parts - minimum phase and non-minimum phase components ie $\tilde{G}_{PZA}(s)=\tilde{G}_{PZA^+}(s)\cdot\tilde{G}_{PZA^-}(s)$. This step ensures that $G_{INV}(s)$ is stable and causal. $\tilde{G}_{PZA^+}(s)$ contains all Non-minimum Phase Elements (Noninvertible) in the process model. i.e. all right half plane (RHP) zeros and time delays. The factor $\tilde{G}_{PZA^-}(s)$ comprise of Minimum Phase Elements and by nature, invertible.

• Model Based Design controller $G_{INV}(s)$, defined as the inverse of minimum phase component $G_{INV}(s)=\tilde{G}^{-1}_{PZA^-}(s)$ is non-causal and stable. We can make the controller causal by choosing the higher order filter in series. The filter of the system is $G_f(s)=\frac{1}{(\alpha s+1)^n}$

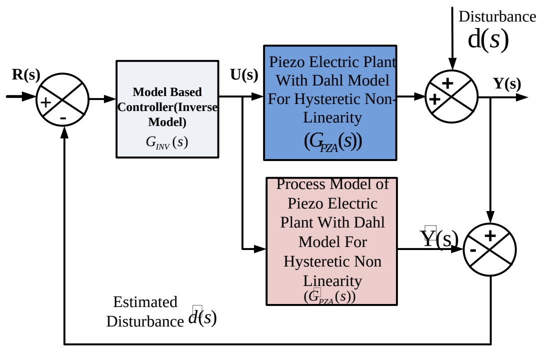


Fig. 5. Block diagram of Classical Model Based Piezo Controller (CMPC)

The filter order n is selected such that to make the system causal and reliable. As the PZA system process model contains components which show stability with no poles in RHP on the s-plane, the model is considered invertible.
The sensitivity function depends on the load disturbance and control input. Thereafter, low pass filter $G_f(s)$ can be incorporated in the model based controller to make ε attenuate the effect of load disturbance at low frequency and to make η reduces the sensor noise at higher frequency.

3.3 ε and η for Classical Model based Piezo Controller(CMPC)

In Laplace domain, the sensitivity function is defined as

$$\varepsilon(s)=\frac{E(s)}{R(s)-d(s)} \tag{9}$$

Which relates the input R(s) and d(s) with the error signal, E(s)
Considering the block diagram in Fig. 5, it follows

$$E(s)=R(s)-Y(s)=R(s)-[G_{PZA}(s)U(s)+d(s)] \tag{10}$$

But, $U(s)=G_{INV}(s)E(s)$ (11)

Therefore

$$E(s)=R(s)-G_{INV}(s)G_{PZA}(s)E(s)-d(s) \quad (12)$$

Or, $E(s)[1+G_{INV}(s)G_{PZA}(s)]=R(s)-d(s)$ (13)

hence, $\frac{E(s)}{R(s)-d(s)}=\frac{1}{1+G_{INV}(s)G_{PZA}(s)}$ (14)

It can be seen concluded from Equation (9) and (14) that

$$\varepsilon(s)=\frac{E(s)}{R(s)-d(s)} \quad (15)$$

$$(15\ \varepsilon(s)=\frac{1}{1+G_{INV}(s)G_{PZA}(s)} \quad (16)$$

Thus, is can be observed that the sensitivity function plays an important role in analyzing the controller performance because it takes into consideration the effects of disturbance, d(s) on the process output, Y(s). For the controller to achieve better disturbance rejection, it is certain that the sensitivity function $\varepsilon(s)$ must be made as small as possible through corresponding controller design.

The PZA physical system is strictly proper with its transfer function representation having a higher order in the denominator than the numerator.

Thus, $\lim_{|s|\to\infty} G_{INV}(s)G_{PZA}(s)=0$ (17)

In frequency domain, we can write

$$\lim_{\omega\to\infty} G_{INV}(j\omega)G_{PZA}(j\omega)=0 \quad (18)$$

Therefore, $\lim_{\omega\to\infty}\varepsilon(j\omega)=\lim_{\omega\to\infty}\frac{1}{1+G_{INV}(j\omega)G_{PZA}(j\omega)}=1$ (19)

Complimentary Sensitivity Function, as the name suggests, is defined as

$\eta(s)=1-\varepsilon(s)$ (20)

It is known, $\varepsilon(s)=\frac{1}{1+G_{INV}(s)G_{PZA}(s)}$ (21)

Hence, $\eta(s)=1-\frac{1}{1+G_{INV}(s)G_{PZA}(s)}=\frac{G_{INV}(s)G_{PZA}(s)}{1+G_{INV}(s)G_{PZA}(s)}$ (22)

3.4 ε and η for Improved Model based Piezo Controller

An improved model based piezo controller is designed based on some minor cosmetic changes in the structure of classical model-based control to lend a robust and PID equivalent form along with reduced hardware logic resource.

As per definition of sensitivity function, we have

$$\varepsilon(s)=\frac{E(s)}{R(s)-d(s)}=\frac{Y(s)}{d(s)}=\frac{1}{1+G_{INV}(s)G_{PZA}(s)} \quad (23)$$

For Improved Model based Control, we get

$$E(s)=R(s)-Y(s) \tag{24}$$

$$Y(s)=G_{M\text{-}IMC}(s)G_{PZA}(s)E(s)+d(s) \tag{25}$$

$$Y(s)=G_{M\text{-}IMC}(s)G_{PZA}(s)(R(s)-Y(s))+d(s) \tag{26}$$

$$Y(s)[1+G_{M\text{-}IMC}(s)G_{PZA}(s)]=G_{M\text{-}IMC}(s)G_{PZA}(s)R(s)+d(s) \tag{27}$$

$$Y(s)=\frac{G_{M\text{-}IMC}(s)G_{PZA}(s)R(s)+d(s)}{[1+G_{M\text{-}IMC}(s)G_{PZA}(s)]} \tag{28}$$

Substituting, $G_{M\text{-}IMC}(s)=\dfrac{G_{INV}(s)}{[1-G_{INV}(s)\tilde{G}_{PZA}(s)}$ (29)

Leads to $Y(s)=\dfrac{G_{INV}(s)G_{PZA}(s)R(s)+[1-G_{INV}(s)\tilde{G}_{PZA}(s)]d(s)}{1+[G_{PZA}(s)-\tilde{G}_{PZA}(s)]G_{INV}(s)}$ (30)

Assuming $R(s)=0$ and $G_{PZA}(s)\approx\tilde{G}_{PZA}(s)$

$$\varepsilon(s)=1-G_{INV}(s)G_{PZA}(s) \tag{31}$$

As per definition, complimentary sensitivity function is

$$\eta(s)=G_{INV}(s)G_{PZA}(s) \tag{32}$$

Hence, in the Improved Model based Control approach, the controller appears linearly in both the sensitivity function as well as the complimentary sensitivity function. Comparing with the respective functions of classical model based control scheme, represented by equation (21) and (22), it is evident that the Model Control based approach is easier to shape the sensitivity function and complimentary sensitivity function. As the sensitivity function relates to system performance and complimentary sensitivity function determines robustness, it is implied that the Improved Model based control approach provides a comparatively much easier framework to design a robust controller

4. Result And Discussion

The modeling and system identification of the piezoelectric system has been carried out to identify the system, as elaborated earlier. Analysis of robustness and closed loop stability is presented in the following section based on sensitivity function magnitude plots and conventional stability analysis techniques.

Closed loop control normally passes signal of low frequencies while it blocks noise of large frequency. Complimentary sensitivity function is associated with the closed loop system equation with large magnitude at small frequencies with the sensitivity function having large value at increased frequencies. However, as $\varepsilon(s)+\eta(s)=1$, if the sensitivity at low frequency is to be increased, then it has to affect $\eta(s)$, and similarly for larger frequencies. Reducing the value of $\eta(s)$, effects in a rise of $\varepsilon(s)$ and if $\varepsilon(s)$ is bumped down, value of $\eta(s)$ increases which brings about a water-bed effect, following Bode sensitivity integral

$$\int_0^\infty \ln|\varepsilon(jw)dw = 0 \tag{33}$$

According to which a region of reduced sensitivity needs to be countered by an equal region of enhanced sensitivity. The equation suggests that if a low sensitivity function is achieved for some particular frequencies, there is bound to be a rise in other frequencies. An enhanced attenuation for a certain frequency range leads to a considerable degraded performance for other range. It is also implicit from the equation that there lies basic shortcomings to benefits attained by control and control design can be presented as a reorientation of disturbance rejection for a frequency range. None of the sensitivity functions can achieve a steady performance at constant frequencies, as it is also seen in Fig. 8 and Fig. 9 that as $|\varepsilon(j\omega)|$ is "pushed down" at lesser frequencies, it "pops up" at larger frequencies.

4.1 Classical Model Based Piezo Controller (CMPC) the sensitivity function for ideal case

The transfer function of piezoelectric system has been obtained in above section. Sensitivity and complimentary sensitivity function following a Classical Model Based Piezo Controller (CMPC) is shown below and the magnitude plot is presented in Fig.6:

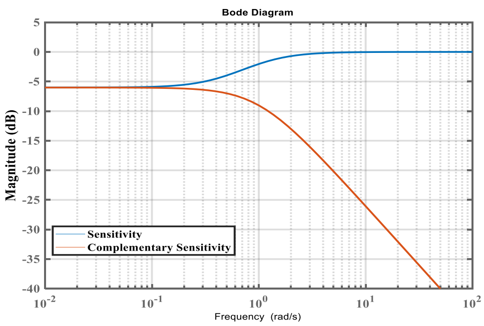


Fig. 6. Magnitude plot of Sensitivity and complementary sensitivity function for classical Model Based Design controller

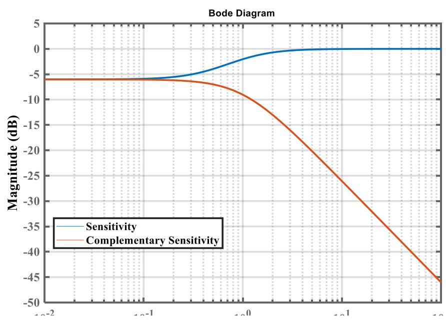


Fig. 7. Magnitude plot of Sensitivity and complementary sensitivity function for classical Model Based Design controller with 2% plant parameters variation.

$$\varepsilon(s) = \frac{1}{1 + G_{INV}(s)G_{PZA}(s)} = \frac{2s+1}{2s+2} \tag{34}$$

Where, $G_f(s) = \dfrac{1}{2s+1}$ is the first order low pass

filter $G_{PZA}(s) = \dfrac{0.0336}{0.1828S^2 + 190.154S + 119206.85}$ plant of piezo electric actuator.

Complimentary sensitivity function of the system is represented as from equation (22)

$$\eta(s)=1-\frac{1}{1+G_{INV}(s)G_{PZA}(s)}=\frac{G_{INV}(s)G_{PZA}(s)}{1+G_{INV}(s)G_{PZA}(s)}=\frac{1}{2s+2} \quad (35)$$

4.2 Classical Model Based Design controller the sensitivity function for plant with parameter variation (2%)

Plant/model mismatch occurs commonly primarily as system modelling due to the resulting approximations when the physical system is transformed into its mathematical equivalent. Model mismatch is difficult to avoid and occurs for reasons such as inaccurate system identification or inappropriate order assumption of the plant. Plant parameter variation due to ageing is also common in piezoelectric actuators where the system parameters vary with time. The non-linear nature of the PZA also enhances possibilities of mismatch. The following section considers a plant with 2% variation as a representation of parametric uncertainly.

The plant model of piezo electric actuator can be represented as

$$G_{PZA}(s)=\frac{0.0342}{0.1864s^2+193.957s+121590.987} \quad (36)$$

As per derivation the transfer function of sensitivity function of the classical model based piezo electric system is

$$\varepsilon(s)=\frac{1}{1+\dfrac{(0.1864s^2+193.957s+121590.987)(0.0336)}{0.0342(2s+1)(0.1828s^2+190.154s+119206.85)}}=\frac{0.3656s^3+380.5s^2+2.386\times10^5s+1.192\times10^5}{0.3656s^3+380.7s^2+2.388\times10^5s+2.384\times10^5} \quad (37)$$

Similarly, complementary sensitivity function with varied plant parameters is shown in equation (38) with the magnitude plot being reflected in Fig.7

$$\eta(s)=\frac{0.1828s^2+190.1s+1.192\times10^5}{0.3656s^3+380.7s^2+2.388\times10^5s+2.384\times10^5} \quad (38)$$

4.3 Improved Model Based Piezo Controller (IMPC) the sensitivity function for ideal case

To overcome the limitation of Classical Model Based Piezo Controller (CMPC), an Improved Model Based Piezo Controller (IMPC) is designed and analysis of its sensitivity function is presented Fig. 8 below.

It is observed from the above magnitude plots that to make the system robust and perform in a better manner, the value of the sensitivity and complementary sensitivity function should be zero. However, it is not possible to simultaneously make both the values to zero. If the system has been made robust at low frequency, then it is seen to suffer from noise at higher frequency.

This is due to the trade-off between ε and η. One possible way to make the system robust for low frequency disturbance is to reduce the sensitivity function value. The sensitivity function attenuates the disturbance in low frequency and complementary sensitivity has the capability to attenuate sensor noise at higher frequency.

$$\varepsilon(s)=1-G_{INV}(s)G_{PZA}(s)==1-\frac{1}{2s+1}=\frac{2s}{2s+1} \tag{39}$$

As per definition, complimentary sensitivity function of the system is represented from equation (32) as

$$\eta(s)=G_{INV}(s)\tilde{G}_{PZA}(s)=\frac{1}{2s+1} \tag{40}$$

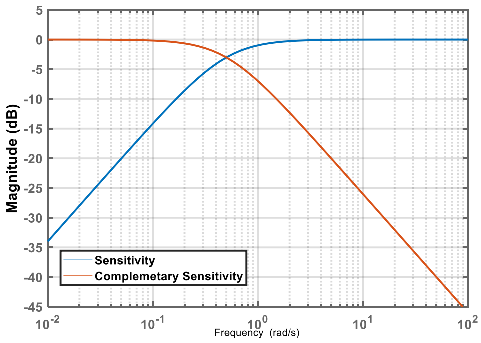


Fig.8. Magnitude plot of Sensitivity and complementary sensitivity function for Improved Model Based Piezo Controller

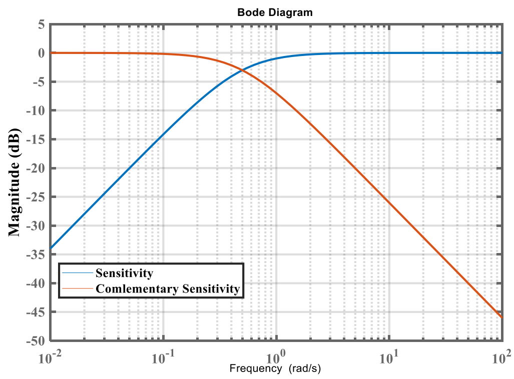


Fig. 9. Magnitude plot of Sensitivity and complementary sensitivity function for Improved Model Based Piezo Controller with 2% plant parameter variation

4.4 Improved Model Based Piezo Controller (IMPC) the sensitivity function for plant with parameter variation (2%)

A similar analysis has been carried out for Improved Model Based Piezo Controller (IMPC) as for CMPC with a plant parameter variation of 2%. It is observed that Sensitivity function is able to shrink the effect of perturbations at low frequencies while the complimentary sensitivity function reduces disturbances associated at larger frequencies is shown in Fig. 9.

The sensitivity transfer function of the IMPC can be represented as

$$\varepsilon(s)=\frac{0.3656s^3+380.3s^2 2.384\times10^5 s+47.68}{0.3656s^3+380.5s^2+2.386\times10^5 s+1.192\times10^5} \tag{41}$$

And complementary sensitivity function

$$\eta(s)=\frac{0.1826s^2+190.1s+1.192\times10^5}{0.3656s^3+380.5s^2+2.386\times10^5 s+1.192\times10^5} \tag{42}$$

4.5 Stability analysis of Improved Model Based Piezo Controller (IMPC)

Analysis of stability closed loop system is carried out through Bode Plot, Pole Zero Plot, Root Locus which are presented below. The closed loop transfer

function of the system has been identified through the input output data of the plant (including hysteresis model) and proposed controller. From the results, it has been verified that real part of all the Eigen values of the system are negative. All the results clearly indicate a stable closed loop system.

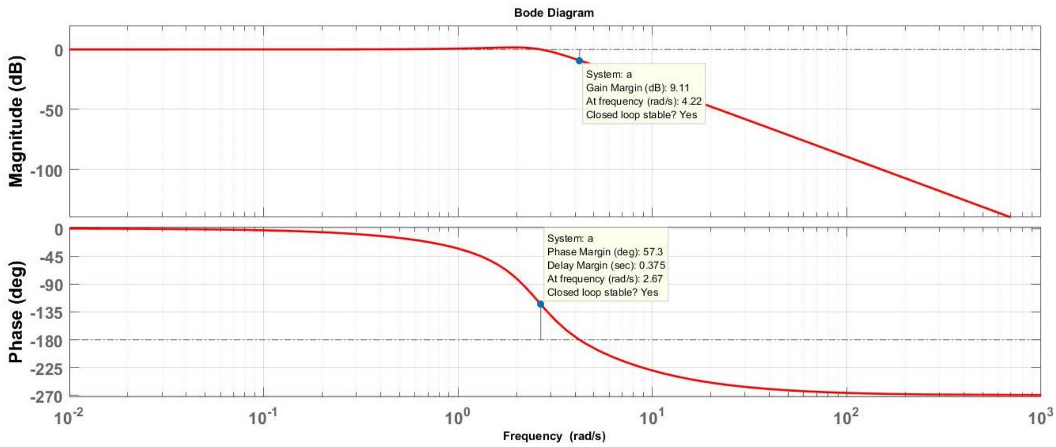


Fig.10. Bode Plot of Closed Loop System (Gain Margin and Phase Margin are positive in nature, suggesting closed loop system stability

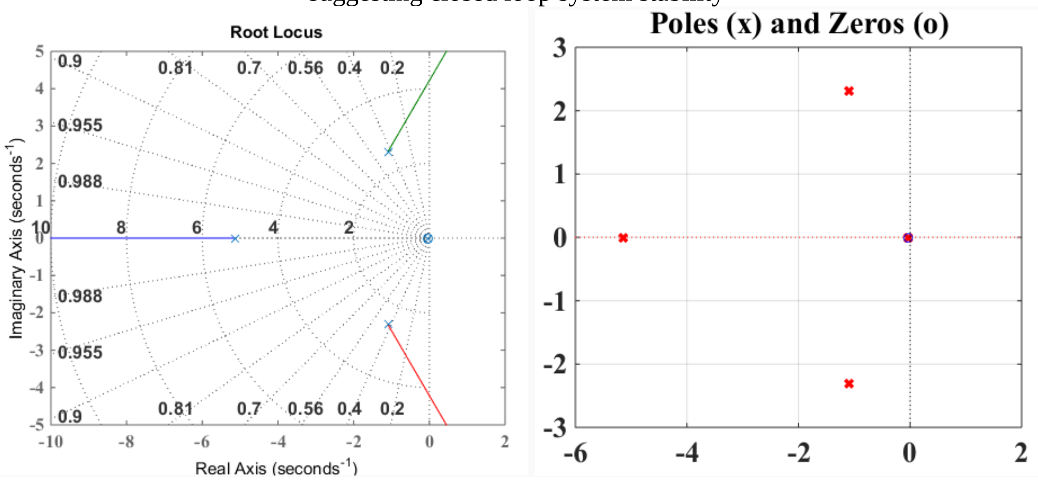


Fig. 11. Pole-Zero and Root Locus plot of the Closed Loop System (Pole: -5.1438, -1.0919 + 2.3173i, -1.0919 - 2.3173i, -0.0334, and Zero : -0.0333)

5. Conclusion

It is seen from the sensitivity and complimentary sensitivity magnitude plots that LMSC there is an uneven compensation with respect to lower and higher frequencies. A better compensation is required for the piezoelectric plant at lower frequencies. IMPC is seen as a promising solution where sensitivity function attenuates the effect of disturbances at lower frequencies while the complimentary sensitivity function plays an important role at higher frequencies. Robustness of IMPC is also established by a relatively unchanged magnitude plot of the sensitivity function when analyzed for a plant with modified parameters. Besides, a formal analysis of the IMPC controller has also been carried out using well established techniques and the results suggest a robust control design.

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