DOUBLE DIFFUSION CONVECTION IN A TILTED SQUARE POROUS DOMAIN UNDER CROSS TEMPERATURE AND CONCENTRATION GRADIENTS

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The present study focuses on double diffusion natural convection in a square cavity saturated by an inclined porous medium subjected to cross temperature and concentration gradients. The Darcy model with the Boussinesq approximation, energy and species transport equations are solved numerically using the classical finite difference method with a time-accurate scheme. The case of equal thermal and solutal buoyancy forces is considered. For this situation, an equilibrium solution corresponding to the rest state is possible and the resulting onset of motion can be either supercritical or subcritical. The study is carried out for an inclination angle of 45°. The results are presented by Nusselt, Sherwood numbers and flow intensity as a function of thermal Rayleigh number. In this study, the beginning of the convection is determined and our study will be limited to the cases of equal and dominant diffusivities.

Keywords: Double diffusion Convection, Porous medium, Supercritical Rayleigh, Subcritical Rayleigh.

1. Introduction

The dynamics of heat and mass transfer can be very different from those driven by the temperature field solely. Interest in coupled heat and mass transfer due to buoyancy forces in porous media has been motivated by such diverse engineering problems related to dispersion of chemical contaminants through water-saturated soil, the exploitation of continental geothermal reservoir, the migration of moisture through the air contained in fibrous insulation, metallurgy, electrochemistry, geophysics, etc.

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A comprehensive review on the phenomena of heat and mass transfer and convection in porous media could be found in the book by Nield and Bejan [1]. Mamou et al. [2] examined the flow in a square cavity subjected to horizontal fluxes of heat and mass. In case where the volume forces are in opposite direction and same order of magnitude, the existence of multiple solutions was demonstrated. The existence of multiple solutions depended heavily on the thermal Rayleigh and Lewis numbers. Mansour et al. [3] studied numerically the Soret effect on multiple solutions in a square cavity. The authors concluded that the Soret parameter might have a strong effect on the convective flow. One, two or three solutions were possible. Mohamad and Bennacer [4] obtained numerical results, on the basis of two- and three-dimensional flows, on heat and mass transfer in a horizontal enclosure with aspect ratio of two filled with a saturated porous medium. The enclosure was heated differentially and a stably stratified species concentration was imposed vertically. It was found that the difference in the rates of heat and mass transfer predicted by the two models was not significant. Mansour et al. [5] studied numerically the Soret effect on fluid flow and heat and mass transfer induced by double diffusive natural convection in a square porous cavity submitted to cross gradients of temperature and concentration. They concluded that the Soret effect might affect considerably the heat and mass transfer as it led to an enhancement or to a reduction of the mass transfer, depending on the flow structure and the sign and magnitude of the Soret coefficient. Bourich et al. [6] studied analytically and numerically the Soret effect on thermal natural convection within a horizontal porous enclosure uniformly heated from below by a constant heat flux using the Brinkman extended Darcy model. It was found that the Soret separation parameter had a strong effect on the thresholds of instabilities and on the heat and mass transfer characteristics. Saeid [7] studied the problem of natural convection in a two-dimensional square porous cavity with the temperature of the hot (left) wall oscillating in time. He finds that during the heat transfer process the hot wall temperature dropped which resulted at some locations inside the cavity with a temperature higher than the hot wall temperature. Also, it is observed that the average Nusselt number had a peak value at the non-dimensional frequency of 450 in the range considered (1–2000) for Rayleigh number 103 because the convection currents are stronger than those at other frequencies. The transient free convection in a two-dimensional square cavity filled with a porous medium was considered by Saeid and Pop [8]. The flow was driven by considering the case when one of the cavity vertical walls is suddenly heated and the other vertical one was suddenly cooled, while the horizontal walls were adiabatic. The results were obtained for the initial transient state up to the steady state, and for Rayleigh number values of 10^2–10^4. It was observed that the average Nusselt number showed an undershoot during the transient period and that the time required to reach the steady state is longer for low Rayleigh number and shorter for high Rayleigh number. Mansour, et al. [9] studied the transient MHD natural convection in an inclined cavity filled with a fluid saturated porous medium by including the effects of both of an inclined magnetic field and heat source in the solid phase. The flow was driven by considering the case when one of the cavity vertical walls was suddenly heated and the other one was suddenly cooled, while the horizontal walls were adiabatic. The authors found that in general, they could increase the temperature of the fluid by increasing both of the Magnetic field force and the inclination angle. Sezai and Mohamad [10] presented results for three-dimensional flow in a cubic cavity filled with porous medium and subjected to opposing thermal and concentration gradients. Their results revealed that for a certain range the controlling parameters of the flow become three-dimensional and multi-solution is possible within this range. The stability of flow structures has been studied by Bergeon et al. [11] where the mechanisms by which stable solutions lose stability or unstable solutions regain stability were determined. They also studied the influence of the cavity inclination on the stability and bifurcation solutions and found that the bifurcation at the critical Rayleigh number is either transcritical or pitchfork, depending on the aspect ratio and the inclination angle of the cavity. Vasseur et al. [12] studied analytically and numerically the flow in a tilted rectangular cavity and observed that the maximum heat transfer, for a given R_T, is obtained when the cavity is heated from below, with θ in the range $90° < \theta < 180°$. They found that this maximum takes place for values of θ approaching 90° whenever R_T increases. Trevisan and Bejan [13] used a numerical method and scale analysis to study double diffusion convection in a porous square cavity, with vertical walls maintained at constant temperatures and concentrations. It was found that the fluid flow was possible beyond a certain number of the critical Rayleigh when $Le \neq 1$. However, the fluid motion disappeared completely for the $Le=1$ and $N=-1$. The results of this analysis were found in agreement with numerical study.

In this work, a numerical study was conducted to examine the effect of Rayleigh number on heat and mass transfer rates in an angled porous square cavity. We have examined the case where the thermal and mass thrust forces are equal for different values of the Lewis number. Darcy's model was used to simulate double diffusive convection inside the cavity. The existence of solution has been demonstrated and the convective threshold has been obtained using a novel method for determining the onset of instabilities in natural convection.

2. Mathematical formulation

The physical configuration considered in this work is a square ($A=L'/H'=1$) porous layer shown in Fig. 1. The origin of the coordinate system is located in the cavity center. The constant thermal and mass flux q' and j' crossed were imposed on the walls of the cavity. The fluid saturating the porous matrix is incompressible, Newtonian and obeys the Boussinesq approximation. The equations describing double diffusion convection are expressed as a function of current function, temperature and concentration, in dimensionless form:

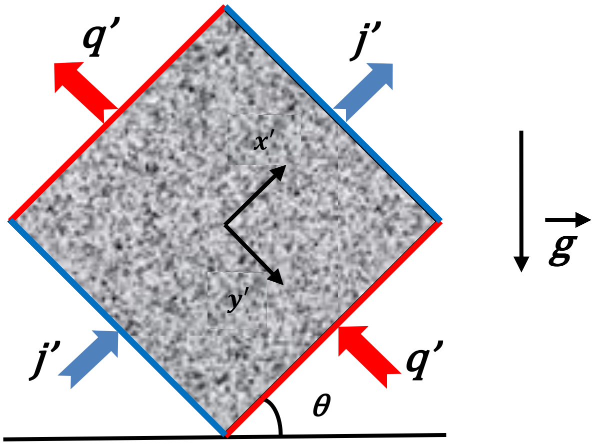

Fig. 1. Geometry of the problem.

The governing equations that describe the double-diffusive convection are expressed in terms of the stream-function, temperature and concentration, in dimensionless form:

$$\nabla^2\Psi = -R_T\mathcal{F}(T+NS) \tag{1}$$

$$\nabla^2 T = \frac{\partial T}{\partial t} - J(\Psi, T) \tag{2}$$

$$\frac{1}{Le}\nabla^2 S = \varepsilon\frac{\partial S}{\partial t} - J(\Psi, S) \tag{3}$$

Where Ψ is the dimensionless current function, defined as follows:

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x}$$

And with the operators $\mathcal{F}$ and J:

$$\mathcal{F}(f) = \sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}, \qquad J(f,g) = \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}$$

u and v dimensionless velocity components, T and S dimensionless temperature and concentration, t dimensionless time, x and y are dimensionless coordinate axes, R_T is the thermal Rayleigh number, *N* the ratio of thrust forces, θ inclination, *Le* is the Lewis number and ε is the normalized porosity of the porous medium. In Darcy's model, the inertia and viscosity forces are negligible and the Reynolds number is assumed to be very low.

Dimensionless boundary conditions are given by:

$$y = \pm\frac{1}{2} : \Psi = 0, \ \ \frac{\partial T}{\partial y} = 0 \ \text{and} \ \ \frac{\partial S}{\partial y} = -1 \tag{4}$$

$$x = \pm\frac{1}{2} : \Psi = 0, \ \ \frac{\partial T}{\partial x} = 1 \ \ \text{and} \ \ \frac{\partial S}{\partial x} = 0 \tag{5}$$

3. Numerical solution

The numerical solution of the equations governing our study (1)-(3) with the boundary conditions (4)-(5) is obtained using a finite difference scheme. The entire domain, as shown in Figure 1, has been discredited with a uniform mesh (101x101). The solution includes the current function, the temperature and concentration fields. The central difference scheme with second order precision is used to transform the basic equations into a set of finite difference equations. The energy and concentration equations, after having been written in conservative form, are solved using the alternating direction implicit (A.D.I) method, while the current function field is obtained from the equation of discredited moment using the method of successive over-relaxation (S.O.R). With a convergence criterion less or equal to 10^{-6}, the integrals in the Nusselt and Sherwood number expressions were computed numerically using the Simpson scheme.

4. Calculation of critical Rayleigh Number

In what follows we present a new method for determining the onset of instabilities in natural convection. The calculation method of the supercritical Rayleigh number (R_{TC}^{sup}) requires at least two flow simulations above and below the threshold on instability. By trial and error procedure, using the numerical code which solves the full governing equations, it was found that for R_T= 5, 10 and 15 the solution is purely conductive (rest state), however, R_T=20, a convective solution was triggered and leads to a convective steady state solution. Even for R_T=18 the solution is convective. Thus, obviously the threshold for the onset of convection must be within the interval [15, 18]. As known, for infinitesimal amplitude convection, the time evolution of the flow intensity is exponential, according to the linear stability analysis, and could be correlated by $\boldsymbol{\Psi_{0max}(t)} = \boldsymbol{\Psi_0 e^{pt}}$, where $\boldsymbol{\Psi_{0max}}$ is the convective flow amplitude and $\boldsymbol{\Psi_0}$ is the initial flow amplitude at *t*=0. The parameter *p* represents the amplitude growth rate. When $p < 0$ the flow is decaying and when $p > 0$ the flow is amplified. Then, $p < 0$ below the threshold of convection and $p > 0$ above the threshold. By performing two simulations for two Rayleigh number numbers below and above the threshold, the growth rate parameter could be computed numerically. Then the threshold of convection could be determined accurately by interpolation for p=0, the situation where the marginal stability occurs. Now, applying this procedure, starting first from a pure conductive state (instable), for R_T=18, a flow simulation was performed. The flow intensity time history is presented in Fig. 2. An excellent exponential curve fit was obtained $\boldsymbol{\Psi_{0max}} < 10^{-2}$. For this case, after the flow amplitude time growth, the solution was converged to a steady state convective solution. Now, for R_T=15, using the converged solution as initial conditions, the flow simulation was carried again and as can be seen from Fig. 2, the flow decays towards the pure conductive state. Focusing only on the infinitesimal curve branch, $10^{-10} < \boldsymbol{\Psi_{0max}} < 10^{-2}$, exponential curve fit is presented. The growth rate parameter, using the exponential curve fit, was obtained as p=1.117 for R_T=18 and p=-0.756 for R_T=15. Using linear interpolation for p=0, the threshold of marginal stability is obtained as R_{TC}^{sup}=16.21.

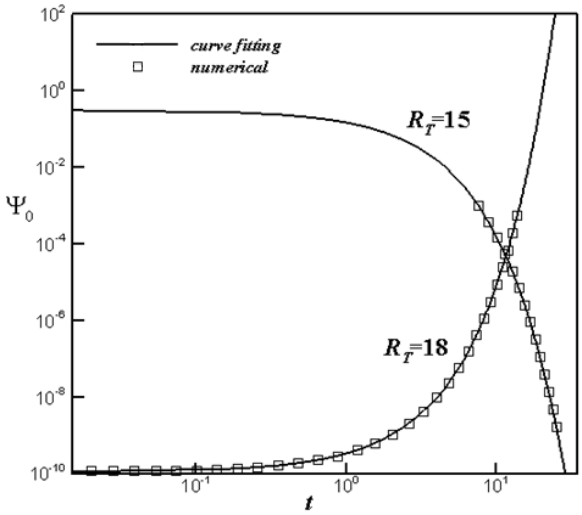

Fig. 2. Flow intensity time histories below and above the threshold of supercritical convection for Le=1.

5. Results and discussion

The present study is limited to the state of equilibrium where the forces of thermal and mass thrust are equal. Under these conditions, a solution in the state of rest is possible. There is a Rayleigh number threshold for the onset of convective flows.

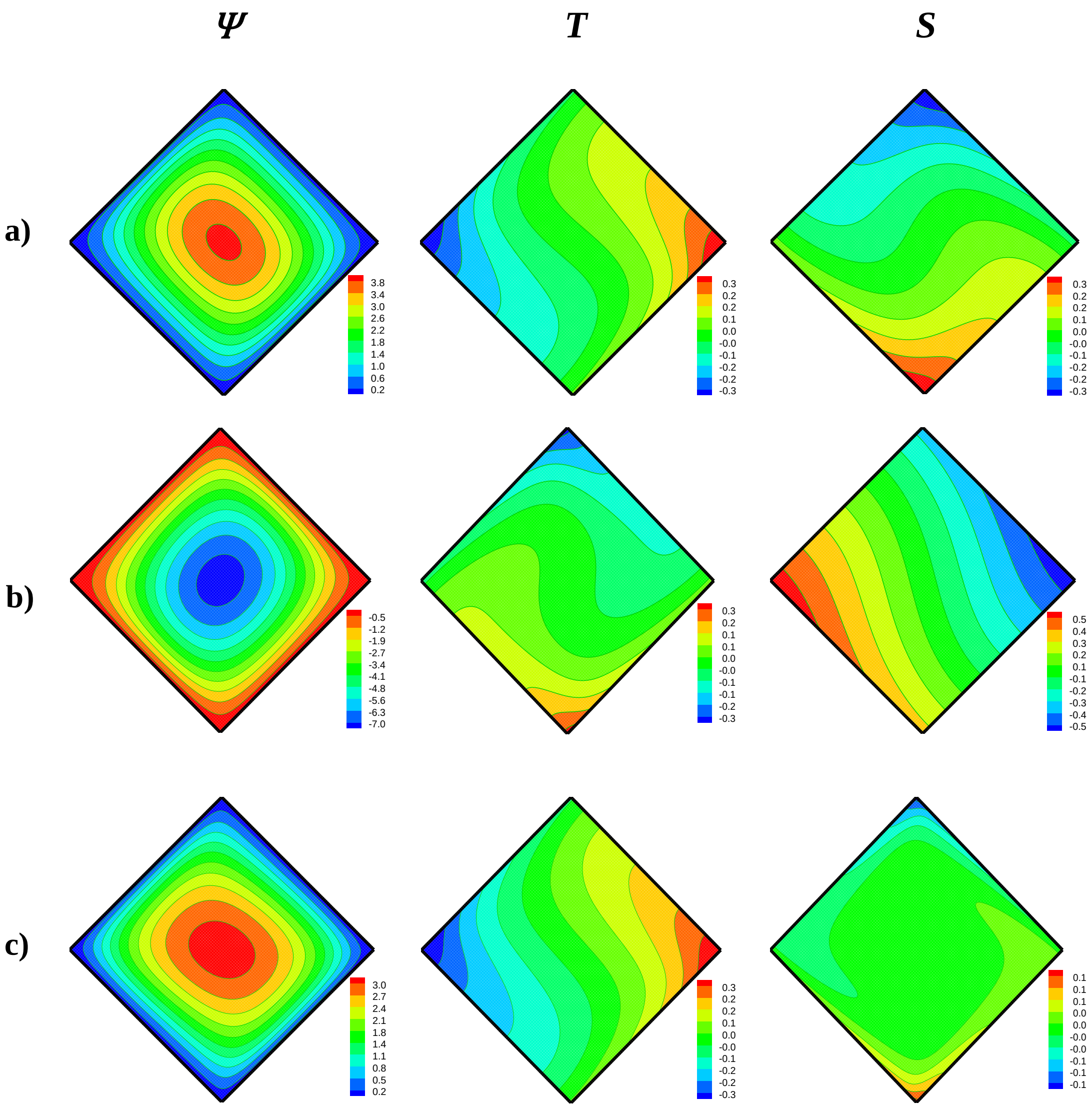

Fig.3. Stream function, temperature and concentration contours obtained for R_T=100:
a) *Le* =1: Ψ_{0min}= 0.00 Ψ_{0max}= 3.89 Nu_m= 3.19 Sh_m= 2.78
b) *Le* =0.1: Ψ_{0min}= -7.37 Ψ_{0max}= 0.00 Nu_m= 4.21 Sh_m= 1.13
c) *Le* =10: Ψ_{0min}= 0.00 Ψ_{0max}= 3.16 Nu_m= 3.07 Sh_m= 9.38

The effect of Rayleigh and Lewis numbers on flow behavior and heat and mass transfer rates is taken into account and convective flow instability thresholds are determined. The threshold of subcritical convection and the onset of natural convective were approximately determined from the numerical solution at finite amplitude convection. The determination of the supercritical Rayleigh number is explained in section 4.

Fig. 3 represents stream function, temperature and concentration contours obtained with the same values R_T =100 and Le = 10, 1 and 0.1. Fig. 4 (a) shows the intensity of the flow. The effect of Rayleigh number on heat and mass transfer rates, *Nu* and *Sh*, is presented in Fig. 4 (b) and (c) for various values of *Le*. Heat and mass transfer rates and flux intensity increase monotonically with R_T. It is observed that, when the R_T is relatively small, the intensity of the flow increases with the number of Lewis, but it seems to decrease for higher R_T, because the curves intersect. The same trend is observed for the number of Nusselt. However, it is found that the number of Sherwood increases monotonically with the number of Lewis.

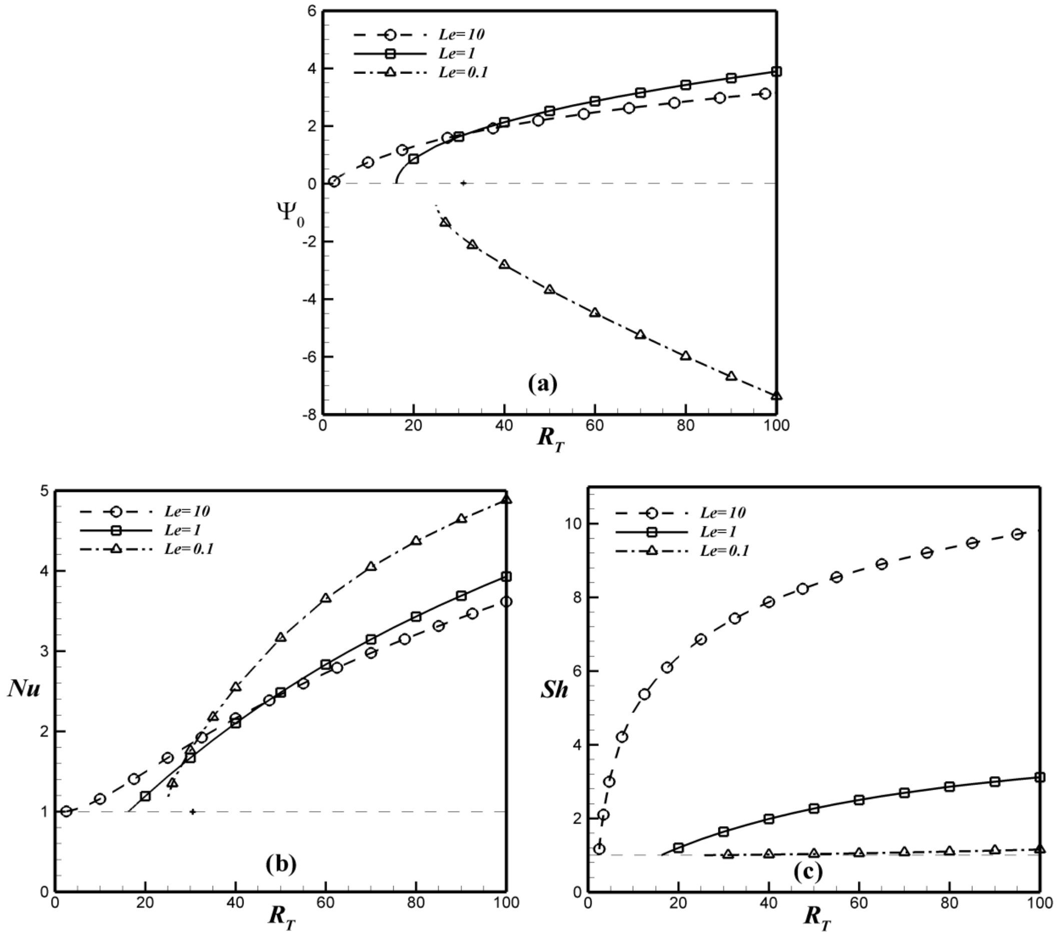

Fig. 4. Bifurcation diagram as a function of R_T and Le: (a) Flow intensity, (b) Nusselt number and (c) Sherwood number.

As shown in Fig. 4 (a). Natural convection is usually the preferable solution when launching the digital code with the solution in the idle state. As the mass diffusivity is greater than the thermal diffusivity (*Le*= 0.1), the heat transfer rate is higher than the mass transfer rate and conversely for the case where the thermal diffusivity exceeds the mass diffusivity (*Le* = 10).

In our study, we have two types of bifurcations, a supercritical bifurcation for *Le* = 1, but for *Le* = 0.1 and *Le* = 10, we have two subcritical bifurcations (see Table 1).

Table 1

Critical values of R_{TC}^{sup}, R_{TC}^{sub} and type of bifurcations

Le	R_{TC}^{sup}	R_{TC}^{sub}	Bifurcation
10	2.86	2.50	Subcritical
1	16.21	…	Supercritical
0.1	29.34	25.07	Subcritical

Starting with the rest state as initial conditions in region (I), the numerical results presented in Fig.5 (a)-(c) indicate that, below the subcritical or supercritical Rayleigh number (region I), the rest state prevails. In region (II), it is observed that the onset of steady motion is supercritical, as illustrated in Fig.5 (a), and subcritical, as illustrated in Fig. 5 (b) and (c), occurring at a Rayleigh numbers, R_{TC}^{sup} and R_{TC}^{sub}, above which the numerical solution bifurcates towards a steady finite amplitude convective regime. Upon increasing R_T above the supercritical Rayleigh number R_{TC}^{sup}, (region III) the strength of the convection is promoted monotonically. As can be seen from the zoom presented in Fig. 5.

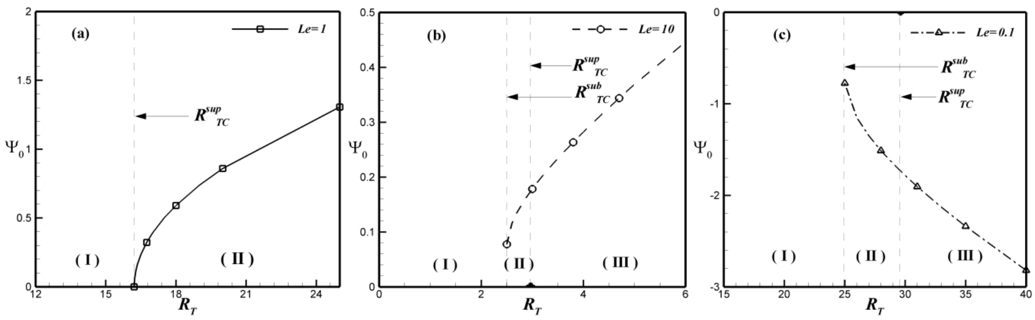

Fig. 5. Bifurcation diagrams in terms of Ψ_0 versus R_T for: (a) *Le*=1, (b) *Le*=10 and (c) *Le*=0.1.

After several tests, we can find a relationship that links supercritical Rayleigh number and Lewis number, also Fig. 6 Showed the Lewis number variation against supercritical Rayleigh number, the analytical expression of R_{TC}^{sup} and the numerical results are a very good agreement:

$$R_{TC}^{sup} = \frac{32.37}{Le+1} \tag{6}$$

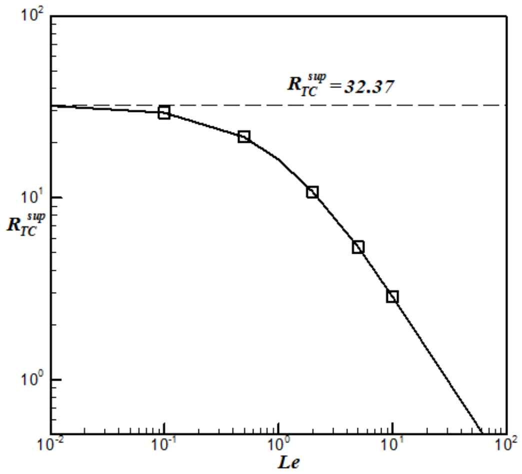

Fig. 6. Supercritical Rayleigh number according to Lewis number

6. Conclusions

In the paper, the double diffusion convection in a tilted porous square cavity, subjected to heat flow and cross-flow, is studied numerically. The cavity inclined at 45°. The conditions under which the forces thermal and mass thrust are equal (N = 1) are taken into account. For this situation, we prove the existence of stable convective solution for the same values of the parameters. The existence of exchange stability is demonstrated, the thresholds for the appearance of supercritical convection are obtained numerically and in great precision using the novel method. Also we can divide the results into two:

- If the thermal diffusivity is equal to the mass diffusivity, we obtain one cell which circulates in the clockwise direction. The intensity of the flow increases by increasing the number of thermal Rayleigh and the heat transfer is slightly larger compared to the mass transfer.

- If the mass diffusivity is large compared to the thermal diffusivity, we obtain a cell that circulates in the opposite direction of the clockwise with very high flux intensity, and each time the mass diffusion becomes greater, we notice that the heat transfer becomes more important than mass transfer.

Nomenclature

A	cavity aspect, L'/H'
D	mass diffusivity of species
H'	height of the layer
j'	constant mass flux per unit area
K	permeability of the porous medium
Le	Lewis number, α/D
N	buoyancy ratio, $\beta_S \Delta S'/\beta_T \Delta T'$
Nu	Nusselt number
q'	constant heat flux per unit area
R_T	thermal Darcy Rayleigh number, $g\beta_T KH'\Delta T'/\alpha\nu$
S	dimensionless concentration, $(S'-S'_0)/\Delta S'$
Sh	Sherwood number
S'_0	reference concentration at x'=0, y'=0
$\Delta S'$	characteristic concentration, $j'H'/D$
ΔS	dimensionless wall-to-wall concentration difference
T	dimensionless temperature, $(T'-T'_0)/\Delta T'$
t	dimensionless time, $t'\alpha/\sigma H'^2$
$\Delta T'$	characteristic temperature, $q'H'/k$
ΔT	dimensionless wall-to-wall temperature difference
u	dimensionless velocity in x-direction, $u'H'/\alpha$
v	dimensionless velocity in y-direction, $v'H'/\alpha$
x	dimensionless coordinate axis, x'/H'
y	dimensionless coordinate axis, y'/H'

Greek symbols

α	thermal diffusivity, $k/(\rho C)_f$
β_S	concentration expansion coefficient
β_T	thermal expansion coefficient
θ	angle of inclination of the cavity
ν	kinematic viscosity of the fluid
μ	dynamic viscosity of fluid
ρ	density of the fluid
$(\rho C)_f$	heat capacity of fluid
$(\rho C)_P$	heat capacity of saturated porous medium
σ	heat capacity ratio $(\rho C)_p/(\rho C)_f$
ε	porosity dimensionless of the porous medium
Ψ	dimensionless stream function, Ψ'/α
Ψ_0	stream function value at center of the enclosure

Superscript

$'$	dimensional variable
sub	subcritical
sup	supercritical

Subscripts

c	critical value
m	average value
max	maximum value
min	minimum value
$\circ$	reference state

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