A THEORETICAL MODEL FOR WELDING PROCESS WITH GAUSSIAN HEAT SOURCE - PART. 1

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În procesul de sudare, cele mai interesante regiuni pentru analiza transferului de căldură sunt zona de topire şi zona influenţată termic unde se ating temperaturi ridicate. Acest nivel ridicat al temperaturii cauzează transformări de fază şi modificări ale proprietăţilor mecanice ale materialului sudat. Calculul pentru estimarea distribuţiei căldurii la sudarea cu treceri multiple este mai complex decât în procesul de sudare cu o singură trecere datărită efectelor termice create de o trecere peste cele precedente. Folosirea surselor de căldură distribuite previne valori ridicate ale temperaturii în aprecierea zonei de topire. Comparaţia arată că ciclurile termice obţinute din modelul de sursă de căldură distribuită (Gaussiană) sunt mai de încredere decât cele obţinute din modelul cu sursă de căldură concentrată.

In the welding process, the most interesting regions for heat transfer analysis are the fusion zone (FZ) and the heat affected zone (HAZ), where high temperatures are reached. These high temperature levels cause phase transformations and alterations in the mechanical properties of the welded metal. The calculations to estimate the temperature distribution in multiple pass welding are more complex than in the single pass processes, due to superimposed thermal effects of one pass over the previous passes. The use of distributed heat source prevents infinite temperatures values near the fusion zone. The comparison shows that the thermal cycles obtained from the distributed (Gaussian) heat source model are more reliabe than those obtained from the concentrated heat source model.

Key words: welding, gaussian, theoretical model, heat source.

Introduction

Most of the published works on heat transfer during welding processes consider that the heat source is concentrated in a very small volume of the material. After such consideration, analytical solutions are obtained assuming a point, a line or a plane heat source, as those proposed by Rosenthal (1941). However, measurements of temperatures in the fusion and heat affected zones differ significantly from the values provided by those solutions, since the singularity located at the source origin results in infinite temperature levels. These

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concentrated source models present higher accuracy in regions where the temperature does not exceed twenty percent of the material melting point.

In order to avoid the occurrence of unrealistic values at the center and in the vicinity of the fusion zone (FZ), it is more adequate to consider a distributed heat source in the model development. In reality, the heat souce is distributed in a finite region of the material, a fact most relevant to the assessment of temperatures near the FZ. There are several models for heat source distribution. The Gaussian distribution, firstly suggested by Pavelic et al. (1969), is the most used. Although solutions considering distributed heat sources can be reached both analytically and numerically, there is an increasing tendency to use numerical methods. This work presents a new theoretical solution to estimate temperature field in multipass welding, as generated by Gaussian heat sources. The solutions were obtained from the known forms for the multipass welding, for point heat sources.

Theoretical model

In the one-dimensional model, the heat flux is considered to occur only in the y direction, as shown in the coordinate system of Fig. 1 The following assumptions are made: the heat source moves at a sufficiently high speed (to neglect heat flux in the x direction), and each weld pass fulfills the whole etched groove (no heat flux in the z direction).

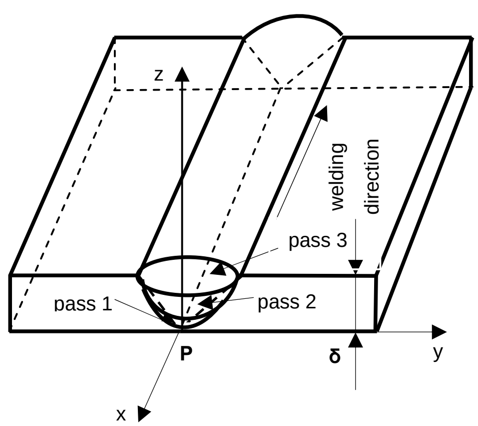

Fig. 1. Coordinate system used in the model.

The formulation of the problem to the first weld pass is made up by the one-dimensional transient heat conduction equation, and its boundary and initial conditions. It is similar to the formulation of the point heat source problem. In terms of θ, it is [4]:

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial y^2} \quad (1)$$

$$\theta(t)_{t=0} = 0 \quad (2)$$

$$\theta(y)_{y\to\infty} = 0 \quad (3)$$

$$\int_{-\infty}^{\infty} \theta dy = \frac{Q_1}{\rho c} \quad (4)$$

where:

T = temperature, (^oC); T_0 = ambient temperature, (^oC)

θ ($\theta = T - T_0$) = temperature difference, (^oC); t = time, (s)

$$\begin{cases} x = x - \text{axis/welding direction, (nm)} \\ y = y - \text{axis/transverse direction, (nm)} \\ \text{z = z - axis/through thickness direction, (nm)} \\ p = \text{ arbitrary point of observation} \end{cases}$$

Q_1 = thermal energy per unit area, (J/m^2); a = thermal diffusivity weld metal, (m^2/s); ρ = density of weld metal, (kg/m^3); c = specific heat weld metal, (J/kg^{o}C); δ = dimension less plate thickness, (m)

The thermal diffusivity is related to the thermal conductivity λ and the volume heat capacity ρc through the following equation: $a = \lambda / \rho c$.

The solution of this problem is known [1, 4], and it is expressed by:

$$\theta(y,t) = \frac{Q_1}{\rho c\sqrt{4\pi a t}}\exp\left(-\frac{y^2}{4at}\right) \quad (5)$$

To take into account the distribution of the heat source, please refer to Fig. 2, where a source with normal or Gaussian distribution is instantaneously at t = 0 to the surface of a plate. The center P of the source coincides with the origin O of the coordinate system xyz. The total power of the source is given by:

$$Q = \int_{-\infty}^{\infty} q_s(y)dy \quad (6)$$

where:

Q = total power of the source (W)

$q_s(y)$ = power density of distributed (Gaussian) heat source (W/m)

In the one-dimensional case, the Gaussian distribution of the heat source along the y direction occurs simultaneously at all points of the x direction of welding. The power $q_s(y)$ can be expressed by:

$$q_s(y) = q_m \exp\left(-ky^2\right) \quad (7)$$

where: q_m = maximum intensity of distributed (Gaussian) heat source (W/m)

k = coefficient of arc concentration (m^{-1}); $dq_s(y')$ = infinitesimal heat source (W)

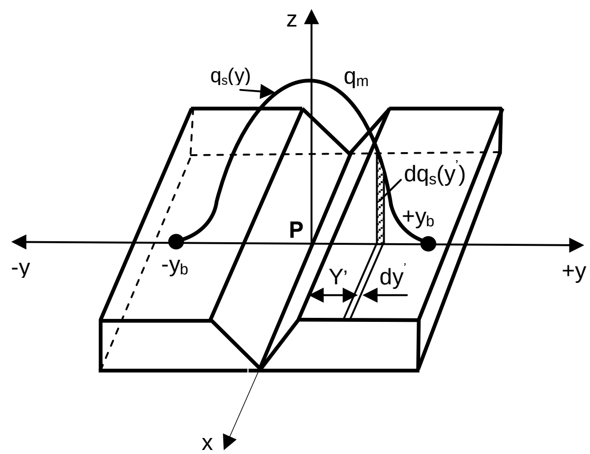

Fig. 2. Gaussian heat source.

Coefficient k is determined considering a distance y_b in Eq. (7), wich corresponds to the distance from the origin to the location where the power is reduced to five percent of its maximum value (Fig. 2). Thus,

$$k = \frac{3}{y_b^2} \tag{8}$$

When y_b is large, $q_s(y)$ decreases slowly with y. Substituting Eq. (8) in Eq. (7) and then in Eq. (6), and integrating this equation between ($-y_b$) and (y_b) limits, one obtains [5,6]:

$$q_m = \frac{\sqrt{3}Q}{\sqrt{\pi}\, y_b\, erf\left(\sqrt{3}\right)} \tag{9}$$

Where:

$erf(u)$ = gaussian error function

$$erf(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp\left(-u^2\right) du \tag{10}$$

Equation (7) may then be written as:

$$q_s(y) = \frac{\sqrt{3}Q}{\sqrt{\pi}\, y_b\, erf\left(\sqrt{3}\right)} \exp\left(-\frac{3y^2}{y_b^2}\right) \tag{11}$$

The diffusion process of an instantaneous Gaussian heat source applied to the surface of the material can be obtained by the source method. Let the y coordinate, along which the heat source varies, be divided in small elements dy. The heat dQ = $q_s(y)$dy is supplied to the element dy at t = 0, and can be regarded as an instantaneous point heat source. According to Eq. (5), the difussion process to an instantaneous heat source is:

$$\mathrm{d}\theta(y,t)=\frac{dQ}{\rho c\sqrt{4\pi at}}\exp\left(-\frac{y^2}{4at}\right)$$

or (12)

$$\mathrm{d}\theta(y,t)=\frac{q_s(y`)dy`}{\rho c\sqrt{4\pi at}}\exp\left(-\frac{d^2}{4at}\right)$$

Where d is the distance between the instantaneous source and a point located on the y axis, that is,

$$d^2=(y-y`)^2 \tag{13}$$

Substituting Eqs. (11) and (13) in Eq. (12) there results:

$$d\theta(y,t)=\frac{\sqrt{3}Q}{\pi y_b\rho c\sqrt{4at}erf\left(\sqrt{3}\right)}\exp\left(-\frac{3y^2}{y_b^2}\right)\exp\left(-\frac{(y-y`)^2}{4at}\right) \tag{14}$$

By the superposition principle, the temperature change in the y point can be obtained by summing the contributions of all instantaneous concentrated sources dQ, acting along the y coordinate of the material, between $-y_b$ and y_b points:

$$\theta(y,t)=\frac{\sqrt{3}Q}{\pi y_b\rho c\sqrt{4at}erf\left(\sqrt{3}\right)}\int_{-y_b}^{y_b}\exp\left(-\frac{3y^2}{y_b^2}\right)\exp\left(-\frac{(y-y`)^2}{4at}\right)\mathrm{d}y' \tag{15}$$

Solving the integral and rearranging the solution, one obtains[1] :

$$\theta(y,t)=\frac{\sqrt{3}Q}{2\rho c\sqrt{\pi\left(12at+y_b^2\right)}erf\left(\sqrt{3}\right)}$$

$$\left\{\begin{array}{l}\exp\left[-\frac{y^2}{4at}+\frac{y^2y_b^2}{4at\left(12at+y_b^2\right)}\right]erf\left(\frac{12at-yy_b+y_b^2}{2\sqrt{at\left(12\alpha t+y_b^2\right)}}\right)+\\ \exp\left[-\frac{y^2}{4at}+\frac{y^2y_b^2}{4at\left(12at+y_b^2\right)}\right]erf\left(\frac{12at+yy_b+y_b^2}{2\sqrt{at\left(12at+y_b^2\right)}}\right)\end{array}\right\} \tag{16}$$

Equation (16) is the solution to the first weld pass, regarding the input of a heat source with Gaussian distribution. The solution to the second pass is obtained from the point heat source solution (Suzuki, 1996):

$$d\vartheta(y,t)=\frac{q_{s1}(y`)dy`}{\rho c\sqrt{4\pi at}}\exp\left(-\frac{d^2}{4at}\right)+\frac{q_{s2}(y`)dy`}{\rho c\sqrt{4\pi a\left(t-t_p\right)}}\exp\left[-\frac{d^2}{4a\left(t-t_p\right)}\right] \tag{17}$$

In Eq. (17), it can be observed that the t variable was displaced by a value t_p, which corresponds to the sum of the welding and waiting times to the beginning of the second pass. The use of indices 1 and 2 in the q_s variable is for possible and sought variations of the heat input between passes. The same steps applied to obtain Eq. (16) are used to reach the solution for the second pass, and so on. Analogously, the general solution to n passes, in terms of T, is given by [2]:

$$T(y,t) = T_o + \frac{\sqrt{3}}{2\rho c\sqrt{\pi}} \sum_{i=1}^{n} \frac{Q_i}{\sqrt{12a\left[t-(i-1)t_p\right]+y_b^2}} \cdot$$
$$\{\exp\left\{-\frac{y^2}{4a[t-(i-1)t_p]} + \frac{y^2 y_b^2}{4a[t-(i-1)t_p]12a[t-(i-1)t_p]}\right\}$$
$$erf\left\{\frac{12a\left[t-(i-1)t_p\right]-yy_b+y_b^2}{2\sqrt{a\left[t-(i-1)t_p\right]\left\{12a\left[t-(i-1)t_p\right]+y_b^2\right\}}}\right\}+$$
$$+\exp\left\{-\frac{y^2}{4a\left[t-(i-1)t_p\right]} + \frac{y^2 y_b^2}{4a\left[t-(i-1)t_p\right]12a\left[t-(i-1)t_p\right]}\right\}$$
$$erf\left\{\frac{12a\left[t-(i-1)t_p\right]+yy_b+y_b^2}{2\sqrt{a\left[t-(i-1)t_p\right]\left\{12a\left[t-(i-1)t_p+z_b^2\right]\right\}}}\right\}\} \tag{18}$$

Conclusions

Equation (18) is the solution to the temperature distribution in one-dimensional multipass welding processes, supplied by Gaussian heat sources. Far from the heat source, i.e., for distances where y is of the same magnitude as y_b, Eq. (17) is similar to the solution obtained for the point heat source. However, near the FZ and HAZ ($y \ll y_b$), the correction introduced by the distributed heat source approach in Eq. (18) allows to better predicting the temperatures in these regions.

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