DISORDER AND COMPLEXITY MEASURES FOR THE STABILITY OF THE DAILY SOLAR RADIATIVE REGIME

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Lucrarea studiază diferite modalitati de caracterizare ale regimului radiativ al unei zile, precum si stabilitatea acestuia. Ierarhizarea zilelor din punctul de vedere al stabilității regimului radiativ poate fi făcută folosind atât valoarea medie zilnică a numărului de stabilitate a strălucirii soarelui cât şi valori ale dezordinii şi complexității acestui regim, definite în mod adecvat. Ierarhizarile rezultate ale zilelor sunt similare pentru toate cele trei criterii.

This paper focuses on different ways of characterizing the solar radiative regime of a day and the stability of this regime. Ranking the days from the view-point of the stability of their radiative regime may be performed by using the daily average value of the sunshine stability number and appropriately defined values of disorder and complexity, respectively. The resulted day hierarchies are similar for all three criteria.

Keywords: Solar raditive regim, sunshine number, disorder, complexity

1. Introduction

Two simple measures related to the state of the sky are often used for the classification of days from the point of view of the solar radiative regime. The most usual indicator is the total cloud cover amount *C*, which represents the fraction of the celestial vault covered by clouds. A second indirect measure of the state of the sky is the daily value of the relative sunshine $\sigma \equiv s / S$, where *s* is the bright sunshine duration during the daytime length *S.*

Solar radiation has alternating nature due to the moving of clouds on the sky. Due to zero transient time of PV modules, it means a noteworthy unstable output power of PV systems, which is of concern in the case of PV plants. As is known, ultraviolet solar radiation affects positively and negatively many natural process in biosphere. Major atmospheric constituents influencing surface UV radiation variability include cloudiness and ozone. Understanding UV radiation radiative regime at the Earth's surface is of significance to a wide range of disciplines, for example agriculture, oceanography as well as human health. Thus,

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the *stability* (or *fluctuation*) of the daily radiative regime might be equally important in some practical cases, such as the operation of photovoltaic arrays [1] or photosynthesis [2]. A recent study on this line [3] concluded that the changing speed of solar irradiance has an exponential density of probability and small- scale and large-scale fluctuations always coexist.

This paper focuses on different ways of characterizing both the radiative regime of a day and the stability of this regime and shows how the sunshine number can be used for day classifications.

Measurements performed in the Romanian town Timisoara (latitude 45°46′N, 21°25′E and 85 m altitude) are used in this study. Global and diffuse solar irradiance (G and G_d, respectively) recorded at the Solar Radiation Monitoring Station (SRMS) of the West University of Timisoara are used here [4]. Measurements were performed at equal time intervals of duration $\Delta\tau = 15\,\text{s}$ from 1st January to 31 December 2009. DeltaOHM first class pyranometers which fully comply with ISO 9060 standards and meet the requirements defined by the World Meteorological Organization are employed [5].

2. Sunshine number and sunshine stability number

The *sunshine number* ξ is a time dependent Boolean variable stating whether the sun is covered or not by clouds [6]:

$$\xi(t) = \begin{cases} 0 & \text{if the sun is covered by clouds at time } t \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

Let us consider a time moment t during the day-time and a time interval of duration Δt centered on t. $\xi(t)$ may be considered as a random variable during the time interval of duration Δt. By calculating the measures for $\xi(t)$ probability a relationship between the total cloud cover amount, relative sunshine and cloud cover can be computed [7]:

$$C(\Delta t) \approx 1 - \sigma(\Delta t) = \kappa(\Delta t) \tag{2a,b}$$

Equation (2a) is a popular relationship used by many models for calculating solar radiation on cloudy sky. Note that the usual attitude is to postulate Eq. (2a) but this relationship and the assumptions necessary to derive it were proved rigorously in [8]. Eq. (2b) is the cloud shade definition.

Series of sunshine number values can be derived from the series of measured solar irradiance values by using the World Meteorological Organization sunshine criterion [9]. Then, the "sun is shining" at time moment t_j if direct solar irradiance exceeds 120 W/m^2. In our notation, the sunshine number at time moment t_j is given by:

$$\xi_{m,j} = \begin{cases} 1 \text{ if } (G_j - G_{d,j})/\sin(h) > 120 \text{ W/m}^2 \\ 0 \text{ otherwise} \end{cases} \tag{3}$$

where the index *m* denotes "measurement" and h the sun's altitude angle.

The number of changes that sunshine number exhibits during a time interval of duration Δt may be used to characterize the stability of the radiative regime. Let us assume an equidistant time moment series t_j $(j = 1,n)$ during the time interval of duration Δt. One denotes $\Delta\tau \equiv t_{j+1} - t_j$ $(j = 1, n-1)$. The fluctuations of the sunshine number ξ during Δt may be described by using the *sunshine stability number* $\zeta(t_j, \Delta\tau)$ $(j = 2,n)$, which is a random Boolean variable defined by:

$$\zeta(t_{j\geq 2}, \Delta\tau) \equiv \begin{cases} 1 \text{ if } \begin{cases} \xi(t_j) < \xi(t_{j-1}) & (\text{when} \quad \xi(t_1) = 1) \quad \text{or} \\ \xi(t_j) > \xi(t_{j-1}) & (\text{when} \quad \xi(t_1) = 0) \end{cases} \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

Depending on the initial value $\xi(t_1) = 1$, Eq. (4) quantifies just one of the two different phenomena: sun appearance and sun disappearance on/from the sky, respectively.

The average value of the sunshine stability number during the interval Δt is denoted $\bar{\zeta}(\Delta\tau, \Delta t)$. Note that $\bar{\zeta}(\Delta\tau, \Delta t)$ is not a Boolean variable. It ranges between 0 (in the extreme case when the instantaneous values of the sunshine number ξ are all 0 or 1, respectively, for all time moments t_j (j=1,n) during Δt) and 1 (in the extreme case when the instantaneous values of the sunshine number ξ change for every two consecutive moments t_{j-1} and t_j during Δt). The radiative regime is *fully stable* in the first case and *fully unstable* in the last case.

Fig. 1 shows the variation of the daily averaged sunshine stability number $\bar{\zeta}(\Delta\tau, \Delta t)$ as a function of day number during the year 2009 at Timisoara. In this particular case, $\Delta\tau = 15$ s and Δt equals the daylight length, $\bar{\zeta}(\Delta\tau, \Delta t)$ ranges between 0 and 0.028. We can conclude that the radiative regime is rather stable.

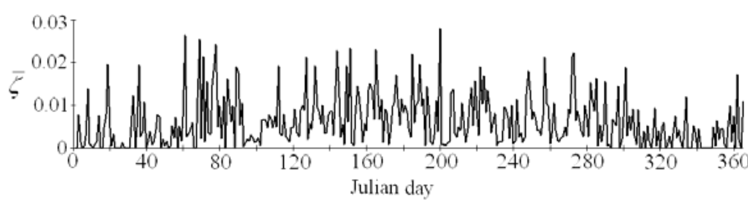

Fig. 1. Variation of the daily averaged sunshine stability number $\bar{\zeta}(\Delta\tau, \Delta t)$ as a function of day number during the year 2009 at Timisoara. In all cases $\Delta\tau = 15$ s and Δt is the daylight length.

3. Disorder and complexity

Entropy is very often used as a measure of disorder. We could define the entropy $S_x(\Delta\tau, \Delta t)$ associated to a random Boolean variable *x* (here *x* stands for either ξ or ζ):

$$S_x(\Delta\tau, \Delta t) \equiv -\sum_{x=0,1} p(x, \Delta t) \ln p(x, \Delta t) \quad (5)$$

However, this tacitly assumes that the size of the system, as measured by the number of states available to it, does not change. In fact, if the number of states of the system increases then the entropy and therefore the disorder of the system will also increase for no other reason than the increase in the number of states. To circumvent this problem, the "disorder" $\Delta_x(\Delta\tau, \Delta t)$ can be defined as [10, 11]:

$$\Delta_x(\Delta\tau, \Delta t) \equiv \frac{S_x(\Delta\tau, \Delta t)}{S_{x,\max}(\Delta\tau, \Delta t)} \quad (6)$$

where $S_{x,\max}(\Delta\tau, \Delta t)$ is the maximum entropy which occurs in the simplest case at the equiprobable distribution. The order $\Omega_x(\Delta\tau, \Delta t)$ is defined as [12]:

$$\Omega_x(\Delta\tau, \Delta t) \equiv 1 - \Delta_x(\Delta\tau, \Delta t) \quad (7)$$

Various complexity measures are defined in literature. Here we are using the simple complexity $\Gamma_x^{ab}(\Delta\tau, \Delta t)$ of disorder strength *a* and order strength *b*, which is defined by [12]:

$$\Gamma_x^{ab}(\Delta\tau, \Delta t) = \Delta_x^a(\Delta\tau, \Delta t)\Omega_x^b(\Delta\tau, \Delta t) \quad (8)$$

The statistical measures defined in this section have been used to classify the days in the database. For a given day, a time interval of duration $\Delta t\ (> \Delta\tau)$ may be adopted. In this paper Δt equals the length of daylight (i.e. the time interval between sunrise and sunset) and depends on day. Time series of sunshine number values are associated to the solar irradiance series during that time interval, by using Eq. (3).

Table 1

Statistical measures for selected days in the cloud shade class 0.5-0.6. In all cases $\Delta\tau = 15$s and Δt is the daylight length. Day rank is calculated according to $\bar{\zeta}(\Delta\tau, \Delta t)$. The day symbol has the general form *yymmdd* where *yy* is the year (09 stands for 2009), *mm* is the month number in the year and *dd* is the day number in the month.

$\bar{\zeta}$	Day symbol	Day rank	$\Delta_\xi(\Delta\tau, \Delta t)$	$\Gamma_\xi^{11}(\Delta\tau, \Delta t)$	$\Delta_\zeta(\Delta\tau, \Delta t)$	$\Gamma_\zeta^{11}(\Delta\tau, \Delta t)$
$0.98\cdot10^{-3}$	091221	1	1	$0.43\cdot10^{-3}$	0.011	0.011
$2.42\cdot10^{-3}$	090703	2	0.994	$5.8\cdot10^{-3}$	0.025	0.024
$3.76\cdot10^{-3}$	091129	3	0.993	$6.9\cdot10^{-3}$	0.036	0.034
0.010	090811	12	0.993	$6.6\cdot10^{-3}$	0.083	0.076

0.01	090624	13	0.989	0.0104	0.078	0.077
0.011	090919	14	0.985	0.0152	0.084	0.077
0.021	090312	24	0.999	$0.636 \cdot 10^{-3}$	0.149	0.127
0.024	090319	25	1	$0.363 \cdot 10^{-3}$	0.164	0.137
0.025	090310	26	0.982	0.0178	0.170	0.141

Table 1 shows results obtained for selected days in the class of daily averaged cloud shade 0.5 – 0.6. Figure 2 displays the diurnal variation of the sunshine stability number in the same days. Three of these days are characterized by the smallest values of the daily averaged sunshine stability number $\bar{\zeta}$ (i.e. these days have smallest solar radiation fluctuations) while the others three days have the largest values of $\bar{\zeta}$ (i.e. largest fluctuations of radiation). The main conclusion obtained from Fig. 2 is that the solar radiation fluctuation in days belonging to the same cloud shade class may be quite different. Table 1 shows that the disorder and the complexity based on sunshine number ξ (i.e. $\Delta_{\xi}(\Delta\tau,\Delta t)$ and $\Gamma_{\xi}^{11}(\Delta\tau,\Delta t)$, respectively) do not differentiate the days with small and large number of sunshine number fluctuations, respectively. On the contrary, both the disorder and the complexity based on the sunshine stability number (i.e. $\Delta_{\zeta}(\Delta\tau,\Delta t)$ and $\Gamma_{\zeta}^{11}(\Delta\tau,\Delta t)$) are obviously smaller in the days with smaller values of the sunshine stability number than in the other days. Any of these last two measures can be used to classify the days from the point of view of the stability of the radiative regime.

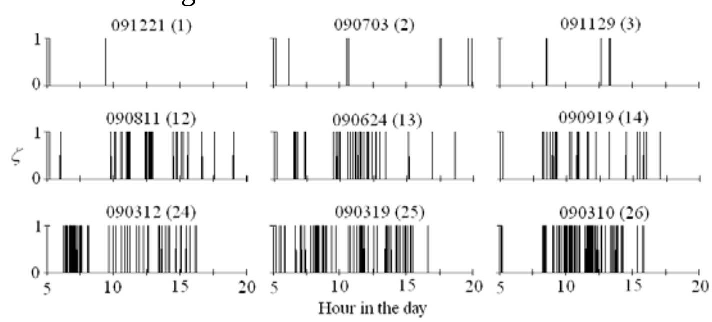

Fig. 2. Diurnal variation of the sunshine stability number ζ in nine days with mean cloud shade 0.5-0.6. In sub-figures hading the day rank according to the daily mean value of sunshine stability number $\bar{\zeta}(\Delta\tau,\Delta t)$ is shown in parentheses. In all cases $\Delta\tau = 15$ s and Δt is the daylight length.

The ξ-based complexity $\Gamma_{\xi}^{11}(\Delta\tau,\Delta t)$ does not scale with the ξ-based disorder $\Delta_{\xi}(\Delta\tau,\Delta t)$ (see Table 1). The complexity $\Gamma_{\xi}^{01}(\Delta\tau,\Delta t)$ is more appropriate to be used for the characterization of days from the point of view of the sunshine number. The ζ-based complexity $\Gamma_{\zeta}^{11}(\Delta\tau,\Delta t)$ scales very well with the ζ-based disorder $\Delta_{\zeta}(\Delta\tau,\Delta t)$. Taking into account Eq. (8), we conclude that the complexity $\Gamma_{\zeta}^{10}(\Delta\tau,\Delta t)$ is a simpler measure of complexity than $\Gamma_{\zeta}^{11}(\Delta\tau,\Delta t)$. However, both measures may be used for the characterization of days from the point of view of the fluctuations of solar global radiation.

Ranking the days from the view-point of the stability of their radiative regime may be performed by using various criteria, such as the daily average value of the sunshine stability number $\bar{\zeta}$ and the ζ-based disorder and complexity. The resulted day hierarchies are similar for all three criteria [13].

4. Conclusion

Sunshine stability ζ number may be used to quantify the stability of the solar radiative regime. The average value of the sunshine stability number $\bar{\zeta}$ during any time interval ranges between 0 and 1. The radiative regime is *fully stable* in the first extreme case and *fully unstable* in the last case. In practice, when applied to the 2009 days in Timisoara, the daily averaged sunshine stability number ranges between 0 and 0.028, demonstrating a rather stable solar radiative regime. Measures based on disorder and complexity concepts have been introduced to properly quantify the daily fluctuations of solar radiation. However, the criterion based on the daily average value of ζ is simpler.

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