ABOUT VISCOUS DISSIPATION ON THE DISKS OF TURBOMACHINES ROTORS

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Se analizează relaţiile de calcul al coeficienţilor de frecare vâscoasă pe discuri în rotaţie obţinute teoretic sau prin formule empirice şi se compară cu cele obţinute de autori în laborator la numere Reynolds $10^2 \div 10^4$. Cercetările experimentale confirmă unele relaţii de calcul şi se prezintă rezultatele originale obţinute.

The calculation relations for the viscous friction coefficients on disks in rotation, obtained theoretically or by empirical formulas, are analyzed and compared with the results obtained by the authors in the laboratory at Reynolds numbers between 10^2 and 10^4. The experimental researches confirm some calculation relations and the original obtained results are exposed.

Keywords: disk in rotation, viscous friction

1. Introduction

The problem of establishing the power consumed through viscous friction at the disks of turbo-machines rotors [10], [11], [12], [13], at mills with disks [8], in installations for blending and homogenization [7], [14], represents, even now, an important research domain [1], [2], [4], [5] where new researchers are beginning to get involved, as, for example, those from NACA [2].

We define the Reynolds number of the flow between the disk and the carcass

$$\mathrm{Re} = \frac{\omega R^2}{\upsilon} = \frac{\omega D_2^2}{4\upsilon} \tag{1}$$

where $\omega=\pi n / 30$, n – rotative speed [rpm], D_2 rotors diameter [m], υ – kinematical viscosity of the working fluid

The friction moment on one of the faces on the disk, M, and respectively the power consumed by viscous friction, N, are given by the relations:

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$$M = C_M \frac{\rho}{2} \omega^2 R^5 \tag{2}$$

$$N = M\omega \tag{3}$$

were ρ- the density of the working fluid and C_M - the friction moment coefficient determined through some relations established by different researchers.The C_M coefficient depends of Reynolds number, relative rugosity k/R and on the distances between the disk and the carcass.

$$C_M = f\left(\mathrm{Re}, k/R, s\right) \tag{4}$$

In the case of disks, the critic Reynolds number is $\mathrm{Re}_{cr} = 3 \cdot 10^5$ [2], [5], [8].

2. Theoretical results

For free disks functioning in unlimited fluid and laminar regime, C_M can be calculated by the relation

$$C_M = \frac{3{,}87}{\sqrt{\mathrm{Re}}} \tag{5}$$

obtained by Cochran, quoted in [3], [6], [9].

In turbulent regime we can use the following relations given by Kàrmàn:

$$C_M = \frac{0{,}46}{\left(\mathrm{Re}\right)^{1/5}}, \quad 3 \cdot 10^5 < \mathrm{Re} < 10^6 \tag{6}$$

or the one given by Goldstein:

$$\frac{1}{\sqrt{C_M}} = \left(\frac{1}{k\sqrt{8}}\right) \ln\left(\mathrm{Re}\sqrt{C_M}\right) + 0{,}03 \ , \quad 10^5 < \mathrm{Re} < 2 \cdot 10^6 \tag{7}$$

In (7), k = 0,41 is Kàrmàn`s constant [3].

In fig. 1 the dependence $C_M(Re)$ and experimental results are presented [2].

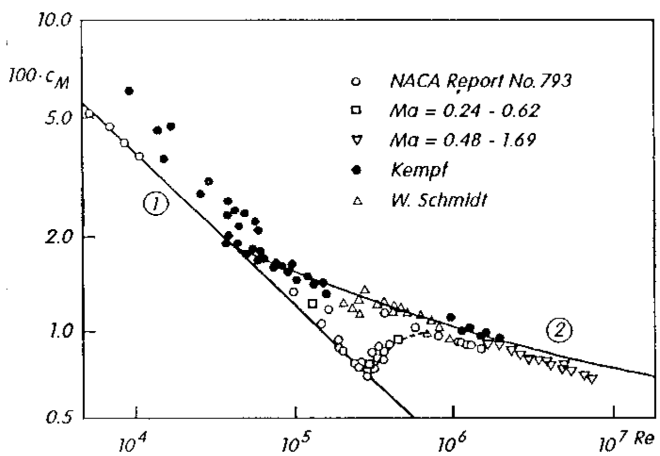

Fig.1 C_M(Re) dependence given in [2]; 1 – equation (5), 2 – equation (7)

For cassed disks, the simplest hypothesis consists in considering the Couette type of flow where the friction tension and the friction moment are given by the following relations:

$$\tau = \eta \frac{r\omega}{s} \; ; \; M = 2\pi \int_0^R \tau r^2 dr = \frac{\pi}{2} \frac{\omega \eta R^4}{s} \tag{8}$$

For the two faces of the disk we obtain

$$C_M = 2\pi \frac{R}{s} \frac{1}{\mathrm{Re}}, \; \mathrm{Re} < \mathrm{Re}_{cr} \tag{9}$$

Other relations proposed [3], [6] are:

$$C_M = \frac{2{,}67}{\sqrt{\mathrm{Re}}} , \quad 10^4 < \mathrm{Re} < 3 \cdot 10^5 \tag{10}$$

for laminar regime and

$$C_M = \frac{0{,}062}{\sqrt[5]{\mathrm{Re}}}, \; \mathrm{Re} > 3 \cdot 10^5 \tag{11}$$

for turbulent regime.

One can notice that the *C_M(Re)* dependence, graphically represented, differs from an edition to another [3] or from a treaty to another [3], [6]. In an older edition [3] we also find the experimental results concerning the cassed disk, which are different from those calculated by equations (10) and (11) with about 17%,on the average. Also, the C_M values for free disk are 45% higher (equations (5) and (10)), which can be explained [5] by the appearance of the three-dimensional secondary flow in the border bed.This statement, is not very plansible, neither is the on according to which the influence of the carcass works for s/R<0,3 [5] .

3. Empirical formulae for turbomachines

The dispersion of the experimental results, especially in the area $Re = 10^5 \div 10^6$ (fig. 1) led to the necessity of getting empirical formulae, specific to the turbomachines.

For centrifugal pumps, Stodola [12] proposes the following formula for powers N in kW:

$$N = \beta \cdot 0{,}376 \cdot 10^{-6} \cdot \rho \cdot u_2^2 D_2^2 \quad (12)$$

in which u_2 is the speed at the rotor exit, in [m/s], while D_2 is the diameter of the rotor at its end, in [m]; the coefficient $\beta = 1{,}1 \div 1{,}2 = f(\mathrm{Re})$, without any other explanations.One can notice that the power N does not depend on the viscosity and on the s/R. ratio. Defining the Reynolds number as:

$$\mathrm{Re} = \frac{u_2 R_2}{\upsilon} = \frac{\omega R_2^2}{\upsilon}$$

and expressing the power N in W we obtain, using the definition (2), the formula:

$$C_M = 3\beta \frac{1}{\omega R^2} 10^{-3} = 3\beta \frac{\upsilon \mathrm{Re}}{R_2} 10^{-3} \quad (13)$$

For water ($\upsilon = 10^{-6}$ m^2/s) at a rotation speed of 3000 rpm, $R_2 = 0.1$ m there results the value $C_M = 3{,}11 \cdot 10^{-3} / 2 = 1{,}555 \cdot 10^{-3}$ for a disk wet on one side, this value differs from the one given by the formula (13) and from the ones published in [3].

For centrifugal pumps, [11] proposes calculation relations which are differentiated by the Reynolds number $\mathrm{Re} = \omega^2 R_2 / \upsilon$:

$$M_f = C_f \rho R_2^5 \omega^2 \quad (14)$$

with $C_f = C_M/2$

The values of C_f are calculated as fallows:

$$C_f = \frac{\pi}{\mathrm{Re}} \frac{R_2}{s} + \mathrm{Re}\left(\frac{s}{R_2}\right)^3 \left[0{,}0146 + \left(\frac{s}{R_2}\right)^2 0{,}1256\right], \quad \text{for } \mathrm{Re} < 2 \cdot 10^4 \quad (15)$$

$$C_f = \frac{C_M}{2} = \frac{1{,}334}{\sqrt{\mathrm{Re}}}; \; C_M = \frac{2{,}668}{\sqrt{\mathrm{Re}}}, \quad \text{for } 2 \cdot 10^4 < \mathrm{Re} < 10^5 \quad (16)$$

These values are close to the ones given by relation (10), while

$$C_f = \frac{0{,}0465}{\mathrm{Re}^{\frac{1}{5}}}, \; \mathrm{Re} > \mathrm{Re}_{cr} \approx 10^5 \quad (17)$$

with values of the same order as the ones given by relation (11).

Returning to the relation (15), for $R_2 = 0{,}1$ m and $s = 2$ mm we obtain

$$C_f = \frac{50\pi}{\mathrm{Re}} + 10^{-6}\,\mathrm{Re}\left(0.1168 + 4{,}019 \cdot 10^{-4}\right) = \frac{50\pi}{\mathrm{Re}} + 1172 \cdot 10^{-10}\,\mathrm{Re} \quad (18)$$

This is practically an inverse dependence of Re, respectively one of type (9).
The great differences between the values of $C_M(C_f)$ coefficients at Reynolds numbers Re>10^6 have recently led [4] to the deduction of a new type of formulae, namely:

$$N = AD_2 z^x n^y (1 + ze) \tag{19}$$

with N in [kW], D_2 in [m] and n in [rpm]; $e = 2s$. The coefficients A and z and the exponents x and y depend on D_2 while n and e are determined in special stalls. Thus, for $D_2 = 0{,}27$ m,
n = 1480 rpm, the working fluid being water ($\mathrm{Re} = 2{,}82 \cdot 10^6$) and e = 0,011 m, we get $A = 4{,}3 \cdot 10^{-9}$,
z = 6, $x = y$ = 3,25 and N = 0,572 kW, a value higher than the measured one with 2% .

For centrifugal compressors we propose the following relation [13]:

$$N = \frac{17}{\mathrm{Re}^{0,179}} \rho_m^{'} D_2^2 \left(\frac{u_2}{100} \right)^3 \cdot X \tag{20}$$

with N in [kW]; X the number of steps; ρ_m the average value of ρ in [kg/m^3] and $\mathrm{Re} = D_2 u_2 / \upsilon$. C_M is identified and we get, for one step, with N in [W]:

$$C_M = \frac{1{,}088}{\mathrm{Re}^{0,179}} \tag{21}$$

with $\mathrm{Re} = \omega D_2^2 / 4$ in order to compare the values given by relation (20) with the published ones.

For air ($\upsilon = 13 \cdot 10^{-6}$ m^2/s), R_2 = 0,1 m, n = 3000 rpm, we get $\mathrm{Re} = 2{,}4 \cdot 10^5$ and
C_M = 0,1184, values which are different from those given by relation (10) ($C_M = 5{,}45 \cdot 10^{-3}$) or by relation (11) ($C_M = 5{,}22 \cdot 10^{-3}$).

We have to mention that at present, for example [5], smaller domains of experimental research are approached, by limiting n_s (n_s = 30 ÷ 80), where, by combining the values of rugosity and those of *s/R,* we obtain diminutions of the power N up to 7%.

4. Experimental research and results

The comparison of the experimental results with the theoretical ones (the relations (5) and (7)), presented in figure 1, leads to same remarks, namely:

- in laminar regime, for $\mathrm{Re} > 5 \cdot 10^4$ the older experimental results (Kempf) as well as the new ones (NACA) are coherent with the theoretical results, meaning that they verify the relation (15) or they are greater;

- in the interval $10^5 < \mathrm{Re} < 10^6$ both relations, (5) and (7) are experimentally confirmed (fig. 1), that leads to a great dispersion of the C_M values;
- at Re>10^6 the experimental values are closed to the ones obtained with relation (9).

The carcass influence over the C_M coefficient is not presented in figure 1; the experimental results for s/R = 0,0199 or for s/R = 0,03 are about 17% greater than those calculated with relation (11) and smaller then those obtained for free disks.

4.1. The experimental installation. Working fluid

The exposed considerations have led to the realization of an experimental installation (fig. 2) which is made from a circular thin disk Φ100 mm, mounted in a cylindrical carcass, which is 100 mm height so that s/R = 1; s/R can be reduced by mounting disks on the carcass cover.

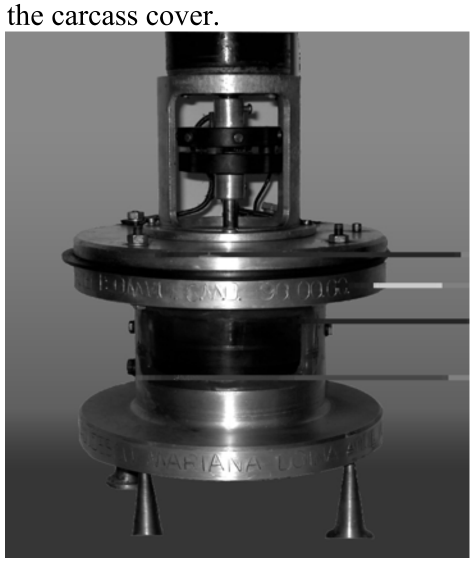

Fig. 2 The experimental installation

For s/R = 1, we can check in order to see if, at this value of the rapport, the presence of the carcass influences the C_M value. The disk is driven by a small engine having the angular velocity of n = 100 ÷ 1400 rpm.

The physical properties of the working liquids, respectively, viscosity and density, in terms of temperature are presented in fig. 3 and in table 1.

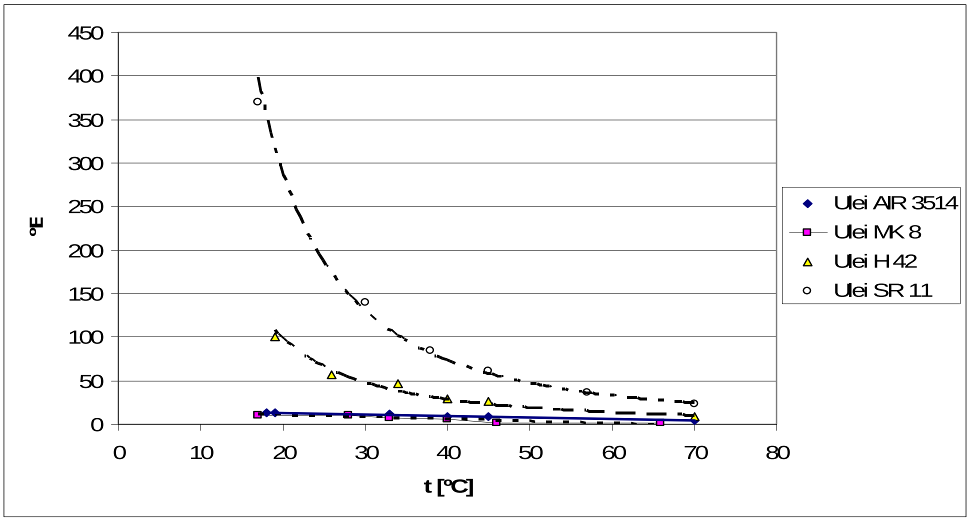

Fig. 3 The viscosity of working fluids

Table 1

The liquids density [kg/m³] in terms of temperature

Liquid | t[◦C]	18	24	34	40	45	50	65	72	80
AIR3514	935,4				912,4		892,3		
H 42	899,5			872,5					841,2
SR 11	896,5		884,2					851,6	
MK 8		875,3				850,4			825,1

4.2. Obtained results

The C_M^e values experimentally obtained for $\mathrm{Re} = 100 \div 3 \cdot 10^4$ are exposed in table 2, as well as those theoretically calculated by relation (5) for free disk C_M and by relation (10) for cassed disks C_M^c.

The values C_M (Re) *Table 2*

Re	109	535	650	915	1228	1581	2195
100 C_M^e	25.00	11.50	10.73	8.82	7.61	6.71	5.69
100 C_M	36.96	16.72	15.16	12.79	11.04	9.73	8.26
100 C_M^c	25.57	11.54	10.47	8.82	7.62	6.71	5.70

Re	2509	4212	10736	14148	20470	24271	28154
100 C_M^e	5.32	4.11	2.87	2.04	1.86	1.71	1.59
100 C_M	7.72	5.96	3.73	3.25	2.70	2.48	2.36
100 C_M^c	5.33	4.11	2.57	2.24	1.86	1.71	1.91

5. Conclusions

1. A comparison between the obtained results with those publish in [3], [6], [9] certifies the experimental installation and the working method, fact that allows the development of researches for $s/R < 1$.
2. The carcass effect over the C_M coefficient is confirmed for $s/R = 1$, the experimental values are with about 45% smaller then those calculated for the free disk.
3. The experimental results obtained for $\mathrm{Re} = 10^2 \div 3 \cdot 10^4$ are original and they confirm the equation (10).
4. The empirical equations (16) and (17) are verified for the turbo-machines disks.

R E F E R E N C E S

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