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ANALYTICAL ANALYSIS OF SQUEEZING FLOW IN POROUS MEDIUM WITH MHD EFFECT

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In this article, the velocity profile of squeezing flow of an incompressible viscous fluid in a porous medium under the influence of uniform transverse magnetic field is argued. Both the continuity and momentum equations, with the help of vorticity and stream functions, are simultaneously transformed to an ordinary differential equation. Boundary conditions are also transformed with the help of transformation $\psi(r, z) = r^2T(z)$. Homotopy Perturbation Method (HPM) is used to solve the boundary value problem obtained. Efficiency of the proposed scheme is examined with the help of residual. Effect of different parameters on the velocity profile is discussed through graphs. It is observed that both imposed magnetic field and electro conductivity are directly proportional to the velocity of fluid.

Keywords: Magnetohydrodynamics, Squeezing flow, Porous Media, Homotopy Perturbation Method
MSC 2010: 76D05, 34B15.

1. Introduction

The concept of squeezing flows is common in moulding and forming operations and therefore the researchers focus these kind of flows for many years. Initially the squeezing of a single film between two parallel plates has been analyzed by stefan in 1874 [1]. Stefan, in his work, developed asymptotic solution for a thin film of Newtonian fluid. Since 1930, the use of plastometer (parallel plate) is common as a rheological measuring device. Rheologists started to look for devices which are in position to impart a controllable extensional flow field to a test sample. The credit goes to Chatraei et al. [2] who initially used such devices. Squeezing flow of two Newtonian films as a guess to the squeeze flow moulding process of polymers between two heated plattes is discussed by Lee et al. [3]. They built up analytic solutions for extreme values of dimensionless group $T = \eta(\phi)^2/u$ by using finite element model. Here the dimensionless half gap is denoted by $\eta(\phi)$ and the ratio of lubricant to sample

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viscosity by u. The extension of their work to oscillatory behaviour of visco-elastic materials under a constant load is considered by them in [4]. Squeeze flows of apparently lubricated thin films are studied by Adam S. Burbidge and Colin Servais [5]. Food industry and chemical engineering are the areas where squeeze flow models are applied widely. Practically such kind of flow-models are used in polymer processing, injection and compression.

Effects of magnetic field on lubrication fluid films has aroused substantial concentration for many years. Hughes and Elco [6] studied the effect of magnetic field on lubrication film dynamics. They investigated the dynamics of an electrically conducting, incompressible, viscous fluid in the presence of a magnetic field between two parallel disks. The analytic solutions for the velocity and pressure fields, considering hydromagnetic slider bearing problem, is studied by Chawla [7]. He proved in his work that by increasing magnetic and electrical fields, the load carrying capacity of the bearing increases. The velocity profile of squeeze flow between two large parallel plates, in the presence of imposed magnetic field and slip boundary condition, is argued in [8]. It is observed in the last work that the velocity of fluid increases by (i) increasing Hartmann number (ii) decreasing Reynold number and (iii) increasing the value of slip parameter respectively. The special case, when the slip vanishes, is also discussed graphically. The effect of external circuit on MHD squeeze film between conducting plates is observed in [9]. The expression for the time of approach is derived in this work and it is proved that by increasing the magnetic field in the free space results the decrease in the response time. Many other researchers considered the magnetic field effect in different aspects which can be found in [10, 11, 12, 13, 14, 15, 16, 17].

The material containing pores filled by fluid is known as porous medium. The ability of allowing fluids to pass through porous material is known as permeability denoted by k. The flow through porous medium is described by Darcy's law presented by Henry Darcy $(1803 - 1858)$. Flow of water through beds and sands is the initiative of this law. The observations in porous medium became more appealing when modified Darcy's law was launched [18]. The discussion of flows through porous medium is made in [19, 20, 21, 22].

The idea of Homotopy Perturbation Method (HPM) is introduced by J. H. He [23, 24, 25, 26, 27, 28, 29]. This scheme uses embedding parameter p and in few iterations the asymptotic solution is obtained. Nonlinear differential equations arise in heat transer and porous media, while modeling, are successfully solved by HPM in [30]. For fourth grade fluid, the thin film flow down a vertical cylinder is considered in [31] and the derived model is then solved by HPM. The model for third grade fluid flow (thin) on a moving belt is also studied by using HPM in [32]. In [33] comparision of Adomian decomposition and Homotopy perturbation method is made to solve Blasius equation.

The aim of present work is to derive the velocity profile, for first grade squeezing fluid flow in a porous medium with imposed magnetic field, by using

Homotopy Perturbation Method. Secondly to argue the effect of various parameters on the velocity profile. The modeled differential equation is derived in the first section. Fundamental idea of HPM and its implementation are discussed in the respective sections. Validity of the solution by HPM is observed by considering the residual of the problem. The influence of magnetic field and electrical conductivity are shown through graphs.

2. **Construction of Mathematical Model**

The continuity and momentum equations [8], for axisymmetric squeeze fluid flow (steady) in a porous medium with imposed magnetic field B_0, constant density ρ and viscosity μ squeezed between two parallel plates detached by a minute distance $2L$ moving towards each other with a stumpy velocity $\mathbf{u} = [u_r, 0, u_z]$ are;

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0, \tag{1}$$

$$\rho(\nabla.\mathbf{u})\mathbf{u} = \nabla.\mathbf{T} + \mathbf{S} + \mathbf{r}. \tag{2}$$

Here $\mathbf{S} = \mathbf{J} \times \mathbf{B}$, $\mathbf{J} = \mathbf{u} \times \mathbf{B}$, ∇ represents the material time derivative, $\mathbf{T}$ is the Cauchy stress tensor expressed by $\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}$ with $\mathbf{A} = \nabla\mathbf{u} + (\nabla\mathbf{u})^T$. $\mathbf{J}$ is the electric current density and $\mathbf{B}$ is the total magnetic field given by $\mathbf{B} = \mathbf{B}_0 + b$. $\mathbf{B}_0$ and b are the imposed and induced magnetic fields respectively. $\mathbf{r}$ is the Darcy's resistance. According to Naduvinamani et al. [34] and Breugem [35], $\mathbf{r}$ can be written as:

$$\mathbf{r} = -\frac{\mu}{k}\left(u_r, 0, u_z\right). \tag{3}$$

Consider displacement currents negligible, The modified Ohm's law and Maxwell's equations can be written as;

$$\mathbf{J} = \sigma[E + \mathbf{u} \times \mathbf{B}], \qquad \nabla.\mathbf{B} = 0,$$
$$\nabla \times \mathbf{B} = \mu_m\mathbf{J}, \qquad curl E = \frac{\partial \mathbf{B}}{\partial t}.$$

The electric field is denoted by E, electrical conductivity by σ and magnetic permeability by μ_m. If b is so small that its value as compared to $\mathbf{B}_0$ is negligible, then the involved magneto hydrodynamic force can be written as:

$$\mathbf{J} \times \mathbf{B} = [-\sigma B_0^2 u_r, 0, 0]. \tag{4}$$

If $P = \frac{\rho}{2}(u_r^2 + u_z^2) + p$ be the generalized pressure and the flow is steady then by comparing components the Navier-Stokes equations (1) and (2) can be written

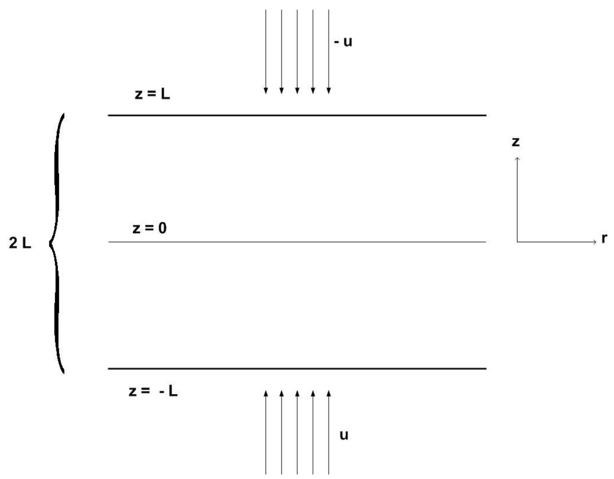

FIGURE 1. Geometry of the flow

as;

$$\frac{\partial P}{\partial r} - \rho\left(\frac{\partial u_z}{\partial r} - \frac{\partial u_r}{\partial z}\right)u_z = -\left(\mu\frac{\partial}{\partial z}\left(\frac{\partial u_z}{\partial r} - \frac{\partial u_r}{\partial z}\right) + \left(\frac{\mu}{k} + \sigma B_0^2\right)u_r\right), \quad (5)$$

$$\frac{\partial P}{\partial z} + \rho\left(\frac{\partial u_z}{\partial r} - \frac{\partial u_r}{\partial z}\right)u_r = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\left(\frac{\partial u_z}{\partial r} - \frac{\partial u_r}{\partial z}\right)\right) - \left(\frac{\mu}{k}\right)u_z. \quad (6)$$

Introducing stream function $\psi(r,z)$ as;

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial z}, \qquad u_z = -\frac{1}{r}\frac{\partial \psi}{\partial r},$$

and eliminating the generalized pressure P from (5) and (6), single partial differetial equation is obtained as;

$$\rho\left(\frac{\partial \psi}{\partial r}\frac{\partial}{\partial z}\left(\frac{E^2\psi}{r^2}\right) - \frac{\partial \psi}{\partial z}\frac{\partial}{\partial r}\left(\frac{E^2\psi}{r^2}\right)\right) = \frac{\mu}{kr}E^2\psi + \frac{\sigma B_0^2}{r}\frac{\partial^2 \psi}{\partial z^2} - \frac{\mu}{r}E^4\psi \quad (7)$$

Where $\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ is denoted by E^2. It is clear from Fig. 1 that;

$$at \quad z = 0, \quad u_z = 0, \quad \frac{\partial u_r}{\partial z} = 0,$$
$$at \quad z = L, \quad u_r = 0, \quad u_z = -u.$$

Using the transformation $\psi(r,z) = r^2 T(z)$, denote $\frac{1}{k} + \frac{\sigma B_0^2}{\mu}$ by S^2 and $\frac{2\rho}{\mu}$ by Q, Eqn. (7) reduces to:

$$T^{(iv)}(z) - S^2 T''(z) + QT(z)T'''(z) = 0, \quad (8)$$

and the corresponding boundary conditions are transformed to;

$$T(0) = 0, \quad T''(0) = 0, \quad T(L) = -\frac{u}{2}, \quad T'(L) = 0. \quad (9)$$

3. **Elementary Theory of HPM**

To illustrate the idea of HPM let us consider the following nonlinear differential equation [24, 25, 26, 27, 28, 29];

$$A(u) - g(s) = 0, \qquad s \in \Omega \tag{10}$$

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad s \in \Gamma. \tag{11}$$

Eqns.(10) and (11) consist of general differential operator $A(u)$ which is usually divided into two parts namely L and N, the linear and nonlinear operators respectively. i.e. $A = L + N$, a known analytic function $g(s)$, the boundary operator B, domain Ω and its boundary Γ. Eqn. (10) can further be written as;

$$L(u) + N(u) - g(s) = 0. \tag{12}$$

For homotopy perturbation method, a homotopy $\mathfrak{H}(v, p) : \Omega \times [0, 1] \to R$ satisfying;

$$\mathfrak{H}(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - g(s)] = 0, \tag{13}$$

is constructed. Here $p \in [0, 1]$ is an embedding parameter, u_0 is the initial approximation satisfying the boundary conditions. From Eqn. (13);

$$\mathfrak{H}(v, 0) = L(v) - L(u_0) = 0 \tag{14}$$

$$\mathfrak{H}(v, 1) = A(v) - g(s) = 0. \tag{15}$$

Eqn. (13) becomes linear equation when $p = 0$ and nonlinear when $p = 1$. It is concluded that when p varies form 0 to 1 then the problem $L(v) - L(u_0) = 0$ is continuously deforms to the problem $A(v) - g(s) = 0$. The solution of Eqn. (13) is written as a power series in p as;

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots . \tag{16}$$

By setting $p = 1$, approximate solution of Eqn. (10) is as under;

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots . \tag{17}$$

4. **Application of HPM**

Following (13) the homotopy construction for (8) is as follows;

$$\mathfrak{H}(z, p) = (1 - p)[T^{(iv)}(z)] + p[T^{(iv)}(z) - S^2 T''(z) + QT(z)T'''(z)] = 0. \tag{18}$$

Where $T^{(iv)}(z)$ and $QT(z)T'''(z)$ are linear and nonlinear terms respectively. If $T(z) = \sum_{m=0}^{\infty} p^m T_m(z)$ be the series solution of (8) then by comparing the coefficients of p^i, $0 \le i \le 5$ and using boundary conditions in (9), the following different order problems are obtained; Zeroth order;

$$T_0^{(iv)}(z) = 0,$$
$$T_0(0) = 0, \quad T_0''(0) = 0, \quad T_0(L) = u/2, \quad T_0'(L) = 0,$$

First order;

$$T_1^{(iv)}(z) = S^2 T_0''(z) - QT_0(z)T_0'''(z)$$
$$T_1(0) = 0, \quad T_1''(0) = 0, \quad T_1(L) = 0, \quad T_1'(L) = 0,$$

Second order;

$$T_2^{(iv)}(z) = S^2 T_1''(z) - QT_1(z)T_0'''(z) - QT_0(z)T_1'''(z)$$
$$T_2(0) = 0, \quad T_2''(0) = 0, \quad T_2(L) = 0, \quad T_2'(L) = 0,$$

Third order;

$$T_3^{(iv)}(z) = S^2 T_2''(z) - QT_2(z)T_0'''(z) - QT_1(z)T_1'''(z) - QT_0(z)T_2'''(z)$$
$$T_3(0) = 0, \quad T_3''(0) = 0, \quad T_3(L) = 0, \quad T_3'(L) = 0,$$

Fourth order;

$$T_4^{(iv)}(z) = S^2 T_3''(z) - QT_3(z)T_0'''(z) - QT_2(z)T_1'''(z) - QT_1(z)T_2'''(z) - QT_0(z)T_3'''(z)$$
$$T_4(0) = 0, \quad T_4''(0) = 0, \quad T_4(L) = 0, \quad T_4'(L) = 0,$$

Fifth order;

$$\begin{aligned} T_5^{(iv)}(z) = {} & S^2 T_4''(z) - QT_4(z)T_0'''(z) - QT_3(z)T_1'''(z) - QT_2(z)T_2'''(z) - QT_1(z)T_3'''(z) \\ & - QT_0(z)T_4'''(z) \end{aligned}$$
$$T_5(0) = 0, \quad T_5''(0) = 0, \quad T_5(L) = 0, \quad T_5'(L) = 0.$$

The command DSolve is used in Mathematica to solve all the above problems. Values of $L,\ u,\ S$ and Q are taken fixed to get the following fifth order solution;

$$\begin{aligned} T(z) &= \sum_{k=0}^{5} T_k(z), \\ &= 1.50813z - 0.518632z^3 + 0.0131766z^5 - 0.00302696z^7 + 0.000424115z^9 \\ &- 0.0000810122z^{11} + 0.0000136556z^{13} - 2.3 \times 10^{-6}z^{15} + 3.35 \times 10^{-7}z^{17} \\ &- 3.75 \times 10^{-8}z^{19} + 2.70 \times 10^{-9}z^{21} - 9.88 \times 10^{-11}z^{23}. \end{aligned}$$

Residuals for different values of permeability					
z	$k = 1$	$k = 5$	$k = 10$	$k = 15$	$k = 20$
0.0	0	0	0	0	0
0.1	7.74×10^{-7}	3.00×10^{-8}	8.00×10^{-8}	9.66×10^{-8}	1.05×10^{-7}
0.2	1.40×10^{-6}	4.50×10^{-8}	1.60×10^{-7}	2.01×10^{-7}	2.22×10^{-7}
0.3	1.55×10^{-6}	9.78×10^{-9}	2.00×10^{-7}	2.80×10^{-7}	3.20×10^{-7}
0.4	5.00×10^{-7}	2.00×10^{-7}	5.00×10^{-8}	1.62×10^{-7}	2.25×10^{-7}
0.5	3.31×10^{-6}	7.00×10^{-7}	6.30×10^{-7}	5.50×10^{-7}	5.00×10^{-7}
0.6	1.30×10^{-5}	1.54×10^{-6}	2.16×10^{-6}	2.30×10^{-6}	2.34×10^{-6}
0.7	3.21×10^{-5}	2.01×10^{-6}	4.00×10^{-6}	4.53×10^{-6}	5.00×10^{-6}
0.8	7.00×10^{-5}	4.00×10^{-7}	2.50×10^{-6}	3.70×10^{-6}	4.30×10^{-6}
0.9	1.30×10^{-4}	1.02×10^{-5}	9.41×10^{-6}	8.60×10^{-6}	8.10×10^{-6}
1.	2.23×10^{-4}	3.22×10^{-4}	4.13×10^{-5}	4.34×10^{-5}	4.43×10^{-5}

TABLE 1. Absolute Residuals for increasing k keeping $B_0,\ h,\ \mu,\ \sigma$ and u fixed

Residuals for different values of viscosity					
z	$\mu = 5$	$\mu = 10$	$\mu = 20$	$\mu = 30$	$\mu = 40$
0.0	0	0	0	0	0
0.1	2.22×10^{-7}	2.00×10^{-7}	1.61×10^{-7}	1.52×10^{-7}	1.50×10^{-7}
0.2	5.00×10^{-7}	3.70×10^{-7}	3.10×10^{-7}	3.00×10^{-7}	3.00×10^{-7}
0.3	8.00×10^{-7}	5.40×10^{-7}	4.20×10^{-7}	4.00×10^{-7}	3.60×10^{-7}
0.4	1.10×10^{-6}	6.70×10^{-7}	5.00×10^{-7}	4.10×10^{-7}	4.00×10^{-7}
0.5	1.40×10^{-6}	7.23×10^{-7}	4.54×10^{-7}	4.00×10^{-7}	3.40×10^{-7}
0.6	1.60×10^{-6}	6.70×10^{-7}	3.53×10^{-7}	3.00×10^{-7}	2.32×10^{-7}
0.7	1.60×10^{-6}	5.00×10^{-7}	2.00×10^{-7}	1.01×10^{-7}	7.14×10^{-7}
0.8	1.20×10^{-6}	1.45×10^{-7}	6.50×10^{-8}	1.01×10^{-7}	1.14×10^{-8}
0.9	4.20×10^{-7}	3.00×10^{-7}	3.20×10^{-7}	3.00×10^{-7}	3.00×10^{-7}
1.	6.64×10^{-7}	7.30×10^{-7}	5.30×10^{-7}	4.50×10^{-7}	4.11×10^{-7}

TABLE 2. Absolute Residuals for increasing μ keeping $B_0,\ h,\ k,\ \sigma$ and u fixed

5. **Numerical Results and Discussion**

Solution obtained using Homotopy Perturbation Method consists of $L,\ Q$, S and u. Table 1 and 2 are constructed to discuss residuals of the problem by considering different values of permeability k and viscosity μ respectively. Residuals for different values of viscosity and permeability in increasing order is considered in Table 3 while in Table 4 the Residuals for various values of viscosity, permeability and imposed magnetic field are shown. All tables show

Residuals for different values of permeability and viscosity					
z	$\mu = 2$ $k = 5$	$\mu = 5$ $k = 7$	$\mu = 10$ $k = 11$	$\mu = 15$ $k = 20$	$\mu = 20$ $k = 30$
0.0	0	0	0	0	0
0.1	2.50×10^{-10}	1.51×10^{-11}	6.50×10^{-13}	2.63×10^{-14}	2.62×10^{-15}
0.2	5.42×10^{-10}	3.21×10^{-11}	1.24×10^{-12}	5.50×10^{-14}	6.00×10^{-15}
0.3	9.00×10^{-10}	5.14×10^{-11}	1.60×10^{-12}	8.44×10^{-14}	9.65×10^{-15}
0.4	1.15×10^{-9}	7.00×10^{-11}	1.30×10^{-12}	1.05×10^{-13}	1.41×10^{-14}
0.5	7.00×10^{-10}	7.00×10^{-11}	8.05×10^{-13}	8.25×10^{-14}	1.63×10^{-14}
0.6	2.60×10^{-9}	1.01×10^{-11}	7.00×10^{-12}	7.34×10^{-14}	7.23×10^{-15}
0.7	1.22×10^{-8}	3.00×10^{-10}	2.02×10^{-11}	6.00×10^{-13}	3.83×10^{-14}
0.8	3.00×10^{-8}	1.04×10^{-9}	5.00×10^{-11}	2.00×10^{-12}	1.76×10^{-13}
0.9	4.30×10^{-8}	3.00×10^{-9}	9.64×10^{-11}	4.42×10^{-12}	5.03×10^{-13}
1.	5.15×10^{-9}	6.00×10^{-9}	2.00×10^{-10}	9.24×10^{-12}	1.16×10^{-12}

TABLE 3. Absolute Residuals for increasing μ, k keeping B_0, h, σ and u fixed

Residuals for various values of μ, k and B_0					
z	$\mu = 5$ $k = 2$ $B_0 = 1$	$\mu = 10$ $k = 5$ $B_0 = 1.23$	$\mu = 15$ $k = 10$ $B_0 = 1.41$	$\mu = 20$ $k = 15$ $B_0 = 1.58$	$\mu = 30$ $k = 25$ $B_0 = 2$
0.0	0	0	0	0	0
0.1	3.00×10^{-9}	4.40×10^{-11}	4.00×10^{-12}	1.62×10^{-12}	1.65×10^{-12}
0.2	8.00×10^{-9}	1.17×10^{-10}	1.02×10^{-11}	4.03×10^{-12}	3.66×10^{-12}
0.3	1.60×10^{-8}	2.46×10^{-10}	2.15×10^{-11}	8.00×10^{-12}	6.26×10^{-12}
0.4	3.00×10^{-8}	4.56×10^{-10}	4.00×10^{-11}	1.40×10^{-11}	9.46×10^{-12}
0.5	5.00×10^{-8}	7.55×10^{-10}	6.63×10^{-11}	2.20×10^{-11}	1.30×10^{-11}
0.6	7.20×10^{-8}	1.12×10^{-9}	9.88×10^{-11}	3.10×10^{-11}	1.57×10^{-11}
0.7	9.54×10^{-8}	1.50×10^{-9}	1.32×10^{-10}	4.00×10^{-11}	1.66×10^{-11}
0.8	1.12×10^{-7}	1.75×10^{-9}	1.54×10^{-10}	4.21×10^{-11}	1.42×10^{-11}
0.9	1.12×10^{-7}	1.75×10^{-9}	1.54×10^{-10}	4.00×10^{-11}	8.00×10^{-12}
1.	9.00×10^{-8}	1.40×10^{-9}	1.23×10^{-10}	2.41×10^{-11}	2.25×10^{-12}

TABLE 4. Absolute Residuals for different increasing values of μ, k and B_0

the efficiency of HPM. It is also investigated that the convergence, using HPM, is not effected by adjusting the convergence region. This method is simple in use and a few iterations are enough to get the required goal.
To discuss the effect of different parameters on the flow four figures are constructed. Fig. 2 presents the velocity profile of fluid for different values of L.

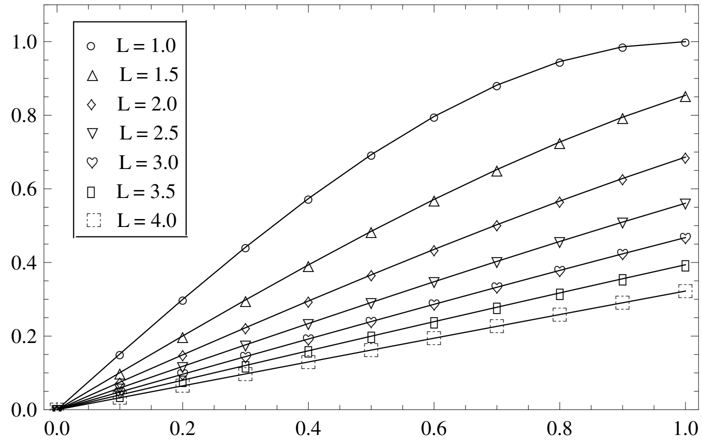

FIGURE 2. Velocity profiles for various values of L

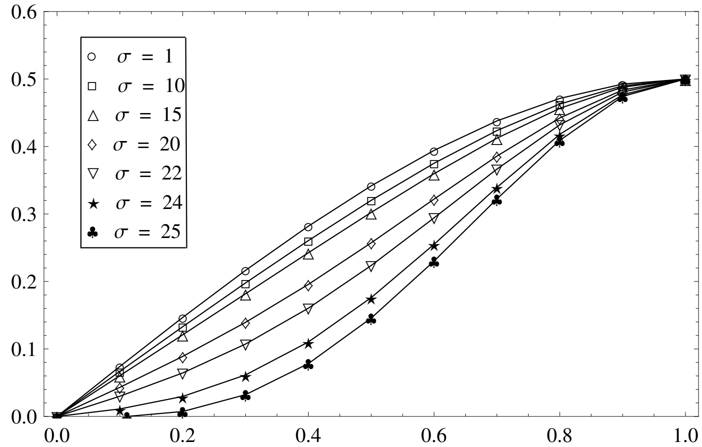

FIGURE 3. Velocity profiles for different values of σ

The velocity of fluid is argued in Fig. 3 for different values of σ while Fig. 4 and Fig. 5 are constructed for various values of v and B_0 respectively.

6. **Conclusion**

Major findings of the current analysis are as follows:

(1) It is observed From Fig. 2 that increase in distance between the plates decreases the velocity of fluid.
(2) It can be seen from Fig. 4 that velocity of plates and that of fluid are directly proportional to each other.
(3) From Fig. 3 and Fig. 5 it is concluded that Electro conductivity σ and magnetic field B_0 have the same effect on the velocity profile of fluid i.e. fluid velocity increases with the increase in σ and B_0.

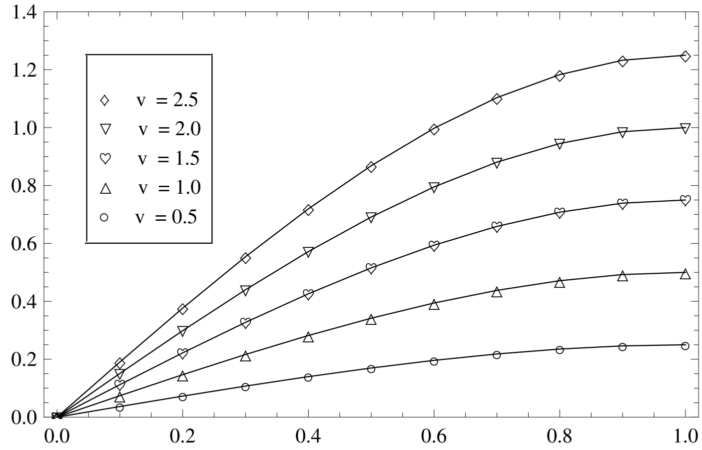

FIGURE 4. Velocity profiles obtained by varying the velocity between plates

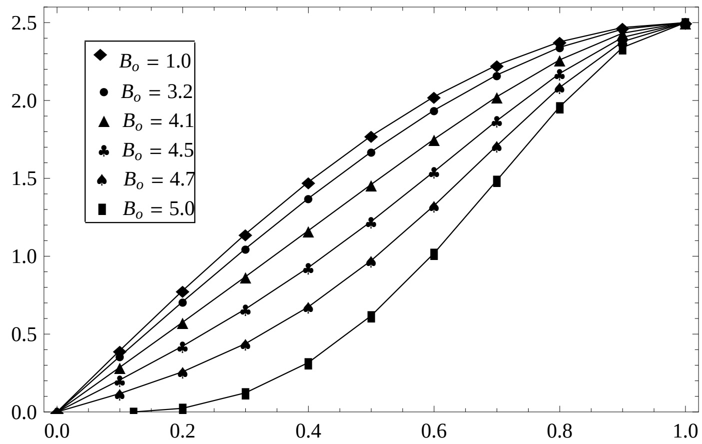

FIGURE 5. Velocity profiles for different values of imposed magnetic field

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