OPTIMAL METHOD FOR CONTROLLED SWITCHING CIRCUITS

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Controlul transferului de putere cu ajutorul circuitelor de comutaţie este o soluţie economică relativ la puterea disipată, dar generează anumite probleme referitoare la calitatea energiei. Filtrarea armonicelor, minimizarea puterii reactive şi amortizarea regimurilor tranzitorii trebuie rezolvate prin strategia de comandă.

Lucrarea prezintă o metodă numerică, verificată prin rezultatele simulării, pentru un exemplu particular al variaţiei semnalului de referinţă. Totuşi, în viitor se pot realiza dezvoltări corespunzătoare unor cazuri mai generale.

The power control, using switching circuits is a power saving solution, but generates some problems concerning the power quality. Harmonic filtering, reactive power minimization and damping transient regimes must be solved by the control strategy.

The paper gives a numerical solution, verified by simulation results, for a particular example of reference signal variation. However, general case developments are possible in the future.

Keywords: optimal control, switching surface, PWM circuits

1. Introduction

In the high current technique the necessity to control the power delivered to a consumer occurs frequently. The switching regime is a good power saving solution. A low-pas filter is necessary, in order to suppress the high order harmonics. When a feedback technique, based on a closed loop is used, an additional delay, due to the reactive filter, may create instability problems. In the case of a single LC filtering cell the stability is on the edge. Even if the filter consists of this structure, only a minor additional delay, caused by the control circuit, will lead to instability.

Thus, the improvement of the control strategy is necessary, in order to eliminate this problem. The main idea is that the control function should, on one hand to prevent instability and on the other assure an acceptable dynamic behavior. In this case, for reasons of respecting the causality principle, the control

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method should use information both on the transfer function of the circuits inside the feedback loop and on the variation mode of reference input signal.

2. The Mathematical background

A rigorous solution corresponding to a class of control problems and easy to implement with a simple electronic schema is based on the switching surface method, described in [1] for a theoretical context. In general, the validity of this method may be demonstrated though the following reasoning.
Consider an n-order differential equation that models the operation of a circuit controlled by the input signal v=v(t) and described by parameter x=x(t), that correspond to a state function.

The input signal is time dependent and may have only the values +M and –M. The evolution of the value of x(t) is represented by a trace (a curve) in phase space, having, as coordinates, the parameter x(t) and its derivatives until order n-1, as time dependent functions: $x'(t), x''(t), \ldots, x^{(n-1)}(t)$. In each moment the state of the circuit is defined by the position on the trace through coordinates in phase space. Consider the function x(t) as solution of an n-order differential equation:

$$a_0 \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_{n-1} \frac{d^n x}{dt^n} + a_n x = v \tag{1}$$

The problem consists of finding the control function v(t) so that signal x(t) comes closer to the origin of the phase space as fast as possible (for the other cases the reference system must be moved). When the generic point follows the trace, the control function changes when the trace intersects a virtual (switching) S. If its position is known, the evolution of the control function may be determined. For this, suppose that the generic point, instead of evolving from a certain initial point towards the origin, will evolve backwards, relative to the flow of time from the origin towards the initial point. Consider a new independent variable denoted $\tau = T - t$, where T is final time of the transient process. A new differential equation is obtained, containing variable τ:

$$a_0 (-1)^n \frac{d^n x}{d\tau^n} + a_1 (-1)^{n-1} \frac{d^{n-1} x}{d\tau^{n-1}} + \ldots + a_{n-1} (-1)^n \frac{dx}{d\tau} + a_n x = v \tag{2}$$

The first trace segment, denoted $x_1(t)$, up to moment τ_1, is described by the above equation with the following initial conditions:

$$(x)_{\tau=0} = \left(\frac{dx}{d\tau} \right)_{\tau=0} = \ldots\ldots\ldots = \left(\frac{d^{n-1} x}{d\tau^{n-1}} \right)_{\tau=0} = 0 \tag{3}$$

The continuation of the evolution backwards to the initial point, after changing the value of the control function, is done by restarting the evolution process with the same differential equation, but with new initial conditions that depend on moment τ_1. Consider that the second trace segment of the solution x(t), that implicitly depends on moment τ_1, may be represented as $x_2(t)=x(\tau,\tau_1)$. If the reasoning is continued in the same manner the solution for other trace segments are obtained: $x_3=$,…, $x_4=$…, each depending on the moment of time in the past, when the control function has changed. Although, in the expression above only the dependence on moments τ, τ_1,…, τ_{n-1} was shown, it is obvious that they also depend on phase space coordinates: $x(\tau)$, $x'(\tau)$, $x''(\tau)$,..., a system of n equations can be assembled with the variables: τ,τ_1,…. Eliminating the variables, the equation of the switching hypersurface can be obtained:

$$S(x(\tau), x(\tau)', x(\tau)'', \ldots , x(\tau)^{(n-1)}) = 0 \tag{4}$$

3. The switching curve method

The most simple close loop structure that uses the control of a switching circuit powered by a bipolar supply is presented, in principle (Fig.1). The Laplace transform of the transfer function of a LC filtering circuit has the expression:

$$H(s) = \frac{1}{1 + s^2 LC} \tag{5}$$

Observe that if the product $s^2LC >> 1$ then the expression can be approximated with the transfer function of an integrator circuit with two cells (Fig.2). If the output signal is considered to be $x_1(t)$ then the input signal will be d^2x_1/dt^2. The scheme in Fig.1 may be represented in open loop, as in Fig.2, with output parameter $x_1(t)$ depending on the control function $u_1(t,x^*,x_1)$, by means of the equation:

$$\frac{d^2 x_1}{dt^2} = u_1(t, x^*, x_1) \tag{6}$$

The function $u_1(t,x^*,x_1)$ has the purpose of approximate signal $x^*(t)$ with signal $x_1(t)$. The error signal at the comparator output is: $x(t) = x^*(t) - x_1(t)$.

The function $u_1(t,x^*,x_1)$ may be written as a new function $u(t,x)$, depending on the error signal $x(t)$. The following differential equation is obtained:

$$\frac{d^2 x}{dt^2} = u(t, x) \tag{7}$$

As shown in Fig.1 the control function $u(t,x)$ must correspond to the action of switching circuit K. This can connect either the positive power supply +V or the negative power supply –V. It results that the control function $u(t,x)$ has the form:

$$u(t,x) = \sigma(t,x)M \tag{8}$$

The σ function may only take positive or negative unitary values (-1,+1), depending on the state of the switching circuit, and M represents the absolute value of the supply voltage. The differential equation that models the circuit may be written as:

$$\frac{d^2x}{dt^2} = \sigma(t,x)M \tag{9}$$

It is desired that the representative differential equation contains a single unknown function, that include information both on the output signal $x_1(t)$ and on the reference signal $x^*(t)$. The most appropriate choice is exactly the error signal x(t). To this end, the second derivative of the error signal is computed, considering that the reference signal has a second order polynomial form, $x^*(t) = C_0 + C_1t + C_2t^2$:

$$\frac{d^2x}{dt^2} = \frac{d^2x^*}{dt^2} - \frac{d^2x_1}{dt^2} = 2C_2 - \sigma(t,x)V \tag{10}$$

Observe that for a second order reference signal, the control's synthesis only depends of the maximal degree term coefficient, denoted above with C_2. Let use the notation $C=C_2$. Thus, the error signal satisfies the differential equation:

$$\frac{d^2x}{dt^2} = 2C - \sigma(t,x)V \tag{11}$$

Using the error signal represents a major advantage because, in the case of a converging approximation process, function x(t) tends to zero (the origin of the referential). Thus, the problem of approximating the reference signal is reduced to that of reaching the origin of the referential. The general theory presented above may be applied, the phase space having only two dimensions, resulting a phase plane and a switching curve. Solving the problem consists in finding the switching function $\sigma(t,x)$, so the output signal $x_1(t)$ gets closer, as fast as possible to reference signal $x^*(t)$ imposed to the input, satisfying, at the same time, the stability criteria. The right term of the differential equation is considered constant until the moment when the trace intersects the switching curve. Thus, for $\sigma(t,x)$=const, the equation giving x(t) has the right term $\mu_0 = 2C-\sigma(t,x)V$:

$$\frac{d^2x}{dt^2} = \mu_0 \tag{12}$$

As in the general case, the reverse evolution is considered, using the new variable τ=T-t. So, in this case, the second derivative of the function x(t) remains the same:

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = -\frac{d}{d\tau}\left(-\frac{dx}{d\tau}\right) = \frac{d^2x}{d\tau^2} = \mu_0 \tag{13}$$

The solution of the differential equation can be found by simple integration:

$$y = \frac{dx}{dt} = -\frac{dx}{d\tau} = -\int \frac{d^2x}{d\tau^2}d\tau = -\mu_1 - \mu_0\tau \tag{14}$$

$$x = -\int y d\tau = \mu_2 + \mu_2\tau + \mu_0\frac{\tau^2}{2} \tag{15}$$

where μ_0, μ_1, μ_2 are integration constants.

Because for τ=0 the variables y and x are null, it results μ_1, μ_2 =0 and:

$$x = \mu_0 \frac{\tau^2}{2}, \quad y = -\mu_0 \tau \tag{16}$$

If variable τ is eliminated, the defining equation for the switching curve is:

$$x = \frac{y^2}{2\mu_0} = \frac{y^2}{2(2C - \sigma V)} \tag{17}$$

The switching curve consists of two braches (for σ(t,x) equal +1 or -1), denoted SP(x), SN(x), having as a common point the origin of the phase plane. It can be demonstrated that the switching curve method offers an optimal solution, both from the point of view of stability and from the point of view of the reference signal approximation speed. In Fig.3 (from [1]) it can be observed intuitively that two branches of the switching curve, SP and SN, actually represent the traces or the possible solution of the problem, corresponding to initial conditions that determine crossing the origin, denoted by Q. The other possible traces will go around the origin, at a greater or smaller distance. In Fig.3 these are represented, partially, until the intersection with the switching curve, by the curves mark with arrows. In order to reach the origin, as fast as possible, and therefore to obtain a minimal error signal, the generic point must follow one of the traces that cross the origin. So, as seen in Fig.3, the traces of the generic point may be partitioned into families, according to the sign of the switching function. In order to reach the origin, the sign of σ(t,x) must be switched, so that the generic point follows traces from different families.

Presume that, for certain initial conditions and for a certain value of the switching function, the generic point moves on a certain trace. It results the approximation process is optimal if the switching function change its sign exactly in the moment when the trace crosses the switching curve. However, if due to various errors (specific to the model or to the numerical calculus) the change of the sign is not perfectly synchronized, the new trace will evolve in the proximity of the switching curve, until it will intersect the other branch of the curve. The process of getting closer to the origin continues step by step. A similar situation may occur in a dynamic regime, when the reference signal, and therefore the phase plane configuration, changes. The stability of the method in inherent, thanks to using directly the phase plane in which any oscillatory process is observable.

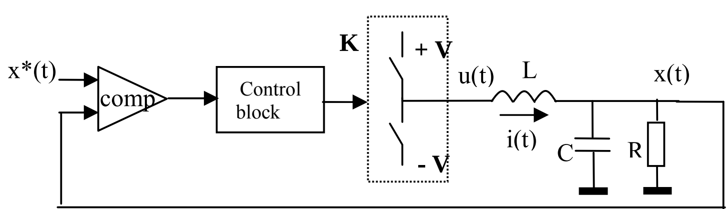

Fig.1.Principial diagram for an controlled switching-mode equipment.

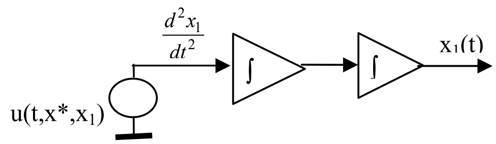

Fig.2.The open loop equivalent diagram.

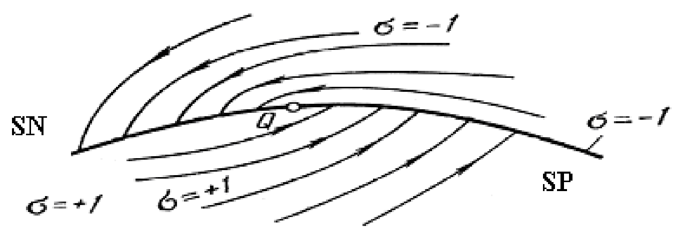

Fig.3. An hypothetical trace family profile, for the two σ values (from [1]).

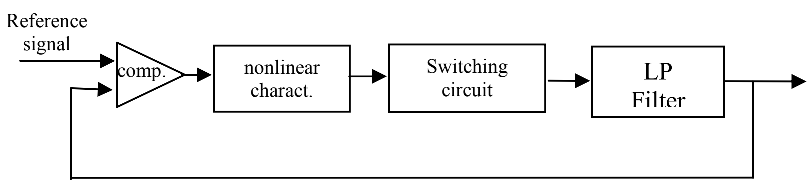

Fig.4. Functional diagram of the control strategy.

4. Implementation and simulation results

Although the mathematical formalism seems complex, implementing the switching curve method may be relatively easy in a digital as in analog technique. The functional diagram (Fig.4) must include besides comparator, switching and filtering blocs, a special nonlinear bloc, containing information on the switching curve. In order to use the presented method, the specific operating characteristics for the given circuit must be known.

The purpose of the numerical simulation was, on one hand, the validation of the reasoning and, on the other, the study of the control process's behavior and stability. Observe that the algorithm may be simplified if a correspondence between the variation domain of function $x(t)$ and the sign of function $\sigma(t,x)$ is found. In this context, due to relation (9), if the condition:

$$|C| < \frac{M}{2} \tag{18}$$

is respected the following dependency results:

$$sign\ y = sign\ \sigma = \sigma \tag{19}$$

The formulated problem may be interpreted either as a control process – having as optimality criterion following the reference signal – or as a process of approximating the value of the reference signal with the output signal.
The study includes two aspects. On one hand, it considers the evolution of the trace in the phase plane, corresponding to error signal $x(t)$ and its derivative $y(t)$. On the other, it considers the evolution of output parameter $x_1(t)$ and reference signal $x^*(t)$, in time domain. Their variation is described by the differential equations derived above:

$$\frac{d^2 x_1}{dt^2} = \sigma(t,x)V \qquad \frac{d^2 x}{dt^2} = 2C - \sigma(t,x)V \tag{20}$$

A general 2-order differential equation is equivalent with a 1-order system:

$$\frac{d^2 v}{dt^2} = f(t,v) \ \Rightarrow \qquad \frac{dw}{dt} = f(t,v) \quad ; \qquad \frac{dv}{dt} = w(t) \tag{21}$$

where v(t) and w(t) denote its solution. This may be solved recursively by means of the Euler method.
For solving the equations above, a step-by-step procedure has been developed. Implemented in Mathcad language, it is based on a first order discrete approximation (Euler method) and a recursive representation:

$$Z^{<k>} := F(Z^{<k-1>}) \tag{22}$$

Including of all information necessary to the recursive solution into a single vector is necessary because, at each step, the calculus must be done simultaneously.
It is important to observe that solving the formulated problems contains two aspects: on one hand, the estimation of the error and output signals and, on the other, determination of the control function by means of switching function $\sigma(t,x)$.
The recursive calculus uses the composed vector Z, calculated at step k as a function of the values at step $k-1$. The vector contains three type of information, necessary for: the recursive calculus of error signal $x(t)$, the estimation of control function $\sigma(t,x)$ and the recursive calculus of output signal $x_1(t)$.
Vector Z contains the five components denoted: Z_0 – the instantaneous value of the error signal derivative; Z_1 – the instantaneous value of the error signal; Z_2 – the value of switching function $\sigma(t,x)$; Z_3 - the instantaneous value of the output signal derivative; Z_4 - the value of the output signal. It results, that the function

that describes the recursive equation, imported from Mathcad program, may be written:

$$F(Z) := \begin{bmatrix} Z_0 + h \cdot Z_1 \\ Z_1 + h \cdot (2 \cdot C - M \cdot Z_2) \\ \mathrm{sign}(Z_1 - S(Z)) \\ Z_3 + h \cdot M \cdot Z_2 \\ Z_4 + h \cdot Z_3 \end{bmatrix} \tag{23}$$

where the branch function *S(z)* is implemented by means of the conditional expression:

$$S(Z) := \mathrm{if}\left[(Z)_0 \geq 0, SP\left[(Z)_0\right], SN\left[(Z)_0\right]\right] \tag{24}$$

and represents, in the phase plane, the ordinate values of the switching curve. According to (16) this is composed of two branches, corresponding to the two variation domains of *x(t)*, for the two possible values of the switching function $\sigma(t,x)$. In this case, these domains are coincident with the abscissa semi-axes, defined as:

$$SP(x) := -\sqrt{2 \cdot (2 \cdot C + M) \cdot x} \qquad SN(x) := \sqrt{2 \cdot (2 \cdot C - M) \cdot x} \tag{25}$$

Finding the switching function σ(t,x) is done by calculating the sign of expression *S(Z)*, by means of the sign(.) function. Thus, according the reasoning above and to equation (*), if the trace in the phase plane is above a branch of the switching curve the value of switching function $\sigma(t,x)$ is +1, otherwise -1.

For the numerical simulation, it is considered that: $C_0 = C_1 = C_2 = 1$ and M =4.

Figure 5, obtained by numeric calculus in Mathcad, presents the evolution of the trace in phase plane, corresponding to error signal *x(t)*, starting at the initial point (x_0, y_0) and going to towards the origin. The changes in the sign of the control function take place at the intersection with the branches of the switching curve (these are asymmetrical, due to the shape of the trace). The dotted curve (corresponding to $S(z^{<k>})$) represents the switching curve. The trace – with coordinates $z^{<k>}{}_0$ and $z^{<k>}{}_1$ – represents the evolution of error signal *x(t)*.

It is important to point out that this control process is, from the point of view of signal theory, an approximation process, where the output parameter $x_1(t)$ is an approximation for the reference signal $x^*(t)$.

Figure 6 presents the process of approximating reference signal (denoted here by *E(t)*) with the output signal with the value $z^{<k>}{}_4$, combined with the evolution of the switching function, with scaled representation by $K(z^{<k>}{}_2)$.

Notice that the starting point for the approximating signal is far from the starting point for the approximating signal and the approximating process contains small oscillations, corresponding to the variation of the sign of switching function.
Observe that as the computing step is smaller, the switching function changes sign more frequently in the vicinity of the origin. Therefore if the value of the computing step is limited, this switching frequency will be limited, which allows to keep stability and approximation error within the desired limits.
Figure 7 shows that, throughout the time evolution, the relative error decreases rapidly at the beginning and is afterwards limited to finite values due to the effects of rounding the numerical results.

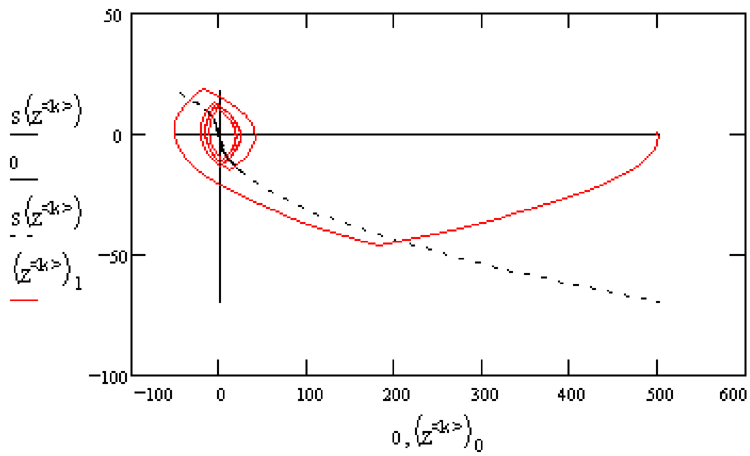

Fig.5. The switching curve and the error function trace represented in phases plane.

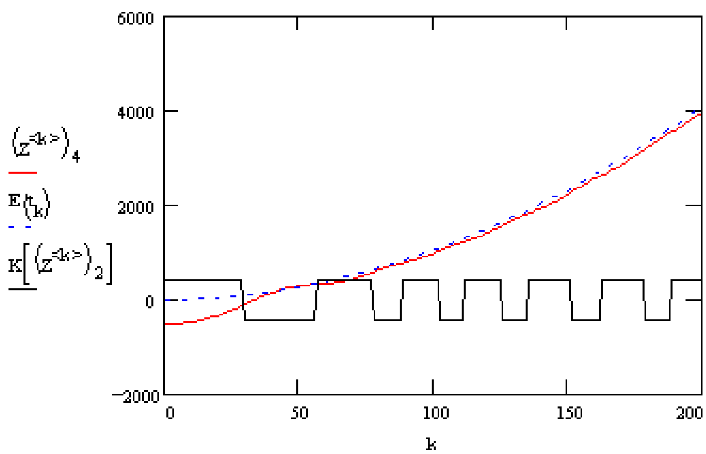

Fig.6. The reference signal, the control function and the obtained solution, in time domain.

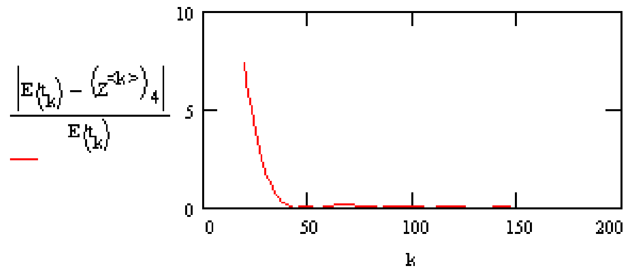

Fig.7. The relative error of the reference signal approximation.

5. Conclusions

The content of the paper creates a conceptual correspondence between abstract mathematical theory and technical problems derived from circuit theory. Some similarities may be established between results belonging to apparently different research domains, like optimal control, approximation methods and stability control. This allows solving, in a rigorous manner, some problems of technical interest:

- optimal approximation process, relative to the approximation speed;
- unconditioned control stability in a feedback loop with delay elements;
- switching frequency limitation of the control function (and active devices).

The reference [1] describes a pure mathematical formalism, corresponding to an abstract representation based on differential equations. The authors have elaborated an original representation and recursive algorithm, simulating the control strategy and its behavior, in a technical context, from electrical engineering domain. Although the numerical simulation was done for the specific case of a second order polynomial, more general cases may be developed, when an analytical solution cannot be found, but a numerical approximation is available.

REFERENCES

[1] *A. Feldbaum* - Principes theoretiques des systemes asservis optimaux, Ed.Mir, 1973.
[2] *V. Zoubov* - Theorie de la comande, Ed.Mir, 1978
[3] *V.I. Utkin* - Sliding modes and their applications in variable structure systems, Mir Publishers, Moscow, 1978
[4] *J. Miret, L. Cruz* - A Simple Sliding Mode Control of an Active Power Filter, Proceedings of IEEE Power Electronics Specialists Conference, pp: 1052-1056
[5] *M. Nayeripour, T. Niknam* - Nonlinear Sliding Mode Control Design for Shunt Active Power Filter with the Minimization of Load Current, World Applied Sciences Journal 4 (1):124-132, 2008
[6] *D. Olaru, D. Floricău* - Qualitative aspects on closed loop power switching system control - EMES'09 International Conference, Oradea 2009.