PUMPING STATIONS SCHEDULING FOR A WATER SUPPLY SYSTEM WITH MULTIPLE TANKS

Sanda-Carmen GEORGESCU[1], Radu POPA[2], Andrei-Mugur GEORGESCU[3]

Fie o rețea inelară de alimentare cu apă cu cerințe de debit variabile, care include două stații de pompare (cu câte 3 pompe funcționând în paralel la turație constantă), un bazin de aspirație cu nivel constant şi două rezervoare cu nivel variabil al apei. Utilizând algoritmul de optimizare bazat pe înmulțirea albinelor melifere (HBMOA), cu penalizări pentru nesatisfacerea restricțiilor hidraulice (legate de nivelul apei în rezervoare), am obținut un program de funcționare zilnică a pompelor, care asigură o valoare minimă a energiei consumate pentru pompare. Soluția suboptimală obținută cu HBMOA a fost ulterior verificată în EPANET, utilizând condiții de control simple (pornirea/oprirea pompelor la anumite ore). Pe baza reglării discrete a funcționării pompelor în stațiile de pompare, am obținut în EPANET o altă soluție, utilizând condiții de control bazate pe reguli de pornire/ oprire a pompelor în funcție de nivelul atins în rezervoare. Ambele soluții conduc la valori minime ale energiei zilnice consumate (eroare relativă de 0.27% între valori).

Let's consider a looped water supply network with variable demand, which includes two pumping stations (each with 3 pumps working in parallel at constant speed), a suction reservoir with constant level and two tanks with variable water level. Using the Honey Bees Mating Optimization Algorithm (HBMOA), with penalty functions for hydraulic constraints violation (related to water levels' in tanks), we obtained a daily scheduling of pumps operation, ensuring a minimum value of the pumping energy consumption. Further, the HBMOA suboptimal solution has been verified in EPANET, using simple controls (pumps starting/stopping at specified hours). According to the pumping stations scheduling for discrete pumped flow rate, we obtained another solution in EPANET, using rule-based controls for the pumps starting/stopping upon the water level in tanks. Both solutions yield minimum values of the daily energy consumption (within a relative error of 0.27%).

Keywords: pumping station scheduling, HBMOA, EPANET

1. Introduction

The proper scheduling of pumps operations in water supply systems yields to energy cost-savings. The pumps schedule is the set of many combinations of

[1] Associate Prof., Hydraulics, Hydraulic Machinery and Environmental Engineering (HHMEE) Department, University POLITEHNICA of Bucharest, Romania, e-mail: carmen@hydrop.pub.ro
[2] Prof., HHMEE Department, University POLITEHNICA of Bucharest, Romania
[3] Associate Prof., Hydraulics and Environmental Protection Department, Technical University of Civil Engineering Bucharest, Romania

pumps operation parameters, variables in time, which must fulfil the system restrictions regarding the: level variation in reservoirs between imposed limits, water demands time pattern, energy cost, reserved power cost [1], pumps maintenance cost etc. In this paper, we consider a looped water supply network with variable demand, which includes two pumping stations (PS), each with 3 identical centrifugal pumps working in parallel at constant speed, one common suction reservoir with constant level and two tanks with variable water level. Within our study, we consider only the above first 3 system restrictions.

Various stochastic methods for combinatorial optimization can be applied to solve optimal pump-scheduling problems, by minimizing (or maximizing) the objective function, while satisfying system constraints, with randomness within the search process [2]. Among them, we selected here the *Honey Bees Mating Optimization Algorithm* (HBMOA), a swarm-based approach, where the search procedure is inspired by the process of mating in a real honey bee colony [3].

In this paper, a solution (*honey bee*) has a number of unknowns (*genes*) equal to the total number of working pumps, at each time moment, defined hourly over one day period. In the classical form of HBMOA [3], all solutions generated and improved during the current iteration (excepting the best solution – the *queen bee*) are completely destroyed at the end of the iteration, and a new swarm of solutions (*drones*) is randomly generated for the next iteration. The modified HBMOA formulation applied in this paper, denoted HBMOA-M2 in Popa and Georgescu [4], uses the solutions improved during the current iteration, ranked after the queen as fitness (performance), and inserts them within the list of drones for the next iteration, thus improving the colony genes in the coming generation. Supplementary, the modified HBMOA improves two classical hypotheses, namely: it uses the *tournament rule* when creating *new brood* (which ensures a greatest chance to available genetic material to produce better *new bees*), and it ensures to new solutions a more intensive performance improvement, in *brood feeding*, where the mutation operator chooses randomly 3 genes from *bee's genome*, and modifies their current values.

In this paper, the above modified HBMOA formulation has been used to find optimal schedule for pumps, in accordance with the water demands time pattern. Being a stochastic approach, the solution found here (corresponding to the minimum value of the pumping energy), is in fact a suboptimal solution, since other better solutions can be found. The HBMOA suboptimal solution has been verified in EPANET, using *simple controls* (pumps starting/stopping at specified hours). By implementing the algorithm that is commonly used for pumping stations scheduling for discrete pumped flow rate [5], we obtained another solution in EPANET, using *rule-based controls* for the pumps starting/stopping upon the water level in tanks. Both solutions yield minimum values of the daily energy consumption.

2. Water supply system description

The configuration of the water supply system considered in this paper is derived from two basic configurations studied by Jeppson [6], namely: a looped hydraulic network supplied from two sources, a tank and a junction [6, page 123], described by a system of head-equations, and a looped hydraulic network supplied by two tanks and one pump fed by a suction reservoir [6, page 93].

The resulting flat network consists of 13 nodes placed at the same level, and 14 main pipelines, labelled with ID numbers as in Figure 1. Geometric data (diameters D and lengths L) of those 14 pipes are summarized in Table 1. Head losses were computed with Darcy-Weissbach formula, where the friction factor λ is defined for fully turbulent flow, with 0.2mm pipe's wall roughness.

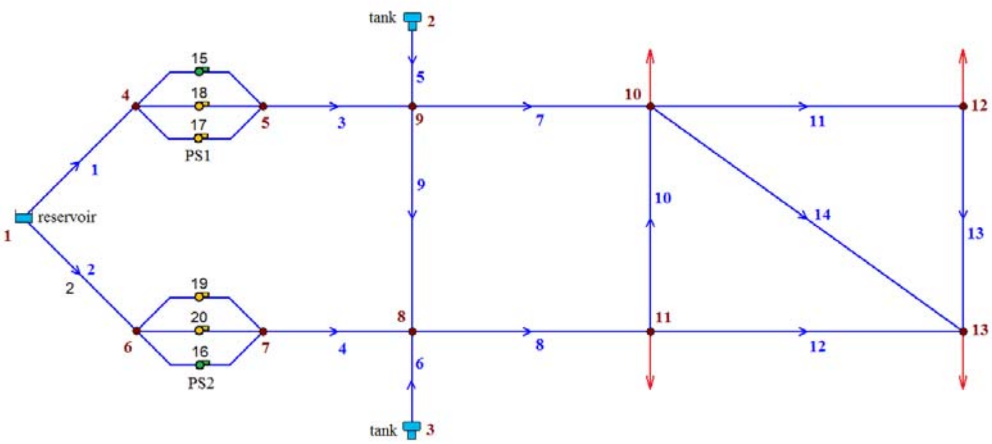


Fig. 1. Studied water supply network: nodes' and pipes' ID labels.

Table 1

Network data: pipe's diameter *D* (in millimetres) and length *L* (in metres) & coefficients of demand pattern *c*(*t*), where *t* is the time moment (in hours)

Pipe	1	2	3	4	5	6	7	8	9	10	11	12	13	14
D	300	300	300	300	300	200	200	250	200	200	200	200	150	150
L	100	100	700	400	300	150	600	600	450	450	600	600	450	1000
t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$c(t)$	.05	.05	.05	.05	.05	.05	0.1	0.5	1	0.8	0.7	0.7	0.6	0.7
t	14	15	16	17	18	19	20	21	22	23	24			
$c(t)$	0.8	0.8	0.7	0.7	0.8	1	1.1	0.7	0.3	0.1	.05			

There are 4 main consumers, placed in nodes $k = 10 \div 13$, which request a variable water demand $Q_{c_k}(t) = c(t) Q_{r_k}$, with respect to the reference values:

$Q_{r_{10}} = Q_{r_{11}} = 14$ l/s, $Q_{r_{12}} = 28$ l/s and $Q_{r_{13}} = 42$ l/s; the demand time pattern is described over a over a 24 hours period by the coefficients $c(t)$, given in Table 1 for each time moment *t*, starting from mid-night. Accordingly, the hydraulic analysis of the resulting network has been performed over one day period, with one hour time step, from mid-night, where $t = 0$, to $t = 24$ h.

The suction reservoir with constant head $H = 65$ m is labelled as node 1. The tanks with variable water level are labelled as node 2 and node 3; at the beginning of computation, where the time clock is 0:00, the initial head of tanks is set to 69.5m; further, at $t > 0$, each tank's head will vary between 69m and 70.5m.

The pumping station PS_1 is equipped with the pumps labelled (as links) by 15, 17 and 18, where the pump 15 is the *basic pump* (with the greatest percent of utilization); PS_2 is equipped with the pumps labelled by 16, 19 and 20, where the pump 16 is the *basic pump*. All computations, using HBMOA or EPANET models, will start at mid-night, with the basic pumps 15 and 16 in operation, while the status of the other pumps will be closed at $t = 0$. All pumps (of centrifugal type) are identical, working in parallel at constant speed. Pump's head curve, usually a 2nd order polynomial $H = H(Q)$ upon the flow rate *Q*, is given as an inverse regression $Q = Q(H)$, of power type:

$$Q = a_1 - a_2 H^{a_3} = 0.03813 - 0.00014 H^{2.7828}, \tag{1}$$

where *Q* values are in m^3/s, for head in metres. Pump's efficiency curve $\eta = \eta(Q)$ is given as a 2nd order polynomial: $\eta = (a_4 - a_5 Q)Q = (55.156 - 1166.25Q)Q$, where $\eta < 1$, for *Q* in m^3/s.

3. Honey Bees Mating Optimization Algorithm (HBMOA) results

Within Honey Bees Mating Optimization, the search algorithm is inspired by the process of mating in a real honey bee colony. The *queen bee*, *drones* (male bees) and *brood* have their own *genome* composed of *genes*. When modelling the mating process, the genome is attached to one solution (to one bee) of the studied optimization problem. One genome is mathematically described by a list of numerical values, where each value is attached to a decision variable (gene) that represents an unknown of the problem. Depending on the values of unknowns from such a list, the performance function of the problem has a greater or smaller value, so the genome of the associated solution (bee) is stronger or weaker. An exhaustive description of HBMOA applied in hydraulic networks can be found in Popa and Georgescu [4]. Particularities of HBMOA applied in pumping station scheduling for water supply are discussed in Georgescu and Popa [2].

For a water supply system with a total number N_{PS} of pumping stations (here $N_{PS} = 2$), and a total number N_R of tanks with variable level (here $N_R = 2$), the proposed objective function F consists of minimizing the daily pumping energy consumption E, while satisfying hydraulic restrictions for the daily variation of the water level in tanks (defined by two penalty functions), as:

$$F = \min\left\{ \Delta t \sum_{t=1}^{T} \sum_{i=1}^{N_{PS}} P_{i,t} + p_1 \sum_{t=1}^{T} \sum_{j=1}^{N_R} \left(\Delta z_{j,t}^{sup} + \Delta z_{j,t}^{inf} \right) + p_2 \sum_{j=1}^{N_R} \Delta z_{j,T}^{f} \right\}, \quad (2)$$

where $P_{i,t}$ is the power (in kW) consumed by the pumping station PS_i (with $i = 1 \div N_{PS}$) over the time step of one hour, elapsed from the time moment $(t - \Delta t)$ to t; p_1 is a penalty coefficient for the level deviation with respect to the maximum value $\Delta z_{j,t}^{sup}$, and minimum value $\Delta z_{j,t}^{inf}$, allowed in the tank j (with $j = 1 \div N_R$) over the same time step elapsed from $(t - \Delta t)$ to t (in this paper, $p_1 = 500$ kWh/m); p_2 is a penalty coefficient for the level deviation $\Delta z_{j,T}^{f}$ at $t = T = 24$, with respect to the final level imposed in the tank j at the end of the computation (here, $p_2 = 1000$ kWh/m). The above limits of level deviations are defined as:

$$\Delta z_{j,t}^{sup} = \begin{cases} 0 & \text{if } z_{j,t} \le z_j^{max} \\ \left(z_{j,t} - z_j^{max} \right) & \text{if } z_{j,t} > z_j^{max} \end{cases},$$
$$\Delta z_{j,t}^{inf} = \begin{cases} 0 & \text{if } z_{j,t} \ge z_j^{min} \\ \left(z_j^{min} - z_{j,t} \right) & \text{if } z_{j,t} < z_j^{min} \end{cases}, \quad (3)$$
$$\Delta z_{j,T}^{f} = \left| z_j^{f} - z_{j,T} \right|,$$

where z_j^{max}, z_j^{min}, and z_j^{f} are the maximum allowed level, the minimum one, and the final imposed level. In this paper, $z_j^{max} = 70.5\,\mathrm{m}$, $z_j^{min} = 69\,\mathrm{m}$, and $z_j^{f} = 69.5\,\mathrm{m}$ (that final level will be not imposed in EPANET models).

As the performance (*bee's fitness*) points to maximisation, we will choose a convenient performance function *f*, defined as: $f = 100 / F$, with F in kWh.

Within the studied problem, a solution (*honey bee*) has a number of unknowns (*genes*) equal to the number $n_{i,t}$ of working pumps, where $i = 1 \div N_{PS}$ indicates the pumping station PS_i, and $t = 0, 1, 2, \ldots, T$ is the time moment, defined with a time step $\Delta t = 1\,\mathrm{h}$; the total number of time steps is $T = 24$. The discrete variable $n_{i,t}$ is upper limited by the number of pumps n_i^* installed in PS_i: $0 \le n_{i,t} \le n_i^*$; here, $n_{i,t} \in \{0; 1; 2; 3\}$, with $i = 1; 2$. As mentioned before, all computations will start with the basic pumps 15 and 16 in operation, meaning: $n_{1,0} = n_{2,0} = 1$. So, any solution (honey bee) has a genome consisting of $N_{PS} \times T$ genes, represented by integers in the range $[0; n_i^*]$. In this problem, the genome can be built either upon the priority of the time step, i.e. $\{n_{1,1}, n_{2,1}, \ldots, n_{N_{PS},1}; n_{1,2}, n_{2,2}, \ldots, n_{N_{PS},2}; \ldots\}$, or upon the priority of the PS, i.e. $\{n_{1,1}, n_{1,2}, \ldots, n_{1,T}; n_{2,1}, n_{2,2}, \ldots, n_{2,T}; \ldots\}$.

Within the water supply system model solved using HBMOA, we assume that in each pumping station *i*, the $n_{i,t}$ number of working pumps are operating in parallel with identical hydraulic parameters, namely with the same amount of the: flow rate $Q_{i,t}$ defined by (1), head $H_{i,t}$ and efficiency $\eta_{i,t}$. Thus, the power consumed by the whole PS_i over the considered time step can be computed as:

$$P_{i,t} = n_{i,t} \frac{9.81\, Q_{i,t} H_{i,t}}{\eta_{i,t}}, \text{ (in kW)}. \tag{4}$$

The hydraulic regime within the network is assumed to be steady over a time step, but the heads of the tanks are iteratively corrected over each time step. The system of equations describing the network operation is expressed as head-equations [6]; i.e. the flow rate in pipe *k* is written as $Q_k = c_k \left(H_{u_k} - H_{d_k}\right)^{0.5}$, where H_{u_k} is the head of pipe's upstream node, H_{d_k} is the head of downstream node, and c_k is the hydraulic conductivity of the pipe: $c_k = \pi D_k^2 \sqrt{g D_k / (8 L_k \lambda_k)}$. The nonlinear system of equations is solved at each time step using the Newton-Raphson method.

At the first iteration, within the initial population of N_{in} solutions/bees (here N_{in} = 80), for each solution we randomly generate a number $N_{PS} \times T$ (here $2 \times 24 = 48$) of integer values in the range $[0; n_i^*]$, with $i = 1 \div N_{PS}$ (here [0; 3]);

then, with those values, we perform the hydraulic analysis of the solution over the T time steps (by solving the nonlinear system of head-equations, then computing the daily consumed power/energy, and finally evaluating the performance function of that solution).

Then, that initial population of bees is ranked decreasingly upon the performance function values, and the best solution (the one with the best performance) is selected as initial *queen bee*. Further, a number N_D of solutions (here N_D = 40), ranked after the queen, forms a *list of drones*, which may mate with the queen during the first *mating-flight*, while the rest of initially generated solutions are ignored. Besides its genome, which is the strongest, the queen is characterised by her speed V, as well as by her spermatheca capacity N_S (that is kept constant during all mating-flights, and equals the maximum number of drones that can mate with the queen during such a flight; here N_S = 30). Queen's speed decays upon time t as: $V(t+1) = \alpha\, V(t)$, down to a minimum value V_{min}; in this paper, $V(0) = 1$, the decay coefficient is $\alpha = 0.97$, and $V_{min} = 0.2$.

The *mating-flight* represents a global iteration, during which the current queen bee Q selects randomly some drones, and by mating, each drone genome is stored in her spermatheca. By crossovering the queen own genome with drones' genomes, a given number N_B of *new bees* appears (here N_B = 30). The new genome creation is made here with a single heuristic crossover operator [4], as: $B_i = Q + \text{round}\left(r\,(Q - D_i)\right)$,where the drone D_i is the solution randomly selected from the spermatheca to generate the new i solution (new bee B_i), and "round" refers to rounding towards the nearest integer. We used the tournament rule when creating new brood, by selecting randomly 3 drone's genomes from the spermatheca, and combining the best of them (the one with best performance) with queen's genome. It ensures a greatest chance to available genetic material to produce better new bees.

Within the phase of improvement of brood's fitness by *worker bees*, workers role is implemented by a single mutation operator, which is applied to a new bee for N_M times (N_M is an imposed number of mutations, equal to the number of worker bees; here N_M = 40), thus simulating the *feeding* with royal jelly, to improve bee's performance. After selecting randomly a new bee (new solution) B_i, 3 of its genes, randomly selected among the $N_{PS} \times T$ genes, are modified to: $v_{new} = \text{round}\,(v_{old} + r_2)$, if $r_1 < 0.5$, and $v_{new} = \text{round}\,(v_{old} - r_2)$, if $r_1 \geq 0.5$, where $r_1, r_2 \in (0;1)$ are random numbers; v_{old} and v_{new} are the old value of the gene, and its new value, altered by mutation. The selection of 3 genes ensures to new solutions a more intensive performance improvement. The modified solution is then used to perform a hydraulic analysis over the T time steps. If the performance of a new solution (modified by mutation) is better than the performance of the current queen, then that new solution will become new queen, replacing the old queen. The above steps are iterated to minimize the objective function (2). Computations stop either when the maximum number of iterations k_{max} is reached (here k_{max} = 1000), or before, at $k < k_{max}$, when an imposed precision criterion for queen's performance function is satisfied [4].

The HBMOA, together with the hydraulic analysis attached to the studied problem, were implemented within a code built in Pascal. We performed 50 runs of the above program, obtaining the values of the pumping energy consumed during a day, as in Table 2. All solutions ensured a final level in tanks equal to the initial one (69.5m).

Table 2

HBMOA results: run number and the corresponding pumping energy *E* in kWh

Run	1	2	3	4	5	6	7	8	9	10
E	95.31	94.15	95.24	95.25	94.25	95.27	95.33	95.32	94.25	95.25
Run	11	12	13	14	15	16	17	18	19	20
E	94.18	95.21	95.30	95.23	94.12	95.22	94.17	**92.97**	94.04	95.28
Run	21	22	23	24	25	26	27	28	29	30
E	94.19	94.06	95.28	95.34	95.28	94.11	94.11	96.37	95.29	94.13
Run	31	32	33	34	35	36	37	38	39	40
E	94.11	95.25	95.24	95.30	95.47	95.22	95.26	96.38	**96.39**	94.15
Run	41	42	43	44	45	46	47	48	49	50
E	95.24	95.30	94.24	94.18	94.15	95.26	95.20	95.27	95.34	95.33

The best solution (suboptimal solution) obtained using the HBMOA is the one of run no. 18, which yields a minimum value of 92.97 kWh for the energy consumed for pumping during a whole day, and 48 pump-working-hours from 144 potential pump-working-hours ($= 2\,\text{pumping stations} \times 3\text{pumps} \times 24\text{h}$). The worst solution is the one of run no. 39, which yields a maximum value of 96.39 kWh for the energy, and 51 pump-working-hours. The above amount of results, namely $n = 50$ energy values, allow performing some statistics: thus, the mean energy is $E_m = 94.915$ kWh, the standard deviation is $\sigma = 0.701$, and the probability-95% confidence interval $[E_m - 1.95996\,\sigma/\sqrt{n}\,; E_m + 1.95996\,\sigma/\sqrt{n}]$, based on the normal distribution, is [94.72; 95.11] kWh (it is the interval in which run's result (energy) falls corresponding to the given probability of 0.95).

4. HBMOA solution verified in EPANET using simple controls

The scheduling of pumps operation (number of working pumps at each time step), corresponding to the best (suboptimal) solution given by HBMOA, which leads to a minimum energy consumption of 92.97 kWh, was implemented in EPANET software by using simple controls. The hydraulic analysis was performed in EPANET as in previous section, over one day period, with one hour time step, starting from mid-night. The simple controls allow to set the daily pumps schedule (the start and stop sequences), at specified hours (time moments), i.e. *link 16 closed at clocktime 2:00*; *link 16 open at clocktime 4:00* (where *link 16* is the pump with ID 16).

In the following, we present some of the results obtained for runs performed in EPANET – they fit the results of the best solution of HBMOA. In Figure 2, we plot the flow rate (in l/s) delivered by each pump, at each time step, over one day period. The pump 18 from PS_1 (see Figure 1) was never opened.

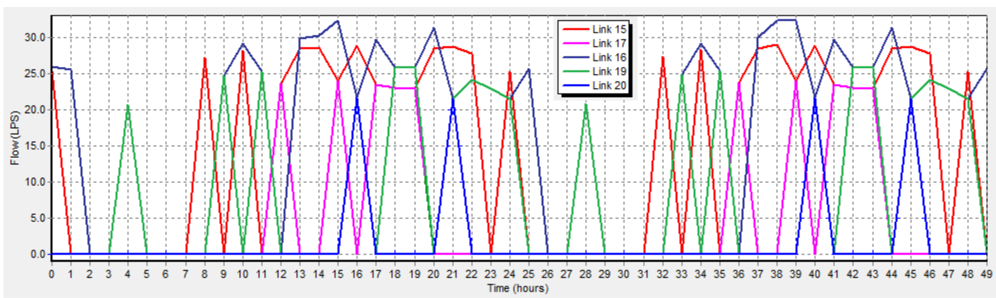


Fig. 2. Flow rate (in l/s) delivered by each pump, at each time step, as in HBMOA.

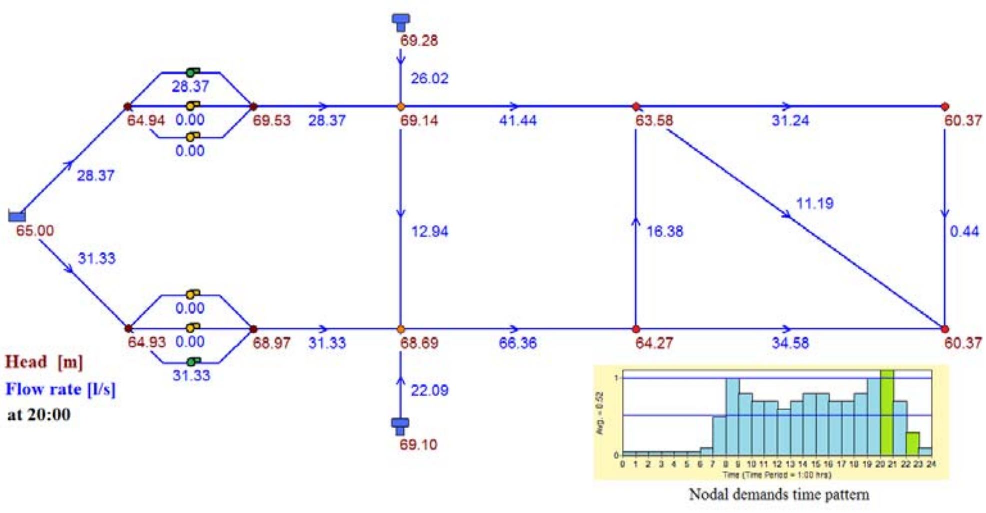
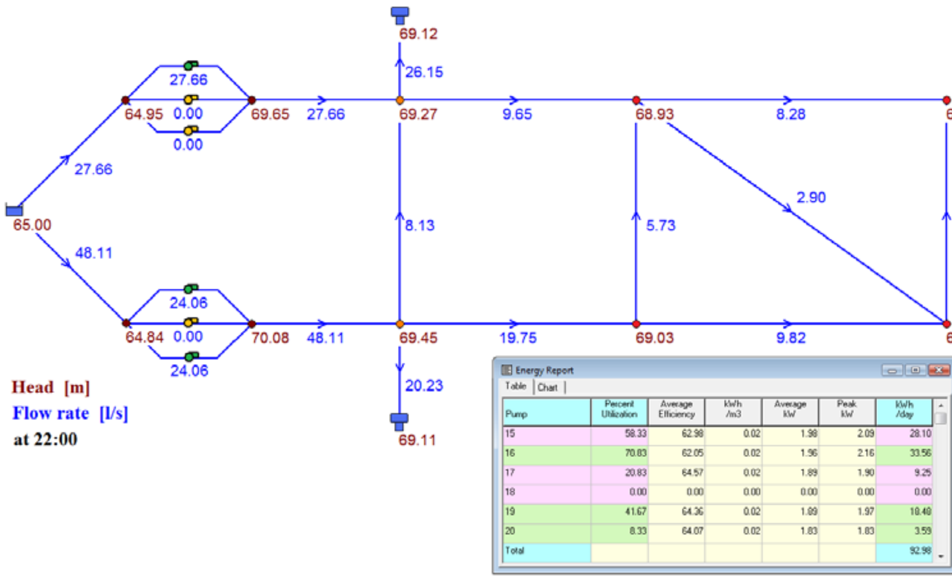


Fig. 3. Head and flow rate distribution at 20:00, as in HBMOA, and water demand time pattern.

In Figures 3 and 4 we present the head and flow rate distribution within the studied water supply system, at two different time moments, namely at 20:00, when the pumping stations and the tanks supply the consumers (placed in the right-hand side of the network), and at 22:00, when the pumping stations fill the tanks and supply simultaneously the consumers.



Pump	Percent Utilization	Average Efficiency	kWh /m3	Average kW	Peak kW	kWh /day
15	58.33	62.98	0.02	1.98	2.09	28.10
16	70.83	62.05	0.02	1.96	2.16	33.56
17	20.83	64.57	0.02	1.89	1.90	9.25
18	0.00	0.00	0.00	0.00	0.00	0.00
19	41.67	64.36	0.02	1.89	1.97	10.48
20	8.33	64.07	0.02	1.83	1.83	3.59
Total						92.98

Fig. 4. Head and flow rate distribution at 22:00 time clock, as in HBMOA, and the Energy Report.

The small graph plotted in Figure 3 show the water demand time pattern, described by the coefficients from Table 1. The small table from Figure 4 is an Energy Report, which gives for each pump: the percent of pump utilization, the pump average efficiency, the specific energy per cubic metre of pumped water, the average power, the power peak, and the daily energy consumed by the pump. The total amount of those final values, meaning $E = 92.98$ kWh, is the total energy consumed for pumping during a day period. That value is almost the same with the one attached to the best solution of HBMOA (92.97 kWh). Since we reproduced in EPANET, at each time step, the same hydraulic conditions as in the runs performed with HBMOA, the slight difference is due to rounding errors.

5. EPANET solution using rule-based controls

In this section, another scheduling of pumps operation was implemented in EPANET, by using 12 rule-based controls, to define the pumps starting/stopping algorithm upon the water level in tanks, in the range 69÷70.5 m. The rule-based controls were defined in accordance with the theory of pumping stations scheduling for discrete pumped flow rate [5].

For example, the rule no. 4 is written as: *if junction 9 pressure below 69*, *and pump 15 status is open*, *and pump 17 status is open*, *and pump 18 status is closed*, *then pump 18 status is open*.

In Figure 5, we plot the flow rate (in l/s) delivered by each pump, at each time step, over one day period. The pumps 17 and 18 of PS_1, as well as the pump 20 of PS_2 (see Figure 1) were never opened.

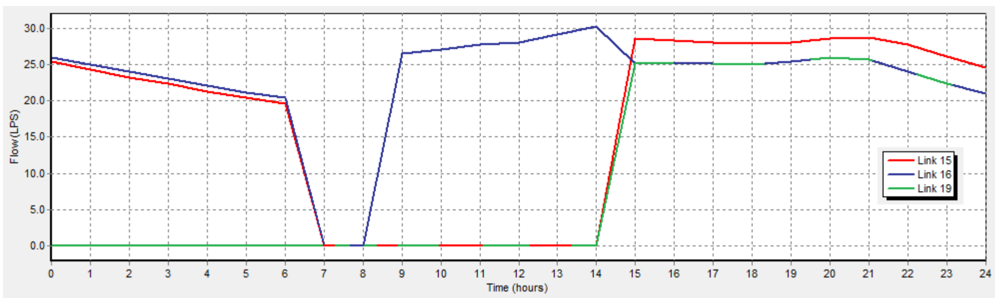


Fig. 5. Flow rate (in l/s) delivered by each pump, at each time step, for the new solution.

In Figures 6 and 7 we present the head and flow rate distribution within the water supply system, at 20:00 and 22:00 time clock, for the new solution obtained in EPANET. The small table from Figure 7 is the Energy Report.

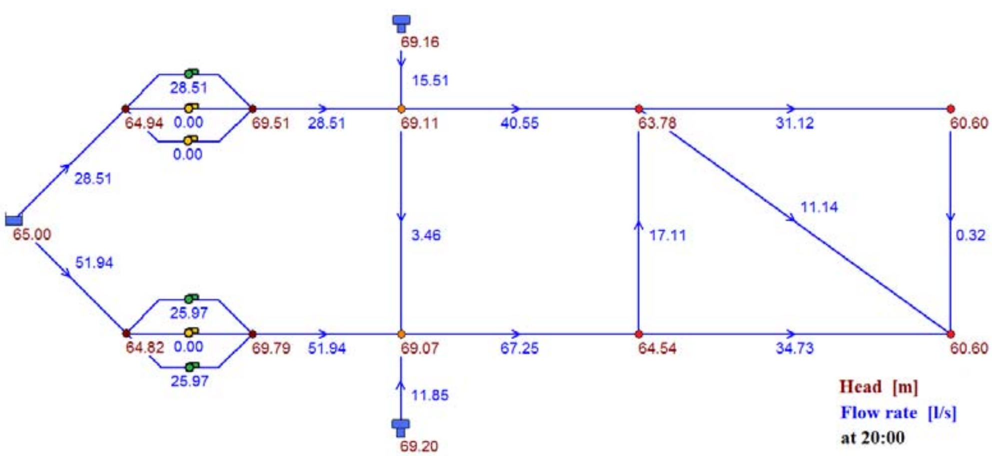


Fig. 6. Head and flow rate distribution at 20:00 (new EPANET solution).

The total energy consumed for pumping during a day period within the new solution is $E = 93.22$ kWh, giving a 0.27% relative error with respect to the best result obtained using HBMOA.

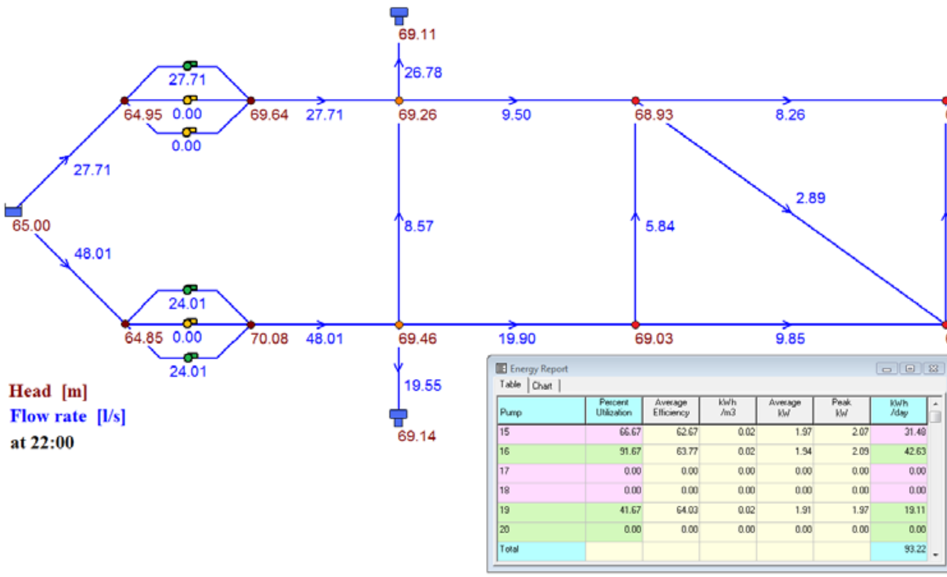


Fig. 7. Head and flow rate distribution at 22:00 time clock (new solution) and the Energy Report.

6. Conclusions

A modified Honey Bees Mating Optimization Algorithm was used to find optimal schedule for pumps upon water demands time pattern. HBMOA's best (suboptimal) solution was verified in EPANET, using simple controls. Another solution was obtained in EPANET, using rule-based controls (written for the pumps operation upon the water level in tanks). Both solutions yield comparable minimum values of the daily energy consumption.

R E F E R E N C E S

[1] *G. McCormick and R. S. Powell*, "Optimal Pump Scheduling in Water Supply Systems with Maximum Demand Charges", J. Water Res. Pl.-ASCE, **vol. 129**, no. 5, 2003, pp. 372-379

[2] *Sanda-Carmen Georgescu and R. Popa*, "Application of HBMOA to Pumping Station Scheduling for Water Supply", U.P.B. Sci. Bull. Series D, **vol. 72**, no. 1, 2010, pp. 77-84

[3] *O. B. Haddad and M. A. Mariño*, "Dynamic Penalty Function as a Strategy in Solving Water Resources Combinatorial Optimization Problems with Honey-Bee Mating Optimization (HBMO) Algorithm", in J. Hydroinform., **vol. 9**, no. 3, 2007, pp. 233–250

[4] *R. Popa and Sanda-Carmen Georgescu*, "Water Distribution Networks Optimal Design Using the HBMOA", Proc. 4th CIEM2009, Ed. Politehnica Press, Bucharest, 2009, S6_10

[5] *A.-M. Georgescu and Sanda-Carmen Georgescu*, Hidraulica rețelelor de conducte şi maşini hidraulice (Hydraulic Networks and Hydraulic Machineries), Ed. Printech, Bucharest, 2007

[6] *R. W. Jeppson*, Analysis of Flow in Pipe Networks, Ann Arbor Science Publishers, Michigan, 1976.