USING BENINI'S ONE PARAMETER DISTRIBUTION TO STUDY THE INCOME VOLATILITY

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Lucrarea precizeaza câteva proprietati statistice ale repartitiei Benini $Ben1(q)$, $q > 0$. În acest context este propus si un estimator W pentru parametrul necunoscut q. Calitatile statistice ale estimatorului W sunt analizate experimental utilizând o tehnica de simulare Monte Carlo. În final este studiata, pentru o valoare q arbitrara, monotonia ratei de hazard în raport cu venitul x.

Deoarece repartitia Benini verifica o lege empirica de tip Pareto privind veniturile, recomandam utilizarea acestei repartitii în cercetarile sociologice, în vederea aprecierii nivelului de prosperitate economica pentru diverse subpopulatii.

In this paper are studied some statistical properties of Benini one parameter distribution $Ben1(q)$, with $q > 0$. The point estimation of the unknown value of the parameter q is suggested too. A Monte Carlo algorithm establishes the accuracy for the parameter estimation procedure. Finaly we analyzed the monotony of the hazard rate function.

This distribution can be used successfuly to compare the income distributions for different human populations.

Keywords : Benini distribution, point estimation, Monte Carlo simulation, hazard rate function.

MSC2000 : primary 60E05 ; secondary 62F10, 65C05, 62N05, 62P25.

Introduction

1. Genesis of Benini distributions

In practice it is very important to establish the theoretical distribution for the income of the individuals from a given population P.

For a fixed income x, $x \geq 0$, we'll denote by n_x the number of individuals from P which have their income greater than x.

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Pareto, the father of the statistical theory of income distributions, observed experimentaly a decreasing linear relationship between the logarithm of income x and the logarithm of n_x. More, the following approximation

$$ln(n_x) \approx a - b\,ln(x) \qquad (1)$$

with $b > 0$, becomes very accurate for high values of x, that is when the income x passes over a given threshold q, $q > 0$ ([2]).

The approximation (1), which is valid for any $x \geq q$, can be rewriten in the form

$$ln(n_x / n_q) \approx \left(a - ln(n_q)\right) - b\,ln(x) = a_1 - b\,ln(x) \qquad (2)$$

We denote by X the random variable (r.v.) having as observations the income x, $x \geq q$, of the individuals from the population P. Let $F_0(x)$ be the cumulative distribution function (c.d.f.) of the r.v. X.

From (2), since the ratio n_x / n_q estimates the probability $Pr(X \geq x) = 1 - F_0(x)$, we can accept the equality

$$ln(1 - F_0(x)) = a_1 - b\,ln(x) \qquad (3)$$

But the equation (3) has the solution

$$F_0(x) = 1 - \left(\frac{x}{q}\right)^{-b} \qquad (4)$$

which is just the Pareto cumulative distribution function (see also [2], p.349-350 , [3]).

Developping the same idea, Benini considered the following approximation ([3] , p.9)

$$ln(n_x) \approx a - b\,ln(x) - c\left(ln(x)\right)^2 \quad , \quad x \geq q \qquad (5)$$

which obviously is more accurate than the relation (1).

For this case, pursuing a similar reasoning, one of the solutions $F(x;q)$, $\theta > 0$, of (5) has the expression

$$F(x;q) = 1 - exp\left(-q.ln^2(x/q)\right) \quad , \quad x \geq q \qquad (6)$$

In the subsequent, for a fixed value of q , q > 0 , we'll denote by X ~ Ben1(θ) if the r.v. X has the c.d.f $F(x;q)$ given by the formula (6).

2. Properties of Ben1(q) distribution

Proposition 1. If X ~ Ben1(θ) then the probabilitity density function (p.d.f.) $f(x;q)$ of X has the form

$$f(x;q) = \frac{2q}{x} ln\left(\frac{x}{q}\right) exp\left(-q\left(ln\left(\frac{x}{q}\right)\right)^2\right) \quad , \qquad x \ge q \ , \ \theta > 0 \qquad (7)$$

Proof. After a direct calculus, since $f(x;q) = \frac{¶ F(x;q)}{¶ x}$ we obtain the expression (7).

Figures 1 and 2 present the graphic of the p.d.f. $f(x;q)$ considering different values for the parameter θ .