A CROSSFEED SOLUTION FOR THE ROLL-COUPLING PROBLEM OF HIGH-PERFORMANCE AIRPLANES

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Lucrarea prezentă tratează problema proiectării unor legi de interconectare ("crossfeed") a comenzilor de eleroane şi de direcție, având drept scop ameliorarea caracteristicilor de ruliu ale avioanelor de mare performanță. În urma analizei dependenței regimurilor "pseudostaționare" de ruliu de variabilele de comandă ale avionului, autorul propune o lege de tip "crossfeed" bazată pe un nou criteriu de corelare a comenzilor, numit "criteriul bifurcației transcritice".

Soluția propusă permite atât evitarea apariției unor fenomene nedorite (regimuri de zbor instabile, discontinuități în răspunsul avionului la comenzi etc.), cât şi creşterea valorii maxime operaționale a vitezei de ruliu a unui avion dat. Eficacitatea legii de tip crossfeed propuse a fost verificată prin efectuarea de studii numerice de caz.

The present paper deals with the problem of designing appropriate aileron-rudder interconnect ("crossfeed") laws for improving the rolling characteristics of high-performance airplanes. As a result of analyzing the dependence of the "pseudosteady" rolling regimes on the airplane control variables, the author proposes a crossfeed law based on a new control correlation criterion, called the "transcritical bifurcation criterion".

The proposed solution allows both avoiding the occurrence of unwanted phenomena (unstable flight conditions, discontinuous airplane response to control inputs etc.), and increasing the maximum operational roll rate of a given airplane. The efficiency of the proposed crossfeed law has been verified by numerical case studies.

Keywords: nonlinear flight dynamics, stability and control

1. Introduction

Typically, high-performance airplane configurations are "inertially slender", i.e. their mass distribution is pronouncedly concentrated in the proximity of the fuselage axis. During high roll rate maneuvers, the discrepancy between the relatively small value of the airplane's rolling moment of inertia (I_x) and the large values of its pitching (I_y) and yawing (I_z) moments of inertia results in significant nonlinear coupling effects involving the longitudinal and the lateral-directional degrees of freedom. As a result, several critical stability and control

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phenomena, which are unpredictable by classic linear analyses, can occur in certain conditions.

The first important contribution to the roll-coupling problem is due to Phillips (1948), [1], who analyzed the stability of a simplified "steady" rolling motion and revealed that aperiodic instabilities concerning the pitch and yaw degrees of freedom occur at certain critical values of the rolling velocity, these values being known as "Phillips' critical roll rates". (It is interesting to remind that Phillips wrote his theoretical note before the first in-flight accidents caused by inertia roll coupling.)

Much of the subsequent contributions in the 1950's (e.g., those due to Stone, [2], Welch and Wilson, [3], Pinsker, [4]) focused on evaluating maximum tail loads in critical rolling conditions.

Rhoads and Schuler, [5], showed that Phillips' critical roll rates could be obtained as steady-state solutions of a simplified system involving, essentially, the assumption that gravitational effects were negligible. Gates and Minka, [6], revealed that inertia roll coupling could generate jump phenomena between two such steady-state solutions.

By means of perturbation analysis methods and numerical simulations, Hacker and Oprisiu, [7], demonstrated that gravitational terms had, indeed, a very limited influence on the airplane's roll-coupling dynamics.

The approach based on neglecting gravitational effects in inertia roll-coupling analyses is generally known today – according to the denomination introduced by Schy and Hannah, [8] – as the *"Pseudo-Steady State"* (PSS) *method.* Using this approach, the above-mentioned authors pointed out (loc. cit.) the existence of *multiple* PSS *solutions* corresponding to a given control configuration (constant aileron, rudder and elevator inputs) and explained, on this basis, the occurrence of jump phenomena at certain critical control values.

The work of Young, Schy and Johnson, [9], showed that nonlinear aerodynamics play a negligible role in generating the jump phenomena associated to the roll-coupling problem.

Guicheteau, [10], and Carroll and Mehra, [11], applied bifurcation methods to airplane dynamics analysis and emphasized the beneficial effects of implementing adequate aileron-rudder crosfeeds for avoiding unwanted roll-coupling phenomena (such as flight stability losses, discontinuous responses to control inputs and inertial auto-rolling). In this context, Ananthkrishnan and Sudhakar considered a simple linear aileron-rudder interconnect (ARI) law, [12], as well as a nonlinear ARI law derived from a PSS coordination (zero sideslip) constraint, [13].

Based on a different control correlation concept, referred to as *"the transcritical bifurcation criterion",* the author of the present paper proposes *a new* *aileron-rudder crossfeed law.* The synthesis of this crossfeed law relies on the position of the transcritical bifurcation points in the PSS solution diagram.

2. Mathematical modeling of roll-coupling dynamics

The following eighth-order differential system is considered for describing the airplane motion ([14], pp. 26-31):

$$\dot{V} = \frac{1}{m}[T\cos(\alpha+\tau)\cos\beta - D] + g(-\cos\alpha\cos\beta\sin\theta + \sin\beta\cos\theta\sin\phi + \sin\alpha\cos\beta\cos\theta\cos\phi), \quad (1)$$

$$\dot{\beta} = p\sin\alpha - r\cos\alpha + \frac{1}{mV}[D tg\beta + \frac{Y}{\cos\beta} - T\cos(\alpha+\tau)\sin\beta] + \frac{g}{V}(\cos\alpha\sin\beta\sin\theta + \cos\beta\cos\theta\sin\phi - \sin\alpha\sin\beta\cos\theta\cos\phi), \quad (2)$$

$$\dot{\alpha} = q - (p\cos\alpha + r\sin\alpha)\tan\beta - \frac{1}{mV\cos\beta}[\mathrm{L} + T\sin(\alpha+\tau)] + \frac{g}{V\cos\beta}(\sin\alpha\sin\theta + \cos\alpha\cos\theta\cos\phi), \quad (3)$$

$$\dot{p} = -\frac{I_z - I_y}{I_x}qr + \frac{1}{I_x}L, \quad (4)$$

$$\dot{q} = \frac{I_z - I_x}{I_y}rp + \frac{1}{I_y}M, \quad (5)$$

$$\dot{r} = -\frac{I_y - I_x}{I_z}pq + \frac{1}{I_z}N. \quad (6)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi, \quad (7)$$

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta, \quad (8)$$

where, as reflected by Eqs.(4)-(6), principal body-axes are used ($I_{xz} = 0$). (To avoid confusion, note that L and L symbolize, respectively, the airplane's lift and rolling moment.)

Since the duration of the analyzed rolling motion does not exceed a few seconds, constant flight speed is assumed ($V = ct.$); in this case ($\dot{V} = 0$), thrust (T) can be expressed from Eq. (1) in the form

$$T = \frac{1}{\cos(\alpha+\tau)\cos\beta}[D - mg(-\cos\alpha\cos\beta\sin\theta + \sin\beta\cos\theta\sin\phi + \sin\alpha\cos\beta\cos\theta\sin\phi)]. \quad (9)$$

Using the above expression (which gives the thrust value required for constant speed maneuvers) and assuming that $\tau = 0$, Eqs. (2) and (3) can be written as follows:

$$\dot{\beta} = p\sin\alpha - r\cos\alpha + \frac{Y}{mV\cos\beta} + \frac{g\cos\theta\sin\phi}{V\cos\beta} , \tag{10}$$

$$\dot{\alpha} = q - (p\cos\alpha + r\sin\alpha)\tan\beta - \frac{1}{mV\cos\alpha\cos\beta}(\mathrm{L}\cos\alpha + D\frac{\sin\alpha}{\cos\beta}) + \\ + \frac{g\cos\theta}{V\cos\alpha\cos\beta}(\cos\phi + \sin\alpha\tan\beta\sin\phi) . \tag{11}$$

Taking into account that

$$\mathrm{L}\cos\alpha + D\frac{\sin\alpha}{\cos\beta} = -(Z + Y\sin\alpha\tan\beta) , \tag{12}$$

the following *seventh-order* model can be derived

$$\dot{\beta} = p\sin\alpha - r\cos\alpha + \frac{1}{\cos\beta}(y + \frac{g}{V}\cos\theta\sin\phi), \tag{13}$$

$$\dot{\alpha} = q - (p\cos\alpha + r\sin\alpha)\tan\beta + \frac{1}{\cos\alpha\cos\beta}[(z + y\sin\alpha\tan\beta) + \\ + \frac{g\cos\theta}{V}(\cos\phi + \sin\alpha\tan\beta\sin\phi)] , \tag{14}$$

$$\dot{p} = l - i_1 qr , \tag{15}$$

$$\dot{q} = m + i_2 pr , \tag{16}$$

$$\dot{r} = n - i_3 pq , \tag{17}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi , \tag{18}$$

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta , \tag{19}$$

where

$$y = \frac{Y}{mV}, \quad z = \frac{Z}{mV}, \quad l = \frac{L}{I_x}, \quad m = \frac{M}{I_y}, \quad n = \frac{N}{I_z} , \tag{20}$$

$$i_1 = \frac{I_z - I_y}{I_x}, \quad i_2 = \frac{I_z - I_x}{I_y}, \quad i_3 = \frac{I_y - I_x}{I_z} . \tag{21}$$

As a result of neglecting the effect of gravity (by setting $g = 0$), Eqs. (18) and (19) decouple from Eqs. (13)-(17). Thus, the following *fifth-order* model is obtained:

$$\dot{\beta} = p\,sin\,\alpha - r\,cos\,\alpha + \frac{y}{cos\,\beta}\,, \tag{22}$$

$$\dot{\alpha} = q - (\,p\,cos\,\alpha + r\,sin\,\alpha\,)\,tan\,\beta + \frac{1}{cos\,\alpha\,cos\,\beta}(\,z + y\,sin\,\alpha\,tan\,\beta\,)\,, \tag{23}$$

$$\dot{p} = l - i_1 qr\,, \tag{24}$$

$$\dot{q} = m + i_2 pr\,, \tag{25}$$

$$\dot{r} = n - i_3 pq\,. \tag{26}$$

As previously remarked, *steady state solutions of such simplified "zero-g" mathematical models* are, usually, referred to as *PSS* ("Pseudo-Steady State") *solutions* of the more complex models that include gravitational terms.

Specifically, the steady state solutions of the fifth-order system (22)-(26) represent the so-called PSS solutions of the seventh-order system (13)-(19).

3. Bifurcation analysis of the PSS solutions

Considering the steady state condition relative to Eqs. (22)-(26), i.e.

$$\dot{\beta} = \dot{\alpha} = \dot{p} = \dot{q} = \dot{r} = 0\,, \tag{27}$$

the PSS solutions of the analyzed problem are obtained by solving the system

$$(\,p\,sin\,\alpha - r\,cos\,\alpha\,)\,cos\,\beta + y = 0\,, \tag{28}$$

$$q\,cos\,\alpha\,cos\,\beta - p\,sin\,\beta + z = 0\,, \tag{29}$$

$$l - i_1 qr = 0\,, \tag{30}$$

$$m + i_2 pr = 0\,, \tag{31}$$

$$n - i_3 pq = 0\,, \tag{32}$$

where Eq. (29) has been derived from Eqs. (23) and (28).

Critical roll-coupling phenomena are essentially generated by the inertial interactions between the longitudinal and lateral-directional degrees of freedom and occur, typically, at low angles-of-attack. Accordingly, retaining only the inertial nonlinearities in the present analysis, the normalized forces and moments y, z, l, m, n are *linearly* represented as functions of the state (β, α, p, q, r) and control (δ_a, δ_e, δ_r) variables in the form

$$y = y_\beta \beta + y_p p + y_r r + y_{\delta_a}\delta_a + y_{\delta_r}\delta_r\,, \tag{33}$$

$$z = z_0 + z_\alpha \alpha + z_{\dot{\alpha}}\dot{\alpha} + z_q q + z_{\delta_e}\delta_e\,, \tag{34}$$

$$l = l_\beta \beta + l_p p + l_r r + l_{\delta_a}\delta_a + l_{\delta_r}\delta_r\,, \tag{35}$$

$$m = m_0 + m_\alpha \alpha + m_{\dot{\alpha}}\dot{\alpha} + m_q q + m_{\delta_e}\delta_e\,, \tag{36}$$

$$n = n_\beta \beta + n_p p + n_r r + n_{\delta_a}\delta_a + n_{\delta_r}\delta_r\,. \tag{37}$$

The following analysis is focused on *the dependence of the pseudo-steady states* β_{ps}, α_{ps}, p_{ps}, q_{ps}, r_{ps} on the values of the aileron input δ_a (which is the characteristic control parameter for the roll-coupling problem). Such dependencies graphically represent *PSS solution branches relative to the control parameter* δ_a and are determined, for specified elevator (δ_e) and rudder (δ_r) deflection values, on the basis of the following *nonlinear algebraic system*

$$(p\sin\alpha - r\cos\alpha)\cos\beta + y_\beta\beta + y_p p + y_r r + y_{\delta_a}\delta_a + y_{\delta_r}\delta_r = 0\,, \tag{38}$$

$$q\cos\alpha\cos\beta - p\sin\beta + z_0 + z_\alpha\alpha + z_q q + z_{\delta_e}\delta_e = 0\,, \tag{39}$$

$$-i_1 qr + l_\beta\beta + l_p p + l_r r + l_{\delta_a}\delta_a + l_{\delta_r}\delta_r = 0\,, \tag{40}$$

$$i_2 pr + m_0 + m_\alpha\alpha + m_q q + m_{\delta_e}\delta_e = 0\,, \tag{41}$$

$$-i_3 pq + n_\beta\beta + n_p p + n_r r + n_{\delta_a}\delta_a + n_{\delta_r}\delta_r = 0\,, \tag{42}$$

using a continuation algorithm.

In order to evaluate the effect of the rudder and elevator deflections on the PSS solution branches, a numerical example has been considered, based on the data given below (which are typical for a fighter airplane configuration):

$$y_\beta = -0{,}196\,(\mathrm{s}^{-1});\qquad z_\alpha = -1{,}329\,(\mathrm{s}^{-1});$$

$$l_\beta = -9{,}99\,(\mathrm{s}^{-2});\quad l_p = -3{,}933\,(\mathrm{s}^{-1});\quad l_r = 0{,}126\,(\mathrm{s}^{-1});$$

$$m_\alpha = -23{,}18\,(\mathrm{s}^{-2});\quad m_{\dot\alpha} = -0{,}173\,(\mathrm{s}^{-1});\quad m_q = -0{,}814\,(\mathrm{s}^{-1});$$

$$n_\beta = 5{,}67\,(\mathrm{s}^{-2});\quad n_p = 0{,}002\,(\mathrm{s}^{-1});\quad n_r = -0{,}235\,(\mathrm{s}^{-1});$$

$$y_{\delta_r} = 0{,}127\,(\mathrm{s}^{-1});\quad z_{\delta_e} = -0{,}168\,(\mathrm{s}^{-1});\quad m_{\delta_e} = -28{,}18\,(\mathrm{s}^{-2});$$

$$l_{\delta_a} = -45{,}83\,(\mathrm{s}^{-2});\qquad l_{\delta_r} = -7{,}64\,(\mathrm{s}^{-2});$$

$$n_{\delta_a} = -0{,}921\,(\mathrm{s}^{-2});\qquad n_{\delta_r} = -6{,}51\,(\mathrm{s}^{-2});$$

$$i_1 = 0{,}727\,;\qquad i_2 = 0{,}949\,;\qquad i_3 = 0{,}716\,.$$

The values of the stability and control derivatives not included in the preceding list are negligible.

The most significant dependence for the present roll-coupling analysis is the dependence of the PSS *values* of the airplane's *roll rate* (p_{ps}) *on the aileron deflection input* (δ_a). (As known, the airplane's roll rate is the vehicle's angular velocity around its longitudinal axis.) Figures 1 and 2 illustrate, in the $p_{ps} - \delta_a$ plane, *the effect of the elevator deflection* δ_e (at $\delta_r = 0^o$) and, respectively, *the* *effect of the rudder deflection* δ_r (at $\delta_e = 0^o$) on the *"primary"* solution branches, i.e. on those solution branches that pass through the origin of the above-mentioned plane (if $\delta_r = 0^o$), or very close to it (if $\delta_r \neq 0^o$). In Figs. 1 and 2 *continuous* lines represent *stable* PSS solutions and *discontinuous* lines indicate *unstable* PSS solutions.

As seen, there have been identified *three types of bifurcation points* (at which stability changes occur), i.e.

- *limit* (turning) points (L);
- *transcritical* bifurcation points (T);
- *Hopf* bifurcation points (H).

Notes:

(a) Each T-type solution branch *separates* the domains of L-type and H-type solution branches, this qualitative aspect of the PSS solution diagram being related to the well-known *structural instability* of transcritical bifurcation points;

(b) The T-type solution branches provide *the extreme PSS roll rate values* that can be obtained by appropriate crossfeed.

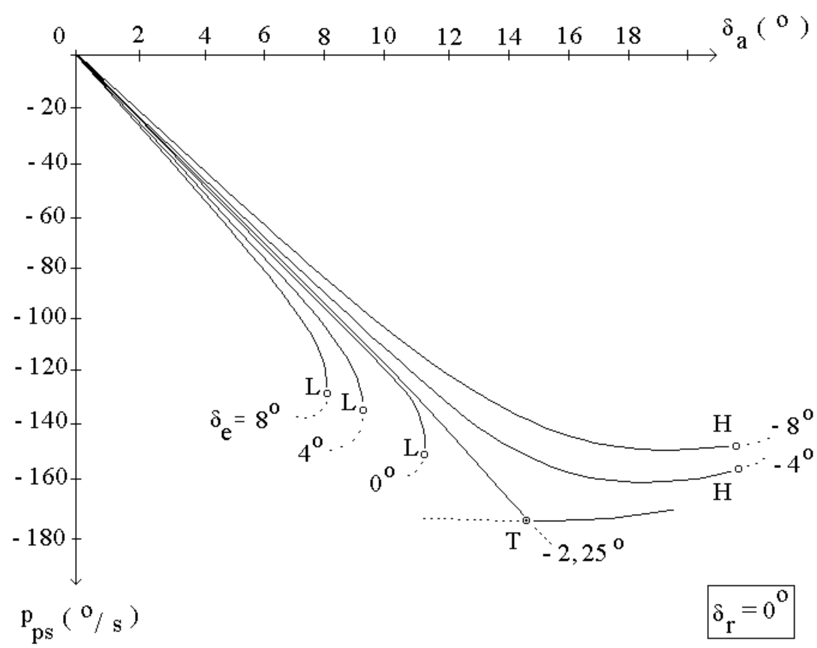

Fig. 1. Primary PSS solution branches corresponding to different δ_e values ($\delta_r = 0^o$)

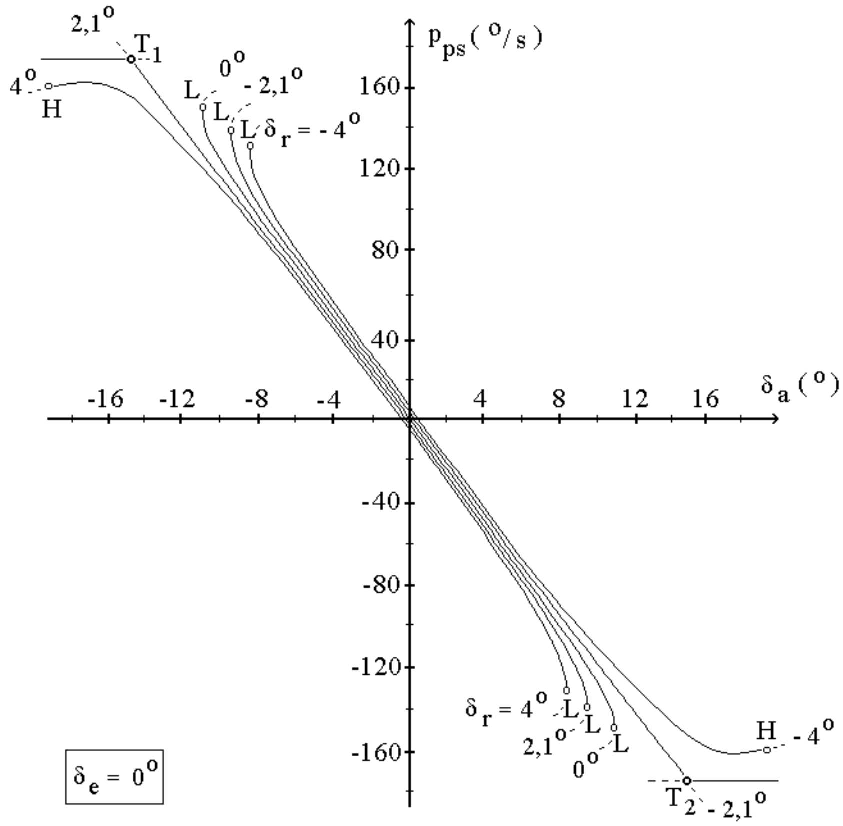

Fig. 2. Primary PSS solution branches corresponding to different δ_r values ($\delta_e = 0^{\circ}$)

4. Crossfeed law synthesis using the "T -criterion"

Determining the primary solution branches and the associated critical control configurations $(\delta_a, \delta_e, \delta_r)_{cr}$, which correspond to the L-, T- and H-bifurcation points (where the PSS solutions become unstable), allows one to predict the airplane response in rolling maneuvers, revealing the potentially dangerous situations. In order to avoid such situations (e.g., divergent and jump-like responses) and enhance the rolling characteristics of an airplane, appropriate correlation laws between its control variables may be used.

Specifically, a *crossfeed law*

$$\delta_r = f(\delta_a, \delta_e), \tag{43}$$

which relate the values of the rudder, aileron and elevator deflections, is considered in the following.

As illustrated in Figs.1 and 2, *the transcritical bifurcation points determine the maximum achievable range* of rolling velocities. Therefore, a so-called *"transcritical bifurcation criterion"* or, briefly, "T-criterion", based on the location of the transcritical bifurcation points, is taken into consideration for synthesizing the crossfeed law. Note, in this context, *the existence of two significant* δ_e*- intervals* (see Fig.1) *defined with respect to the value* $\delta^0_{e_T}$, which is the "transcritical" value of δ_e for $\delta_r = 0^o$ (in the considered numerical example, $\delta^0_{e_T} = -2.25\,^o$). Specifically, one can distinguish the following intervals: (a) $\delta_e < \delta^0_{e_T}$, where the $\delta_r = 0^o$ solution branches (i.e. the solution branches corresponding to $\delta_r = 0^o$) are *H-type solution branches;* (b) $\delta_e > \delta^0_{e_T}$, where the $\delta_r = 0^o$ solution branches are *L-type solution branches.* Hence, the author proposes the following "T-criterion" crossfeed law:

$$
\begin{aligned}
&\text{- if } \delta_e \le \delta^0_{e_T}, \quad \delta_r = \kappa_T \cdot \delta_a, \qquad \delta_{a_{min}} \le \delta_a \le \delta_{a_{max}}, \\
&\text{- if } \delta_e > \delta^0_{e_T}, \quad \delta_r = \begin{cases} \delta_{r_{T_1}}, & \delta_{a_{min}} \le \delta_a \le \delta_{a_{P_1}}, \\ \kappa^*_T \cdot \delta_a, & \delta_{a_{P_1}} < \delta_a < \delta_{a_{P_2}}, \\ \delta_{r_{T_2}}, & \delta_{a_{P_2}} \le \delta_a \le \delta_{a_{max}}, \end{cases}
\end{aligned}
\tag{44}
$$

where the gains κ_T and κ^*_T are defined in the form

$$\kappa_T = \delta_{r_{T_1}} / \delta_{a_{T_1}} \; ; \; \kappa^*_T = \delta_{r_{T_1}} / \delta_{a_{P_1}} \tag{45}$$

or, equivalently, $$\kappa_T = \delta_{r_{T_2}} / \delta_{a_{T_2}} \; ; \; \kappa^*_T = \delta_{r_{T_2}} / \delta_{a_{P_2}} . \tag{46}$$

The limit values $\delta_{a_{min}}$, $\delta_{a_{max}}$ of the aileron control input are set to satisfy -at all operational δ_e values- prescribed state, input and output restrictions (e.g., for stability reasons, $\delta_{a_{T_1}} < \delta_{a_{min}}$, $\delta_{a_{max}} < \delta_{a_{T_2}}$).

Using the proposed crossfeed law, the obtained dependence of the pseudo-steady values of the airplane's roll rate on the aileron input has the aspect illustrated in Fig. 3 (bold lines).

The points P_1 and P_2 are determined by the "transcritical" values of the rudder control input ($\delta_{r_{T_1}}$, $\delta_{r_{T_2}}$) and by the relationships

$$p_{ps}(P_1) = p_{ps}(L_1^0), \qquad p_{ps}(P_2) = p_{ps}(L_2^0), \tag{47}$$

where L_1^0 and L_2^0 are the limit points situated on the primary solution branch corresponding to $\delta_r = 0^o$.

Obviously, in order to implement the proposed crossfeed law, the location of the transcritical bifurcation points in the control space $(\delta_a, \delta_e, \delta_r)$ must be previously determined in an accurate manner (using an appropriate computer algorithm).

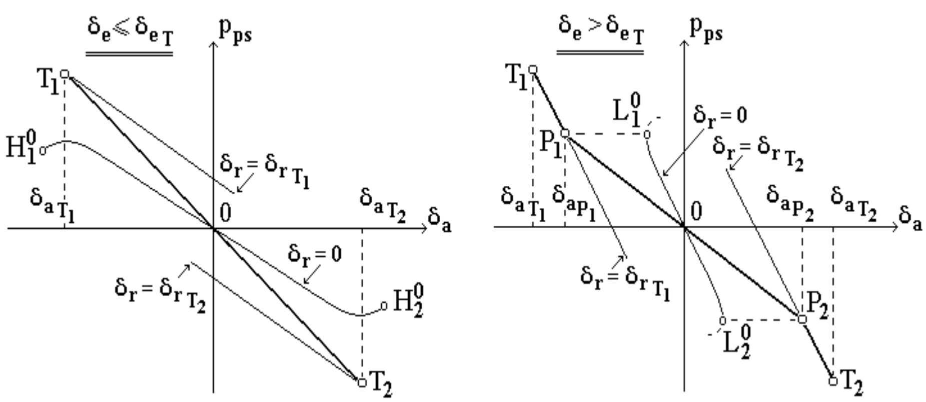

Fig. 3. $p_{ps}(\delta_a)$ dependence corresponding to the proposed crossfeed law

5. Numerical study of the proposed "T -criterion" crossfeed law

The effect of the proposed control crossfeed on the dynamic response to simultaneous step-aileron and step-elevator inputs has been studied numerically. Thus, the case of a complex rotational maneuver initiated, at $t = t_0$, from a *symmetric, rectilinear and horizontal* flight condition (i.e., $p(t_0) = q(t_0) = r(t_0) = 0^o / s$, $\beta(t_0) = 0^o$, $\alpha(t_0) = 1.49^o$, $\theta(t_0) = 1.49^o$, $\phi(t_0) = 0^o$; $\delta_a(t_0) = 0^o$, $\delta_e(t_0) = -1.23^o$, $\delta_r(t_0) = 0^o$) by the simultaneous *step inputs* $\Delta\delta_a = 14^o$, $\Delta\delta_e = 1.23^o$ has been considered.

The airplane's dynamic response has been simulated by solving *the seventh-order* differential system (13)-(19), which includes gravitational effects.

The results concerning the time variations of the *roll rate* ($p(t)$), *angle of attack* ($\alpha(t)$), *sideslip angle* ($\beta(t)$) and *pitch rate* ($q(t)$) are illustrated in Figs. 4-7, where the symbols C and N denote, respectively, the case of using the proposed crossfeed law and the non-correlated case (without crossfeed).

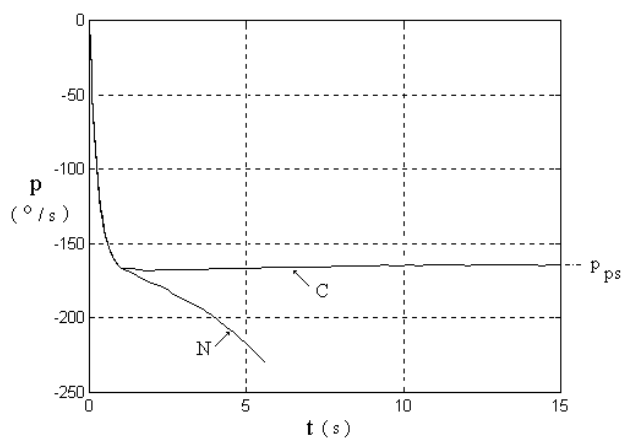

Fig. 4. Roll rate response

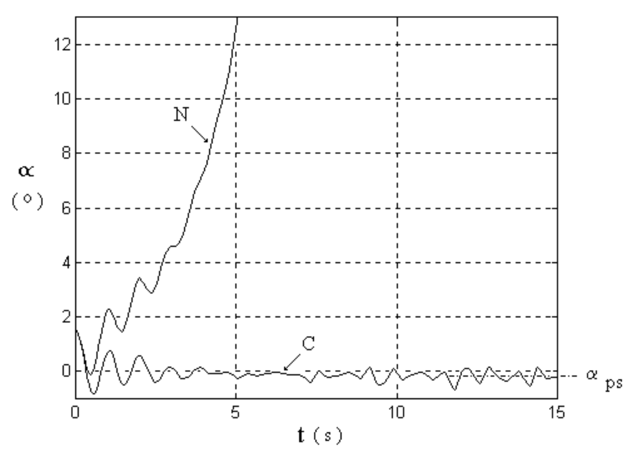

Fig. 5. Angle-of-attack response

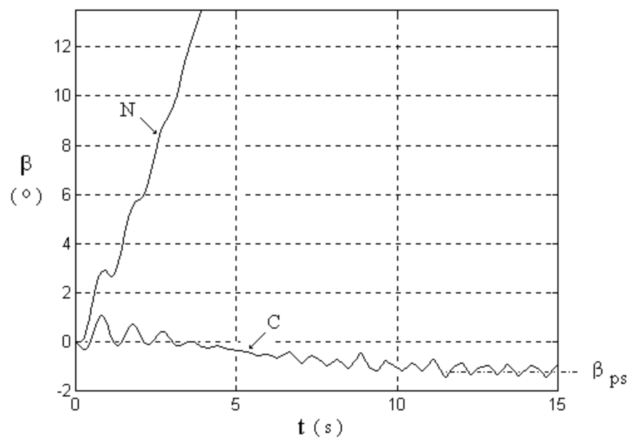

Fig. 6. Sideslip-angle response

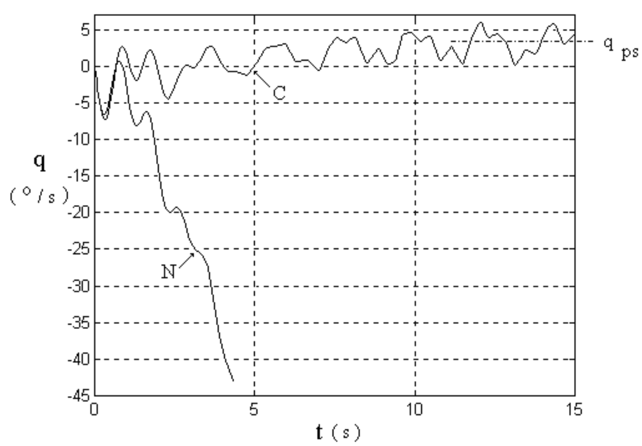

Fig. 7. Pitch rate response

Note that if the proposed crossfeed law is used, the illustrated time dependencies have favorable characteristics, in contrast with the rather abrupt, undesirable variations corresponding to the non-correlated case. With increasing time, the roll rate *p,* the angle of attack α, the sideslip angle β and the pitch rate q vary toward their predicted PSS values ($p_{ps} = -163.98\ ^{\mathrm{o}}/s$; $\alpha_{ps} = -0.17\ ^{\mathrm{o}}$; $\beta_{ps} = -1.26\ ^{\mathrm{o}}$; $q_{ps} = 3.37\ ^{\mathrm{o}}/s$) and oscillate around these values.

The mentioned fluctuations around the PSS values are essentially due to the gravitational terms.

6. Conclusions

A new crossfeed law has been proposed for improving the rolling characteristics of high-performance airplanes.

The synthesis procedure of the proposed crossfeed law involves a preliminary bifurcation analysis of airplane's pseudosteady-state rolling regimes in order to determine the critical control configurations; it is shown that these control configurations are related to three types of bifurcation points: limit points, transcritical bifurcation points and Hopf bifurcation points.

Using the results of the bifurcation analysis, the values of the aileron, rudder and elevator deflections are correlated according to the so-called *"transcritical bifurcation criterion" or T-criterion"*.

The obtained dependence of the airplane's pseudosteady-state rolling velocity on the aileron deflection, $p_{ps}(\delta_a)$, is linear or piecewise linear (see figure 3), which is a desirable characteristic from the pilot's standpoint. As illustrated in Figs. 4-7, the dynamic response -calculated by solving the seventh-order differential system (13-19)- is also favorable and confirms the predictions of the PSS method.

A significant advantage of the present crossfeed law with respect to other previously developed control laws (e.g., [12], [13]) consists in the *maximization of the interval of achievable controlled rolling velocities.* Thus, implementing the proposed crossfeed law results in a superior exploitation of the intrinsic rolling capability of a given airplane and, consequently, in *maneuverability and agility enhancements.*

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