TRANSMISSION LOSS ALLOCATION IN POOL-BASED ELECTRICITY MARKET BASED ON INCREMENTAL LOSS INDEX

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In this paper, a method is proposed to assign transmission losses costs in pool-based electricity markets. This method is based on using the impedance matrix of the network and partial derivatives of the active power losses respect to bus currents coefficients. After performing load flow equations, the losses of each bus are calculated using the impedance matrix of the network and the injected currents from each bus. These losses are properly and fairly shared between network buses for fair loss allocation, in proportion to partial derivatives of the active power losses respect to bus currents coefficients. Finally, this method has been tested on a benchmark IEEE 14-bus network and the results are compared with the other existing methods.

Keywords: loss allocation, pool-based electricity market, admittance equivalent circuit, loss matrix

1. Introduction

In power networks, a few percentage of the transmission power is always lost. The main part of these losses is due to the flow of current in ohmic resistance of the transmission lines. In traditional power systems with uniform structure, all the attempts were made in order to minimize the network losses and in terms of costs. The overall cost of losses is added to other generation and transmission costs and forms the total operation cost of the network. But in deregulated power systems, every player of the system possesses the separate legal character and therefore it is independent in terms of income and costs. Thus, determining their share in total network costs including the losses is unavoidable [1]. On the other hand, in deregulated power systems, regardless of losses optimization, another serious question is posed that how the total cost of losses should be paid by the power market players. In the pool-based electricity market, the loss allocation helps to recognize the share of each generation or consumption unit from the total network losses. So, the ISO can receive the losses costs from each of the market participants and return it to the generation companies [2].

In the markets which are based on bilateral contracts, the losses of each contract should be specified in the contract content and its support source should be

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determined. In spite of the high importance of loss allocation to the participants, technically and economically, due to complexity, nonlinear nature and high dependence of loss function on the different variables, no comprehensive and precise method which can be practically employed has been presented hitherto. But due to significance of this issue, the various methods have been published in the papers which most of them have used simple assumptions. In Pro rata method [3], that is the most popular ones, the loss is allocated to each generator or load, regarding their power injection to network, rather than total network power injection. In fact, this method doesn't consider the location of them or network topology. So, a remote generator or load that certainly causes more power losses treats the same as other near network players. Proportional sharing principle is based on a non-provable or disprovable theorem that assumes the inflow powers are proportionally shared between the outflows power at each network bus [4]-[5]. This method uses an additional assumption, which losses of each branch allocate in 50 percent to its sending and ending nodes.

Ref [6] suggests a radial equivalent network for transmission system that each generator may have an individual connection to all loads, and thus enabling the allocation of system loss, but total losses may not equal to real system loss and also it is too complicated for real power systems. References [7-9] trace losses back from the network branch to the load. These strategies generally involve an algorithm to determine how the losses are attributed to generators/loads as one traverse through the network. Either the algorithm allows loss attribution to be specified according to a user-defined formula, or a loss sharing formula is implicitly included. The cooperative game theory was utilized to allocate transmission costs to wheeling transactions [10]. A method, based on circuit theory, has also been proposed to trace power from either the seller's and/or the buyer's point of view [11]. In [12], line power flows are first unbundled into a sum of components, each corresponding to a bilateral transaction. In these schemes, the coupling terms among the components appeared in the line losses can be allocated to individual bilateral transactions. In [13], a process is used whereby individual bilateral transactions are gradually incremented along a given path of variation. Each bilateral transaction may elect to have its losses supplied by a separate slack generator. In [9] starting from an AC load flow solution, the contributions of all generators to the flow in each circuit are evaluated and the same proportion is used to share circuit losses among them. The Z-bus loss allocation uses the total system loss formula and tries to write it in the summation form of each bus complex current injection [14]. In [15] a loss allocation method has been introduced in bilateral markets. In order to apply the loss allocation to contracts, this method uses the branch current circuit equations. In this paper, each contract contains a sending bus (seller) and several receiving buses (buyers). The loss allocation problem in multi-area transmission networks is studied in [16].

In this paper, the share of each of the players from the network losses has been proposed using the network's impedance matrix and the partial derivatives of the active power losses respect to bus currents coefficients. In the next section, the share of each of the buses of the network from the transmission losses is determined using these coefficients. In the third section, this method is tested on the IEEE 14-bus system. Finally, the concluding results are presented. Moreover, the share of each of the players from the network losses has been proposed using the network's impedance matrix and the partial derivatives of the active power losses respect to bus currents coefficients. In the next section, the share of each of the buses of the network from the transmission losses is determined using these coefficients. In the third section, this method is tested on the IEEE 14-bus system. Finally, the concluding results are presented.

2. Proposed Method

The loss allocation problem is intrinsically different from the loss compensation problem. In a pool-based market, ISO performs an economic load dispatch after the reception of other players' cost suggestions in order to minimize the operational costs of the system. In the loss allocation problem, it is tried to divide the loss costs between all of the parts of the system fairly. This cost allocation is performed after a complete load flow run. Supposing that the economic load dispatch has been done, the total losses of a network with n- buses can be expressed as follows:

$$P_{loss} = \sum_{k=1}^{n} P_k = \mathrm{Real}\left\{ \sum_{k=1}^{n} V_k I_k^* \right\} = \mathrm{Real}\left\{ \sum_{k=1}^{n} I_k^* \sum_{j=1}^{n} Z_{kj} I_j \right\} \quad (1)$$

The Z-bus matrix can be written in the form of equation (2):

$$Z_{kj} = R_{kj} + j\, X_{kj} \quad (2)$$

Replacing this equation in (1) and expressing the values in terms of their magnitude and angle, the total losses can be obtained as:

$$P_{loss} = \sum_{k=1}^{n}\sum_{j=1}^{n} |I_k| \times |I_j| \times R_{kj} \times Cos(\delta_k - \delta_j) \quad (3)$$

The above equation can be written in a matrix form:

$$P_{loss} = \sum \begin{bmatrix} P_{loss1,1} & P_{loss1,2} & \cdots & P_{loss1,n-1} & P_{loss1,n} \\ P_{loss2,1} & P_{loss2,2} & \cdots & P_{loss2,n-1} & P_{loss2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{loss\,n-1,1} & P_{loss\,n-1,2} & \cdots & P_{loss\,n-1,n-1} & P_{loss\,n-1,n} \\ P_{loss\,n,1} & P_{loss\,n,2} & \cdots & P_{loss\,n,n-1} & P_{loss\,n,n} \end{bmatrix} \quad (4)$$

The above loss matrix has diagonal and no diagonal elements which are as equation respectively (5) and (6):

$$P_{lossk,k} = R_{kk} \times |I_k|^2 \tag{5}$$

$$P_{lossk,j} = R_{kj} \times |I_k| \times |I_j| \times \cos(\delta_k - \delta_j) \tag{6}$$

Equation (5) which shows the current injection just by the k-th bus shows the self-loss of the k-th bus. On the other hand, equation (6) is a part of the network losses which happen due to the interaction of current injection by the k-th and j-th buses, which is called mutual loss between the k-th and j-th buses. Using partial derivatives of the active power losses respect to bus currents coefficients of the equation, the sensitivity of losses to the injected currents of buses are given as (7):

$$\partial P_{loss} / \partial I_k = \{\cos\delta_k - j\sin\delta_k\} \times \left\{ R_{kk} I_k + \sum_{j=1}^{n} R_{kj} \times |I_j| \times \cos(\delta_i - \delta_j) \right\} \tag{7}$$

The magnitude of the above equation is as (8):

$$\left| \partial P_{loss} / \partial I_k \right| = \left| R_{kk} I_k + \sum_{j=1}^{n} R_{kj} \times |I_j| \times \cos(\delta_i - \delta_j) \right| \tag{8}$$

Table IV shows these partial derivatives of the active power losses respect to bus currents coefficients. These losses are properly and fairly shared between network buses for fair loss allocation, in proportion to partial derivatives of the active power losses respect to bus currents coefficients. Using equation (8), the share of each of the k-th and j-th buses from the mutual losses can be expresses as:

$$P_{lossk,j}^{k} = P_{lossk,j} \times \frac{\left| \partial P_{loss} / \partial I_k \right|}{\left| \partial P_{loss} / \partial I_k \right| + \left| \partial P_{loss} / \partial I_j \right|} \tag{9}$$

$$P_{lossk,j}^{j} = P_{lossk,j} \times \frac{\left| \partial P_{loss} / \partial I_j \right|}{\left| \partial P_{loss} / \partial I_k \right| + \left| \partial P_{loss} / \partial I_j \right|} \tag{10}$$

Separation of the losses of each bus from the mutual loss elements is based on the coefficients of equation (8). Considering the above equations, the share of the k-th bus from the total network losses can be stated as follows:

$$P_{loss}^{k} = P_{lossk,k} + 2 \times \sum_{j=1}^{n} P_{lossk,j}^{k} \tag{11}$$

On the other hand, the total losses of the network are:

$$P_{loss} = \sum_{k=1}^{n} P_{loss}^{k} \tag{12}$$

3. Worked Example

A simple example without fixed losses is selected to show the application of the proposed allocation method. Fig. 1 shows a 3-bus system and Table I shows its transmission line data which is used for this purpose. Generator (located at buses 1) supplies the power demand located at buses 2 and 3.

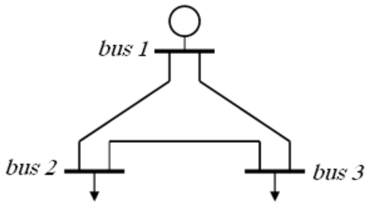

Fig.1: 3-Bus System

Table I

Three-bus system: transmission line data

Line From Bus to Bus	R (%)	X (%)	B (%)
1-2	0.0200	0.040	0.025
1-3	0.0100	0.030	0.025
2-3	0.0125	0.025	0.025

Table II summarizes the power flow solution by the Newton–Raphson method. Columns 2, 3, 4, 5, 6 and 7 show respectively, bus magnitude voltages, bus angle voltages, active generated powers, reactive generated powers, active demand powers and reactive demand powers.

Table II

Three-bus system, power flow results

Bus. No	Vol	Ang	PG (MW)	QG (MVAr)	PD (MW)	QD (MVAr)
1	1.05	0.0000	409.2289	172.963	0.000	0.000
2	0.984	-3.539	0.0000	0.00000	256.6	110.2
3	1.003	-2.892	0.0000	0.00000	138.6	45.20
Total Sum			409.2289	172.963	395.20	155.40

Loss allocation to each bus of the typical 3-bus network is illustrated in Table III. As shown in Fig. 1, bus 3 injects the current in the opposite direction with respect to the resultant current of the network in line 2-3. So, the allocated loss of the line 2-3 to the bus 3 has a negative value. The negative allocated loss to the bus 3 is due to its decreasing role in reduction of the network losses. On the other hand, if this bus increases the network losses, it receives the positive loss allocation cost.

Table III

The allocated loss of transmission lines to each bus of the typical 3-bus network

Line	Line loss(MW)	Share bus 1	Share bus 2	Share bus 3
1-2	8.3400	5.0339	3.2328	0.0663
1-3	4.8943	3.5211	0.3369	1.0363
2-3	0.7946	0.1895	1.0811	-0.4760

For more description, the active load of all buses has increased. For each case, the loss allocation by using the proposed method has been done. The variations of allocated loss to each bus and the network line losses due to the load increase of bus 2 from zero to 1000 MW have been illustrated in Figs. 2 and 3 respectively.

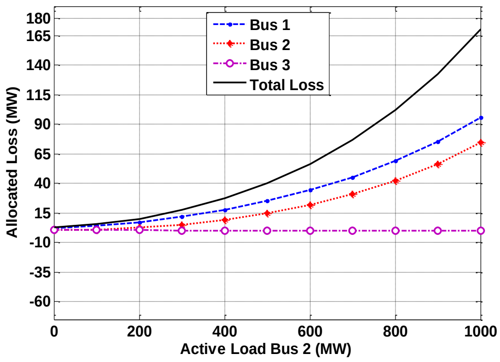

Fig.2: Variation of allocated loss to each bus due to the load increase of the bus

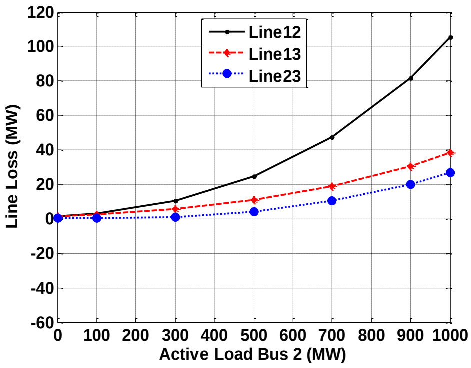

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Fig.3: Variation of the network line losses due to the load increase of the bus 2

As shown in Figs.2 and 3, by increasing the load in bus 2, the power flows in lines and proportionally the network line losses have increased. Thus, the allocated losses to buses 1 and 2 from the line losses have been increased. By increasing the load of bus 2, the power flow in line 3-2 from bus 3 toward bus 2 has increased. Therefore, the load of bus 3 has a decreasing role in flowing power of line 3-2. So, the share of bus 3 in the allocated loss should be constant that has been yield by the proposed method.

4. Numerical Study

In order to show the results of the proposed method and compare it with other methods, the IEEE 14-bus system has been chosen. As can be observed in Fig. 4, the IEEE 14-bus system has 5 voltage controlled buses and 2 generators. The bus no.1 is chosen as the slack bus.

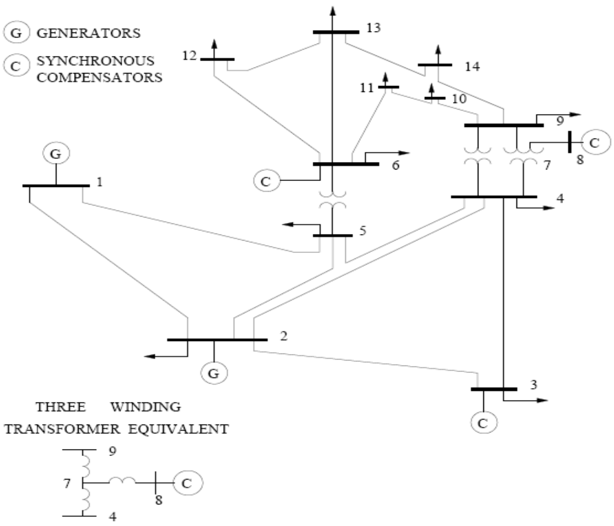

Fig..4: 14-Bus IEEE test system

The load flow results of Table IV show that 13.4 MW of losses are supplied by bus no.1 which should be divided between the market customers.

Table VI

Results of a normal load flow analysis

Bus Number	Z-bus method	ITL method	Pro-rata method	Proposed method
1	7.800	6.14	6.46	9.68
2	0.155	0.96	0.50	0.04
3	2.698	2.92	2.62	1.55
4	0.9056	1.26	1.36	0.57
5	0.0903	0.18	0.22	0.01
6	0.6783	0.32	0.32	0.39
7	0.000	0.000	0.000	0.000
8	0.0258	0.000	0.000	0.09
9	0.4484	0.68	0.82	0.20
10	0.1690	0.20	0.24	0.08
11	0.0620	0.08	0.10	0.01
12	0.1385	0.18	0.16	0.07
13	0.3412	0.32	0.38	0.26
14	0.4689	0.32	0.42	0.44
sum	13.39	13.39	13.39	13.39

Table V shows the results of the proposed method in comparison to the other methods. Considering this equation and the values of Table IV, it is seen that the buses no. 5, 8, 11 have the minimum rate of loss changes in response to current injection which shows that the corresponding buses act in the direction of loss reduction. Therefore, fewer shares of losses should be assigned to these buses. On the other hand, Table VI shows that these buses have the least share of allocated losses in the proposed method which proves the fairness of this method in comparison to the other methods. Furthermore, bus no.1 has the largest rate of loss changes to current injection and in most of the methods; this bus has the largest share of losses. As can be seen in Fig. 2, by changing the generation of bus no.8 from zero to 300 MW the network total loss has been decreased at first and then increased. Similarly, the allocated loss to bus no.8 has been decreased and then increased. This fact shows that the proposed method considers the network topology and the injected current. The other note has been involved in this paper and other papers, is the negative loss allocation to some network buses. This subject is due to the mutual and dominant flows. Here, this question is introduced if the negative loss allocation can be accepted or not. In fact, the answer of this question depends to kind of market and the available players in it. If the negative loss allocation to some buses be accepted, these negative loss allocation signals can be used in order to reduce the total network losses. In real power systems, there are some loads with low power factor in which cause to increase the network losses and reduction transmission line capacity. To illustrate the proposed method can consider these conditions, the reactive load of bus no.14 increased from 5 MVAR to 50 MVAR. The effect of this increase on the connected line, line 13-14, and the allocated loss to bus no.14 has been studied. As can be seen from Fig. 3, the losses of line 13-14 and the allocated loss to the bus no.14 is increased with the increase the reactive load of bus no.14.

Table V

Results of loss allocation from different methods and proposed method

Bus Num	voltage	angle	Pg	Qg	Pd	Qd
1	1.060	0.000	232.4	-16.5	0.00	0.00
2	1.045	-4.983	40.00	30.86	21.7	12.7
3	1.010	-12.72	0.000	6.000	94.2	19.0
4	1.018	-10.31	0.000	-3.90	47.8	-3.90
5	1.020	-8.774	0.000	-1.60	7.60	1.60
6	1.070	-14.22	0.000	5.000	11.2	7.50
7	1.062	-13.36	0.000	0.000	0.00	0.00
8	1.090	-13.36	0.000	17.62	0.00	0.00
9	1.056	-14.93	0.000	-16.6	29.5	16.6
10	1.051	-15.09	0.000	-5.80	9.00	5.80
11	1.057	-14.79	0.000	-1.80	3.50	1.80
12	1.055	-15.07	0.000	-1.60	6.10	1.60
13	1.050	-15.15	0.000	-5.80	13.5	5.80
14	1.036	-16.03	0.000	-5.00	14.9	5.00
sum			272.4	82.44	259.0	73.5

Table IV

Partial derivatives of the active power losses with respect to the bus currents coefficients

Bus Number	$\left\| \partial P_{loss} / \partial I_k \right\|$
1	0.2500
2	0.0076
3	0.0244
4	0.0273
5	0.0045
6	0.0358
7	0.0000
8	0.0007
9	0.0156
10	0.0120
11	0.0066
12	0.0176
13	0.0294
14	0.0374

5. Conclusion

In this paper, a fair method has been proposed to allocate the transmission losses in a power system using its circuit equations and simplifying them. This method divided the losses between the players of a pool-based market using the network impedance matrix and the partial derivatives of the active power losses respect to bus currents coefficients. The method is based on load flow and the following principles:

a) Incorporates the main equations of the power system in conjunction with the network impedance matrix and the vectors of the injected currents of the buses.
b) Uses the partial derivatives of the active power losses respect to bus current coefficients for fair allocation of losses between the network customers.
c) It is a simple and easily understandable method.

The proposed method in this paper doesn’t consider any bus or buses compensating the network total losses. It is actually independent of the slack bus and divides the losses between the market players considering the penetration percent of them in the network. The method separates the self and mutual losses and is therefore applicable in other forms of the power system such as multi-transaction contract markets. It can actually be used to compensate the losses by the buses, themselves. Finally, the proposed method has been tested on the IEEE 14-bus system and fair results have been achieved in comparison to the other methods.

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