MULTIPLE CLASS SYNCHRONIZATION OF FRACTIONAL-ORDER UNCERTAIN CHAOTIC SYSTEMS

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In this paper, a sliding mode controller is proposed for synchronization of fractional-order chaotic systems in the presence of uncertainty. First, a class of three-dimensional fractional-order chaotic systems has been studied and then sliding mode controller is designed to guarantee asymptotically stable presence of uncertainty. In addition, control of fractional-order Lu & Chen system is implemented by this method. Simulation results confirm numerical results.

Keywords: fractional-order chaotic systems, uncertainty, chaos synchronization

1. Introduction

Chaos phenomenon can be considered as one of the hot issues in many applications such as medical [1-3], pharmaceutical [4], laser [5, 6], and economic systems [7, 8]. So chaos can be part of a group of the most fascinating subjects which has attracted wide attention in recent years. Fractional calculus was introduced almost 300 years ago. But, in recent decades, the study of fractional calculus has attracted the wide attention of researchers as a branch of mathematics [9-13]. Fractional-order chaotic systems occur as nonlinear phenomena in many scientific fields, such as chaotic behavior in financial systems [14, 15] and many articles were published in the field of fractional-order chaotic systems [16-18]. Today, control and synchronization of fractional-order chaotic systems is one of the most interesting topics that attracted the attention of researchers in the past decade. For example, in [19], a fractional order sliding mode controller is implemented for unknown chaotic fractional order systems. Synchronization strategies of a three-dimensional chaotic finance system are investigated in [20]. In [21] a non-fragile state feedback controller is applied for the fractional order synchronization of a new chaotic system. Synchronization of chaos has been studied for fractional-order Liu system [22]. Also, in [23] the chaotic synchronization of Genesio-Tesi system is studied utilizing two strategies; active control and sliding mode. Projective synchronization of fractional order chaotic systems with non-identical orders is investigated in [24]. In [25] a robust observer

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was used for synchronization of integer order and fractional order Chua's systems. Concept of synchronization of different fractional order chaotic systems using active control technique is demonstrated in [26]. In [28] a fractional-order scalar controller which involves only one state variable is proposed. Also, some circuits are designed to realize proposed control schemes.

In this paper, a set of fractional order chaotic systems with uncertainty is considered, which can include a variety of fractional order chaotic systems such as Chen, Lorenz, Lu, Liu and financial. To control and synchronize fractional order uncertain chaotic systems, a fractional order sliding mode controller is proposed. The fractional-order sliding mode controller investigated is asymptotically stable in the presence of uncertainty. Simulation results clearly show that the proposed sliding mode controller has the ability to eliminate chaos and mitigate Chattering phenomenon.

2. Fractional-order calculus

Derivative operator - integrator is characterized by ${}_aD_t^q$, a combination of differential-integral operator used in the calculations. The operator is a symbol to represent the fractional integral and fractional derivative expressed in a phrase and is defined as follows:

$$ {}_aD_t^q = \begin{cases} \dfrac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int_a^t (dt)^{-q} & q < 0 \end{cases} $$

where q is the fractional order. There are various definitions for fractional derivative and integral. The most common definitions are Grunwald–Letnikov definition, Riemann–Liouville definition and Caputo definition. In the rest of this paper, Riemann-Liouville (RL) definition of derivative is used. RL derivative in the order of q is explained below ([27]):

$$ {}_0D_t^q f(t) = D^q f(t) = \frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma(m-q)} \frac{d^m}{dt^m} \int_0^t \frac{f(\tau)}{(t-\tau)^{q+m-1}} d\tau $$

where m is the first integer which is not less than q, i.e. $m-1 \le q < m$ and $\Gamma(.)$ is the well-known Euler's gamma function

$$\Gamma(P)=\int_0^{\infty} t^{p-1}e^{-t}dt \quad ; \quad \Gamma(P+1)=P\Gamma(P)$$

3. System description

A class of three-dimensional fractional-order chaotic systems is given by:

$$\begin{cases} \dfrac{d^{q_1}x}{dt^{q_1}} = y.f\ (x,y,z)+z.\phi(x,y,z)-\alpha x, \\ \dfrac{d^{q_2}y}{dt^{q_2}} = g(x,y,z)-\beta y, \\ \dfrac{d^{q_3}z}{dt^{q_3}} = y.h(x,y,z)-x.\phi(x,y,z)-\gamma z, \end{cases} \tag{1}$$

where $q_i \quad (i=1,2,3)$ are fractional orders satisfying $0<q_i<1$; x, y and z are state variables. Each of the four functions $f(.), g(.), h(.)$ and $\phi(.)$ is considered as continuation nonlinear vector functions belonging to $R^3 \to R$ space, and α, β, γ are known constants, for any negative or positive values.

Remark 3.1. If $q_1=q_2=q_3=q$, fractional-order system (1), is called a commensurate fractional-order system. Otherwise, it is called incommensurate fractional-order system.

Remark 3.2. Note that many fractional-order chaotic systems belong to the class characterized by (1). Examples include the fractional-order financial system, the unified chaotic system of fractional-order version (including the fractional-order Chen system, fractional-order Lu's system). Table 1 shows that these fractional-order chaotic models can be described by the proposed system (1).

In this paper, we consider the master system in the presence of uncertainty (h_1) as follows:

$$\begin{aligned} \frac{d^{q_1}x_1}{dt^{q_1}} &= x_2 f\ (x_1,x_2,x_3)+x_3\phi(x_1,x_2,x_3)-\alpha x_1 \\ \frac{d^{q_2}x_2}{dt^{q_2}} &= g(x_1,x_2,x_3)-\beta x_2+h_1 \\ \frac{d^{q_3}x_3}{dt^{q_3}} &= x_2 h(x_1,x_2,x_3)-x_1\phi(x_1,x_2,x_3)-\gamma x_3 \end{aligned} \tag{2}$$

where x_1, x_2, x_3 are state variables. Also, adding a control input $u(t)$ and h_2 uncertainty to the system (1), the slave system would be as follows:

$$\begin{aligned}
\frac{d^{q_1} y_1}{dt^{q_1}} &= y_2 f\ (y_1, y_2, y_3) + y_3 \phi(y_1, y_2, y_3) - \alpha y_1 \\
\frac{d^{q_2} y_2}{dt^{q_2}} &= g(y_1, y_2, y_3) - \beta y_2 + h_2 + u(t) \\
\frac{d^{q_3} y_3}{dt^{q_3}} &= y_2 h(y_1, y_2, y_3) - y_1 \phi(y_1, y_2, y_3) - \gamma y_3
\end{aligned} \tag{3}$$

where y_1, y_2, y_3 are state variables.

Remark 3.3. We assume that $f(.), g(.), h(.)$ and $\phi(.)$ are required to ensure that the fractional-order system (3) with control input $u(t)$ has a unique solution in the time interval $[T, +\infty]$, $T > 0$ for any given initial condition.

Table 1.

List of published fractional-order chaotic systems, which can be described by the proposed general model.

Name	Model	$f(x,y,z)$	$g(x,y,z)$	$h(x,y,z)$	$\phi(x,y,z)$
Chen's system	$\begin{cases} D^{q_1} x = a(y-x) \\ D^{q_2} y = dx - xz + cy \\ D^{q_3} z = xy - bz \end{cases}$	a	$dx - xz$	x	0
Lorenz model	$\begin{cases} D^{q_1} x = a(y-x) \\ D^{q_2} y = x(b-z) - y \\ D^{q_3} z = xy - cz \end{cases}$	a	$x(b-z)$	x	0
Financial system	$\begin{cases} D^{q_1} x = z + (y-a)x \\ D^{q_2} y = 1 - by - x^2 \\ D^{q_3} z = -x - cz \end{cases}$	x	$1 - x^2$	0	1
Lu's model	$\begin{cases} D^{q_1} x = a(y-x) \\ D^{q_2} y = -xz + cy \\ D^{q_3} z = xy - bz \end{cases}$	a	$-xz$	x	0
Liu system	$\begin{cases} D^{q_1} x = -ax - ey^2 \\ D^{q_2} y = -kxz + by \\ D^{q_3} z = mxy - cz \end{cases}$	$-ey$	$-kxz$	mx	0

4. Fractional-order sliding mode controller design

One of the control methods for nonlinear systems is sliding mode control. This approach is usually utilized to face indefinite and uncertain systems. System design using sliding mode control method consists of two steps: first, the vector of sliding surface is defined, second, the control signal to reach sliding surface is determined. Sliding surface chosen is as follows:

$$s(t) = D^{q_2-1}e_2(t) + D^{-1}\psi(t) = D^{q_2-1}e_2(t) + \int_0^t \psi(\tau)d\tau \tag{4}$$

$D^{q_2-1}e_2(t)$ and $\psi(t)$ functions are described:

$$D^{q_2-1}e_2(t) = D^{q_2-1}y_2(t) - D^{q_2-1}x_2(t) \tag{5}$$

$$\begin{aligned}\psi(t) &= [y_1 f(y_1, y_2, y_3) + y_3 h(y_1, y_2, y_3) + \beta y_2] \\ &-[x_1 f(x_1, x_2, x_3) + x_3 h(x_1, x_2, x_3) + \beta x_2]\end{aligned} \tag{6}$$

In the sliding mode, the sliding surface and its derivative must satisfy:

$$s(t) = 0, \qquad \dot{s}(t) = 0 \tag{7}$$

So we conclude from (7):

$$\dot{s}(t) = \frac{d}{dt}s(t) = \frac{d}{dt}[D^{q_2-1}e_2(t) + D^{-1}\psi(t)] = D^{q_2}e_2(t) + \psi(t) = 0 \tag{8}$$

From Equation (8), we have:

$$\begin{aligned}&D^{q_2}y_2(t) - D^{q_2}x_2(t) = -\psi(t) = \\ &-[y_1 f(y_1, y_2, y_3) + y_3 h(y_1, y_2, y_3) + \beta y_2] \\ &+[x_1 f(x_1, x_2, x_3) + x_3 h(x_1, x_2, x_3) + \beta x_2]\end{aligned} \tag{9}$$

Thus, u_{eq} will be as follows:

$$\begin{aligned}u_{eq}(t) &= -g(y_1, y_2, y_3) + g(x_1, x_2, x_3) - y_1 f(y_1, y_2, y_3) - y_3 h(y_1, y_2, y_3) \\ &+x_1 f(x_1, x_2, x_3) + x_3 h(x_1, x_2, x_3) - \Delta h\end{aligned} \tag{10}$$

where $\Delta h = h_2 - h_1$.

The next step to satisfy the sliding condition, the discontinuous reaching law is chosen as follows:

$$u_r(t) = -k_r sign(s) \tag{11}$$

where

$$sign(s)=\begin{cases}1 & s>0\\ 0 & s=0\\ -1 & s<0\end{cases}$$

and k_r is the gain of the controller. Finally, the switching control is calculated from (10) and (11) and we get

$$\begin{aligned}&u(t)=u_{eq}(t)+u_r(t)\\&=-g(y_1,y_2,y_3)+g(x_1,x_2,x_3)-y_1f(y_1,y_2,y_3)-y_3h(y_1,y_2,y_3)\\&+x_1f(x_1,x_2,x_3)+x_3h(x_1,x_2,x_3)-\Delta h-k_r sign(s)\end{aligned}\tag{12}$$

Theorem1. *Exploiting control law (12) and selecting gain properly* $k_r>0$*, proposed sliding surface is asymptotically stable in presence of uncertainty.*

Proof. Selecting a Lyapunov candidate $V=\frac{1}{2}s^2(t)$. We have

$$\begin{aligned}&\dot{V}=s\dot{s}=s[D^{q_2}e_2(t)+\psi(t)]\\&=s[g(y_1,y_2,y_3)-\beta y_2-g(y_1,y_2,y_3)+g(x_1,x_2,x_3)-y_1f(y_1,y_2,y_3)\\&-y_3h(y_1,y_2,y_3)+x_1f(x_1,x_2,x_3)+x_3h(x_1,x_2,x_3)-\Delta h-k_r sign(s)\\&-g(x_1,x_2,x_3)+\beta x_2+\Delta h+y_1f(y_1,y_2,y_3)+y_3h(y_1,y_2,y_3)+\beta y_2\\&-x_1f(x_1,x_2,x_3)-x_3h(x_1,x_2,x_3)-\beta x_2]=-k_r|s|<0\end{aligned}\tag{13}$$

As it was observed, to obtain $s\dot{s}<0$, choose the gain as $k_r>0$.

5. Simulation result

First, the electrical circuit realizing the multi fractional-order chaotic system has been designed in reference [28], where the existence of chaos in this system has been demonstrated in an analytical, simulated form. The circuit is designed as mentioned blew to realize the fractional-order Chen system (14).

In this section, the effect of sliding mode control for synchronization of chaotic systems of Chen and Lu is shown.

Example 1. In this example, the fractional-order Chen system (1) is taken, we have master system in the presence of uncertainty $h_1=0.4\sin x_1\cos x_2$ and slave system in $h_2=0.35\,\mathrm{siny}_1\,\mathrm{siny}_2$ uncertainty as below.

$$\begin{aligned} D^{q_1}x_1 &= a(x_2 - x_1) \\ D^{q_1}x_2 &= dx_1 - x_1x_3 + cx_2 + h_1 \\ D^{q_1}x_3 &= x_1x_2 - bx_3 \end{aligned} \tag{14}$$

$$\begin{aligned} D^{q_1}y_1 &= a(y_2 - y_1) \\ D^{q_1}y_2 &= dy_1 - y_1y_3 + cy_2 + h_2 + u(t) \\ D^{q_1}y_3 &= y_1y_2 - by_3 \end{aligned} \tag{15}$$

where $(a,b,c,d) = (35,3,28,-7)$. Simulation was done with initial conditions $\left[{}_0D_t^{-0.05}x_1(0),\ {}_0D_t^{-0.05}x_2(0),\ {}_0D_t^{-0.05}x_3(0)\right]^T = [-9,-5,14]^T$ in system (14) and $\left[{}_0D_t^{-0.05}y_1(0),\ {}_0D_t^{-0.05}y_2(0),\ {}_0D_t^{-0.05}y_3(0)\right]^T = [10,-14,-8]^T$ in system (15) and fractional-order $q = [0.95, 0.95, 0.95]$ and controller $k_r = 2$. Chaotic behavior of the system (14) without uncertainty is shown in Fig. 1. Given the choice of sliding surface (4) and the control law (12) for the fractional-order Chen system, we have:

$$\begin{aligned} s(t) &= D^{q_2-1}y_2(t) - D^{q_2-1}x_2(t) + \int_0^t [ay_1(\tau) + y_1(\tau)y_3(\tau) - \mathrm{c}\,y_2(\tau)] \\ &-[ax_1(\tau) + x_1(\tau)x_3(\tau) - \mathrm{c}\,x_2(\tau)]d(\tau) \end{aligned} \tag{16}$$

$$\begin{aligned} u(t) &= u_{eq}(t) + u_r(t) = -dy_1 + y_1y_3 + dx_1 - x_1x_3 - ay_1 - y_1y_3 \\ &+ax_1 + x_1x_3 - \Delta h - k_r sign(s) = (a+d)(x_1 - y_1) - \Delta h - k_r sign(s) \end{aligned} \tag{17}$$

The simulation results are shown in Figs. 2-4. Fig. 2 gives error states in synchronization. Fig. 3. shows Sync on the effectiveness of the sliding mode control system variables $x_1, y_1, x_2, y_2, x_3, y_3$. Fig. 4 shows control input.

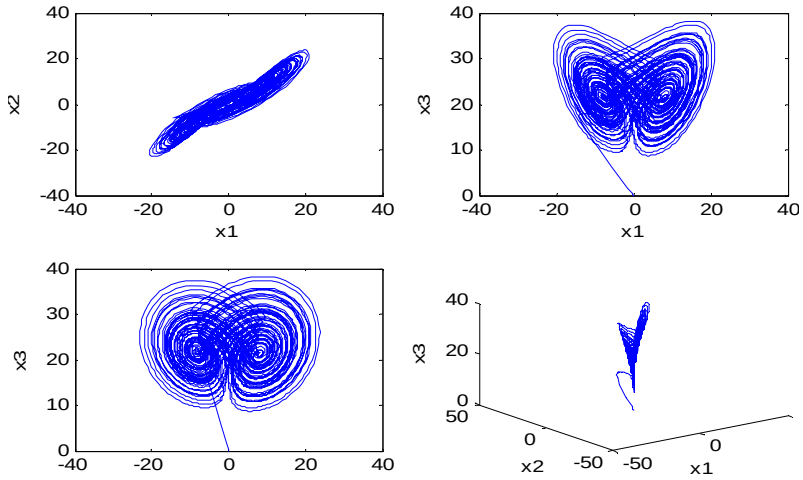

Fig. 1. Phase diagram of Chen system with fractional order q= [0.95, 0.95, 0.95]

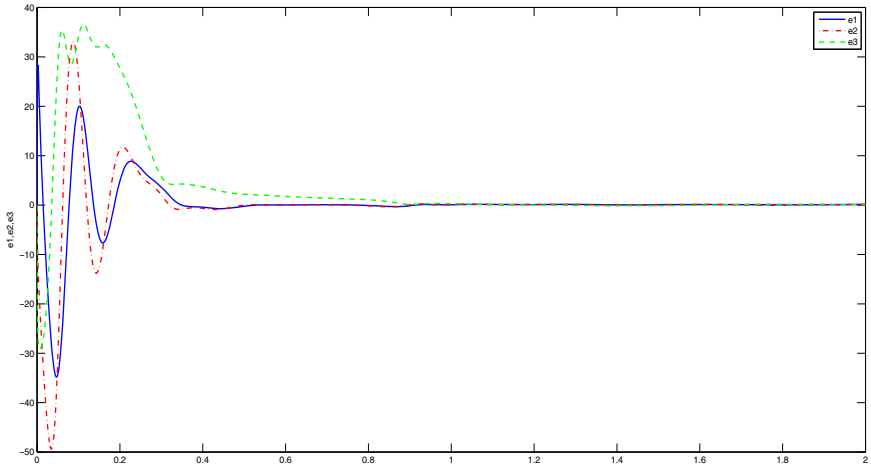

Fig. 2 error states in synchronization

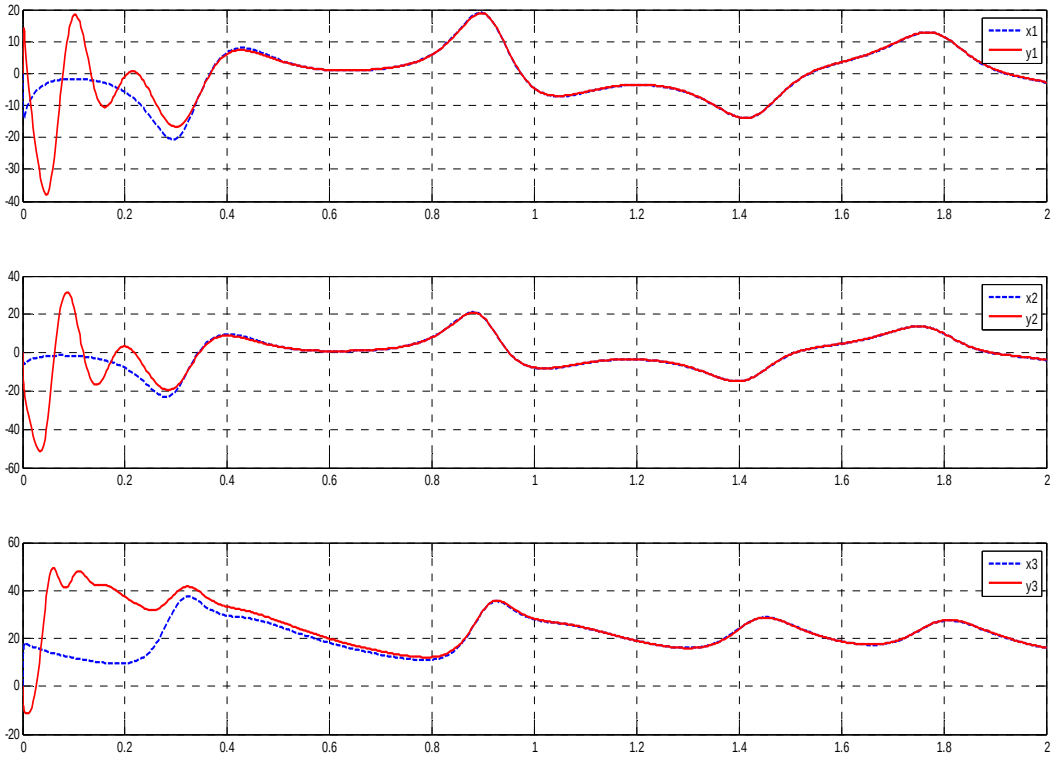

Fig. 3.State trajectories of master and slave in the synchronization

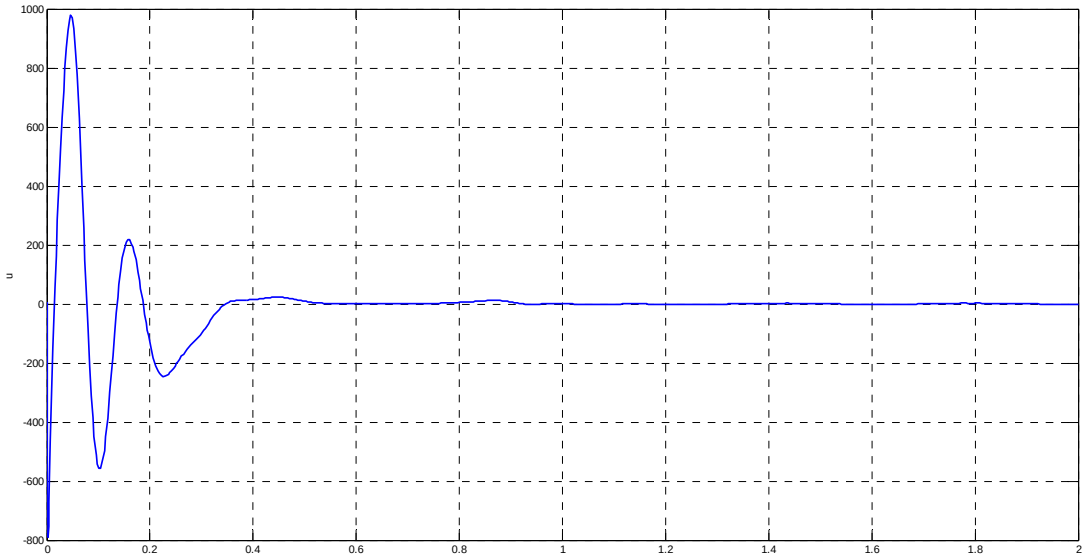

Fig. 4. Control function in synchronization procedure

Example2. In this example, the fractional-order Lu system (1) is taken, we have master system in the presence of uncertainty $h_1 = 0.4\sin x_1 \cos x_2$ and slave system in $h_2 = 0.45\,\mathrm{siny}_1\,\mathrm{cosy}_2$ uncertainty as below.

$$\begin{aligned} D^{q_1}x_1 &= a(x_2 - x_1) \\ D^{q_2}x_2 &= -x_1x_3 + cx_2 + h_1 \\ D^{q_2}x_3 &= x_1x_2 - bx_3 \end{aligned} \tag{18}$$

$$\begin{aligned} D^{q_1}y_1 &= a(y_2 - y_1) \\ D^{q_2}y_2 &= -y_1y_3 + cy_2 + h_2 + u(t) \\ D^{q_2}y_3 &= y_1y_2 - by_3 \end{aligned} \tag{19}$$

where $(a,b,c) = (36,3,20)$. Simulation was done with initial conditions $\left[{}_0D_t^{-0.08}x_1(0),\ {}_0D_t^{-0.07}x_2(0),\ {}_0D_t^{-0.07}x_3(0)\right]^T = [5,-9,9]^T$ in system (18) and $\left[{}_0D_t^{-0.08}y_1(0),\ {}_0D_t^{-0.07}y_2(0),\ {}_0D_t^{-0.07}y_3(0)\right]^T = [-7,6,3]^T$ in system (19) and fractional-order $q = [0.92, 0.93, 0.93]$ and controller $k_r = 2$. Chaotic behavior of the system (18) without uncertainty is shown in Fig. 5. Given the choice of sliding surface (4) and the control law (12) for the fractional-order Lu system, we have:

$$\begin{aligned} s(t) &= D^{q_2-1}y_2(t) - D^{q_2-1}x_2(t) + \int_0^t [ay_1(\tau) + y_1(\tau)y_3(\tau) - \mathrm{c}\,y_2(\tau)] \\ &-[ax_1(\tau) + x_1(\tau)x_3(\tau) - \mathrm{c}\,x_2(\tau)]d(\tau) \end{aligned} \tag{20}$$

$$\begin{aligned} u(t) &= u_{eq}(t) + u_r(t) = y_1y_3 - x_1x_3 - ay_1 - y_1y_3 + ax_1 + x_1x_3 - \Delta h \\ &-k_r\,sign(s) = a(x_1 - y_1) - \Delta h - k_r\,sign(s) \end{aligned} \tag{21}$$

Fig. 6 gives error states in synchronization. Fig. 7. shows Sync on the effectiveness of the sliding mode control system variables $x_1, y_1, x_2, y_2, x_3, y_3$. Fig. 8 shows control input. Simulation results represent the effectiveness of the sliding mode controller on fractional-order Chen and Lu systems in master-slave structure.

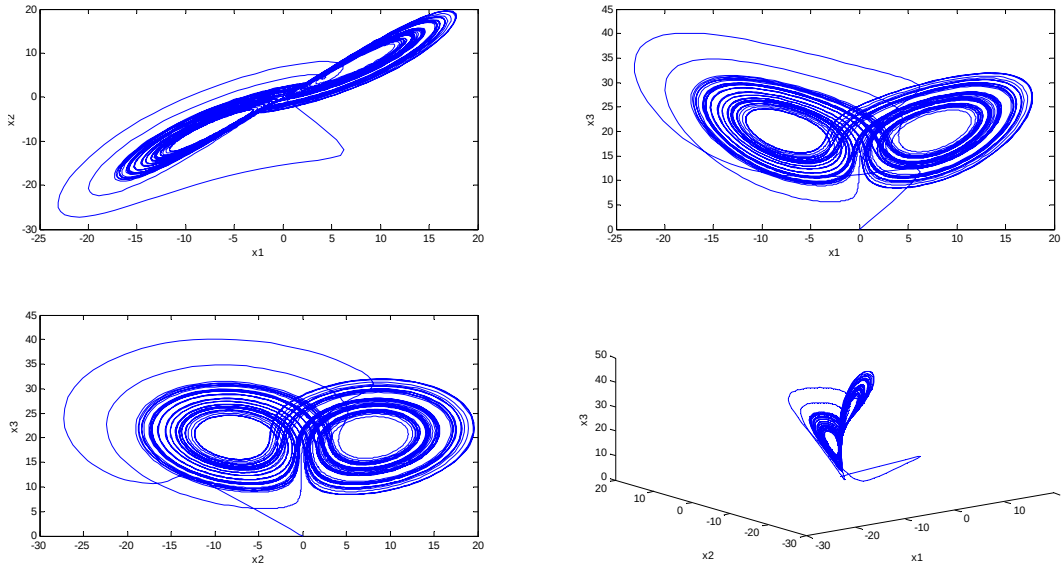

Fig. 5. Phase diagram of Lu system with fractional order q= [0.92, 0.93, 0.93]

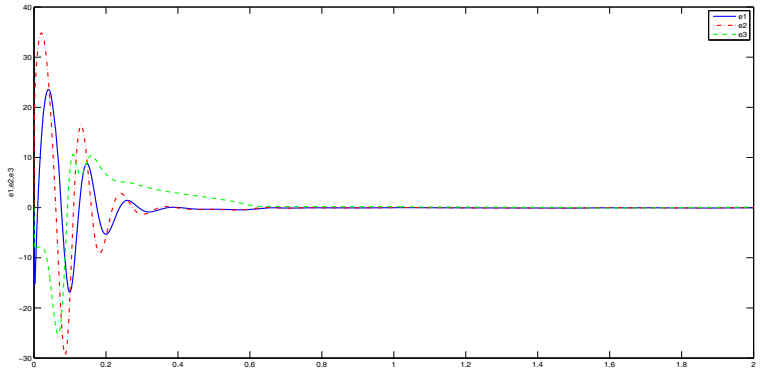

Fig. 6 error states in synchronization

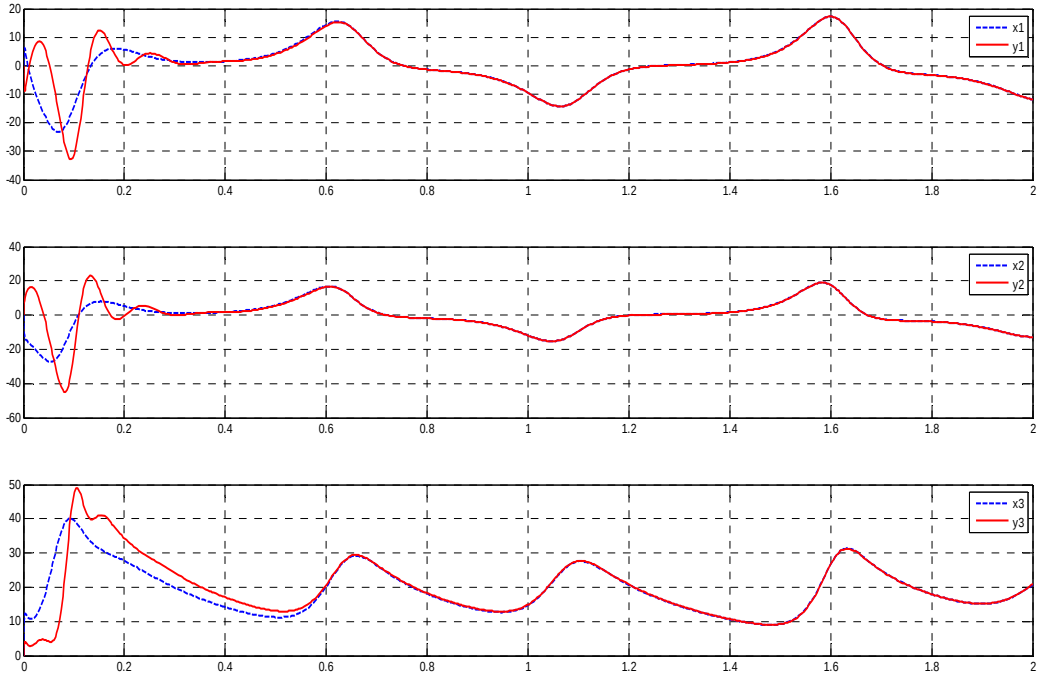

Fig. 7.State trajectories of master and slave in the synchronization

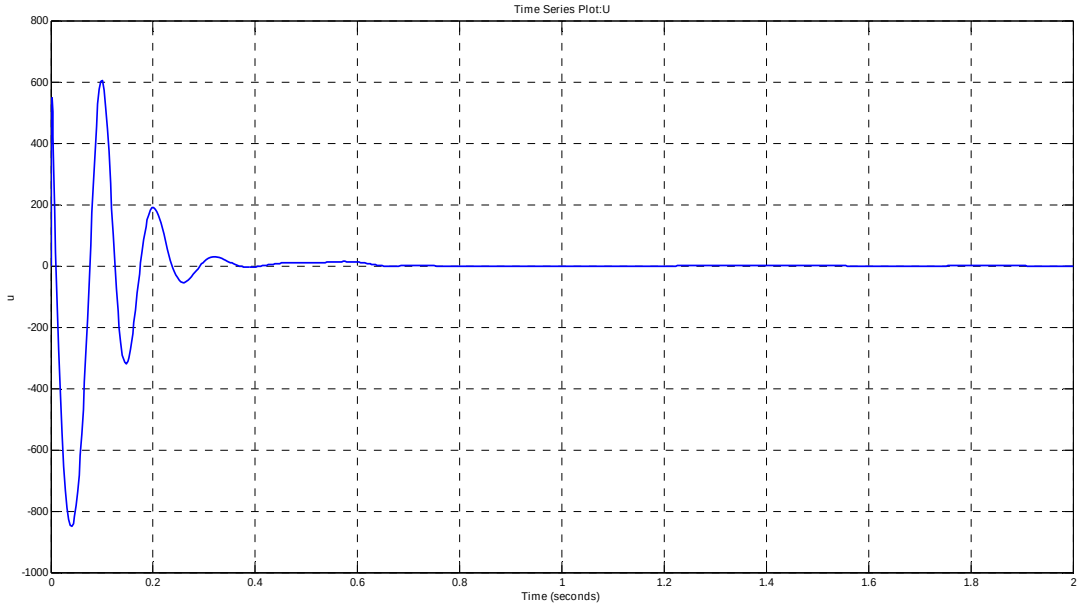

Fig. 8. Control function in synchronization procedure

6. Conclusions

In this paper, a fractional order sliding mode controller was proposed for fractional-order chaotic systems in the presence of uncertainty. The asymptotic stability of the proposed controller is investigated. Effectiveness of the sliding mode controller is clearly evident in eliminating errors and reducing chattering phenomenon. Numerical results confirm the theoretical hypotheses.

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