U.P.B. Sci. Bull., Series D, Vol. 74, Iss.4, 2012 ISSN 1454-2358

PERIPHERAL SURFACE PROFILING OF THE HOB CUTTER FOR SCREW COMPRESSOR ROTORS COMPONENTS I. FEMALE ROTOR

Camelia POPA[1], Nicolae OANCEA[2]

Profilarea suprafeţelor periferice a sculei melc, reciproc înfăşurătoare cu suprafeţele elicoidale ale flancurilor rotoarelor compresorului elicoidal, constituie o problemă de generare de speţa a II-a referitoare la suprafeţele reciproc înfăşurătoare cu contact punctiform. O astfel de problemă se rezolvă apelând la metoda suprafetei intermediare.

Autorii au propus o metodă de profilare a suprafeţei periferice primare a sculei melc generatoare a rotorului condus, pe baza teoremelor referitoare la descompunerea unei mişcări elicoidale.

Sunt prezentate formele constructive ale lobilor rotorului condus, forma cremalierei generatoare şi forma profilului axial al sculei melc, reciproc înfăşurătoare acesteia. De asemenea, sunt prezentate exemple numerice ale formelor axiale ale sculelor melc generatoare pentru rotorul condus, pentru compresoare cu raport de transmitere 4/6, respectiv 3/5.

Peripheral surface profiling of the hob cutter represents a problem of second degree, referring to the contact point of surfaces.

In this paper, according to theorems referring to the decomposition of a helical motion, we propose a profiling method of the peripheral surface of the hob cutter for the female rotor from the screw compressor component.

The constructive form of the female rotor lobes, the shape of the generating rack and the shape of the axial profile of the hob cutter are presented. Also, based on a soft product, we present frontal applications of the axial shapes of the generating hob cutter for the female rotor, component of screw compressor, gear ratio 4/6, and 3/5.

Keywords: helical surfaces, generating rack, polynomials Bézier, hob cutter.

1. Introduction

Methods of profiling tool drives, envelope a vortex of surfaces associated with a couple in rolling axodes, are: Oliver II theorem, for enveloping surfaces with contact point, and Gohman theorem, through the intermediate surface; helical movement decomposition method, Nicolaev [1], [4], [6], [8]. We also proposed 2D and 3D graphic solutions in order to solve these problems [8],[10].The generation process of the surfaces constituent of the rotor lobes, can start by

[1] Assist., Department of Machine Elements and Graphics, "Dunărea de Jos" University of Galati, Romania, e-mail: lpopa@ugal.ro

[2] Prof., Department of Manufacturing, Robotics and Welding Engineering, "Dunărea de Jos" University of Galati, Romania, e-mail: noancea@ugal.ro

imposing a known shape of the generating rack , which must meet some requests, as follows: a convenient shape of the contact line [9]; a reduced length of the contact line [10],[11]; technological demands – lack of critical points to the profile of the generating rack , that would lead to discontinuities on the generated profile. In Fig. 1, the shape and size of the parameters of the generating rack in cross section are presented.

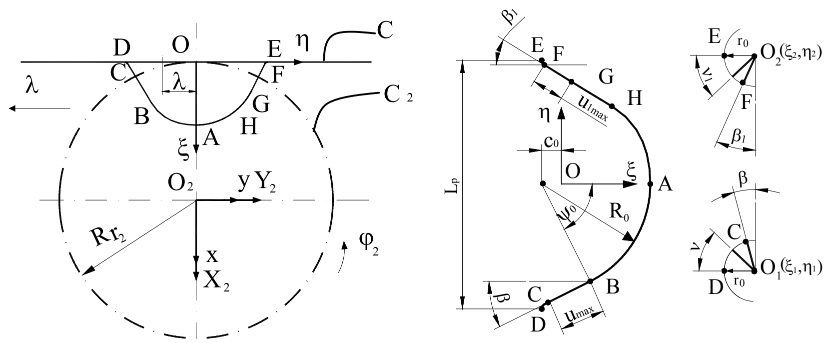

Fig. 1. Generation the transverse profile of female rotor; the generating rack

2. Coordinates systems; generation motion

In Fig. 2, the coordinates systems are presented:

-$X_2Y_2Z_2$ relative system jointly to the axode of the rolling surface generating, A_1;

-$\xi\eta\zeta$ - relative system jointly to the axode of the generating rack (flat surface superposed to the plane $\eta\zeta$), A_2;

-$X_{2S}Y_{2S}Z_{2S}$ - relative system associated to the peripheral surface of the hob cutter.

The fixed coordinates systems are the following:

-$x_0\ y_0\ z_0$ is the global system with axis z_0, superposed to the axis of the female rotor;

-$x_{02}y_{02}z_{02}$ – global system, with axis y_{02} superposed to the peripheral surface axis of the hob cutter.

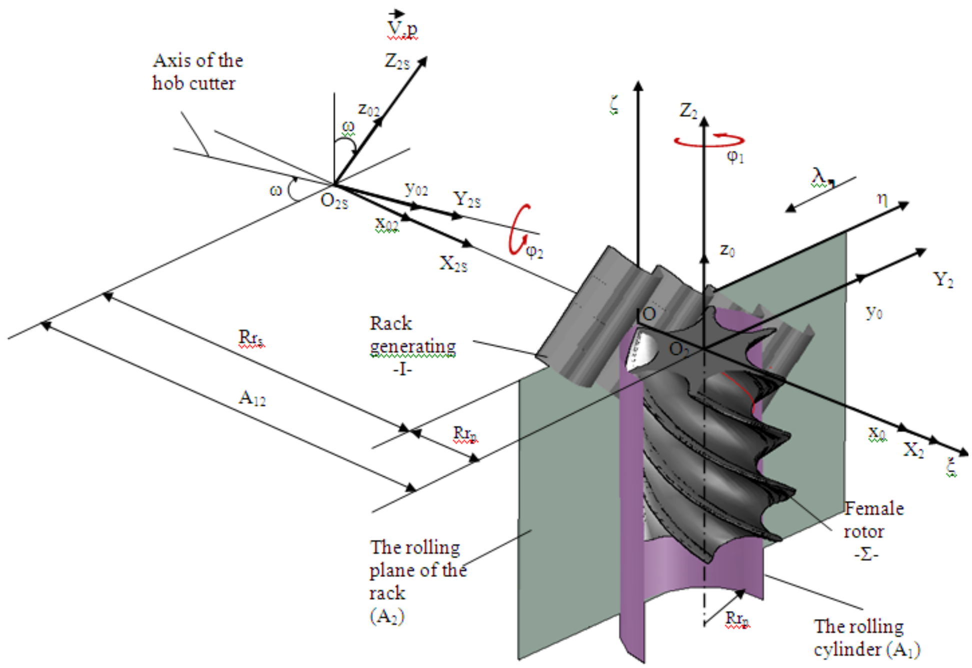

Fig. 2. Female rotor; coordinates systems

The kinematics of the hob cutter generation process includes:

- rotation of axode A_1 (radius R_{rp}), parameter φ_1 - angular motion parameter;
- translation of axode A_2, parameter l_η - parameter motion;
- rotation of system $X_{2S}Y_{2S}Z_{2S}$ around the axis Y_{02}, φ_2 - angular motion parameter.

3. The shape of the generating rack

The generating rack is a complex cylindrical surface, having the profile of the rack as imposed directrix, see Fig. 3.

The element generatrix of the rack is a line g, of unit vector $\vec{t}$.

$$\vec{t} = sin\beta_d \cdot \vec{j} + cos\beta_d \cdot \vec{k} \quad (1)$$

where, β_d is the angle between the tangent to the helical line of the hob cutter, on the pitch circle Rr_p, and the rotor axis (the value is known through the process of generation of rotor).

Therefore, for a current point of the generating rack surface, we can determine the following equations, see Fig. 3:

$$\begin{aligned} &\xi = \xi(\theta); \\ &\eta = \eta(\theta) + t \cdot sin\beta_d; \\ &\zeta = \zeta(\theta) + t \cdot cos\beta_d, \end{aligned} \tag{2}$$

where, θ is the generic parameter of the profiles belonging to the plan ξη, components of the generating rack and t is a scalar, along unit vector $\vec{t}$.

The parametric equations of the components of the cylindrical surfaces of the rack are determined as follows:

-cylindrical surface, corresponding to the arc $\overset{\frown}{AB}$

$$I_{AB}\left|\begin{aligned} &\xi = R_0 \cdot cos\psi - c_0; \\ &\eta = -R_0 \cdot sin\psi + t \cdot sin\beta_d; \\ &\zeta = t \cdot \cos\beta_d, \end{aligned}\right. \tag{3}$$

c_0 is constant, t and ψ are the variable parameters; t is measured along the generatrix of the cylindrical surface , and the angle ψ varies between:

$$0 \le \psi \le \psi_{max}; \psi_{max} = \frac{\pi}{2} - \beta; \tag{4}$$

-plane surface, corresponding to the segment $\overline{BC}$:

$$I_{BC}\left|\begin{aligned} &\xi = \xi_B - u \cdot cos\beta_d; \\ &\eta = \eta_B - u \cdot sin\beta + t \cdot sin\beta_d; \\ &\zeta = t \cdot cos\beta_d, \end{aligned}\right. \tag{5}$$

t, u – variable parameters; $\beta = \frac{\pi}{2} - \psi_{max}$; ξ_B, η_B - are determined from equation (5) for $\psi = \psi_{max}$; similar for $\overline{FG}$;

-cylindrical surface corresponding to the arc $\overset{\frown}{CD}$:

$$I_{CD}\left|\begin{aligned} &\xi = -r_0 \cdot cos\nu + \xi_{0_1}; \\ &\eta = +r_0 \cdot sin\nu + \eta_{0_1} + t \cdot sin\beta_d;\ 0 \le \nu \le \frac{\pi}{2} - \beta; \\ &\zeta = t \cdot \cos\beta_d, \end{aligned}\right. \tag{6}$$

t and ν – variable parameters; ξ_{O_1}, η_{O_1} - coordinates of the centre of circle O_1 having constructive values; similar for $\overset{\frown}{EF}$.

-cylindrical surfaces corresponding to arcs $\overset{\frown}{AH}$ and $\overset{\frown}{HG}$ polynomial Bézier 2nd degree is considered:

$$I_{AH}\left|\begin{aligned} &P_{\xi_{AH}} = \xi = \lambda_1^2 A_\xi + 2(1-\lambda_1)\lambda_1 B_\xi + (1-\lambda_1)^2 C_\xi; \\ &P_{\eta_{AH}} = \eta = \lambda_1^2 A_\eta + 2(1-\lambda_1)\lambda_1 B_\eta + (1-\lambda_1)^2 C_\eta + t \cdot \sin\beta_d; \\ &\zeta = t \cdot \cos\beta_d, \end{aligned}\right. \tag{7}$$

with $0 \le \lambda_1 \le 1$; similar for $\overset{\frown}{HG}$.

4. The characteristic curve

Knowing the surface of the rack flank (in the approximated form as Bézier polynomials), it is possible to determine the contact curve, using the method of the decomposition helical motion [8], see Fig. 3.

The generation of the peripheral helical surface $(\vec{V}, p)$ of the hob cutter, decomposes into a sum of equivalent movements: a translation, on the direction of unit vector generators of the cylindrical surfaces - rack flank - and the rotation of the axis $\vec{A}$, parallel to $\vec{V}$, at the distance:

$$a = p \cdot tan\left(\frac{\pi}{2} - \omega\right), \tag{8}$$

from the helical axis surface, $\vec{V}$, Fig. 3.

Thus, the characteristic surface I, in composed motion, do not depend on the surface component which is self generating, the folowing identity being fulfilled:

$$\vec{N} \cdot \vec{t} \equiv 0, \tag{9}$$

(normal to the surface I, cylindrical surface, is always perpendicular to its own generator); the characteristic used for determination helical movement $(\vec{V}, p)$, will depend only on the rotational movement around the axis $\vec{A}$. So, Fig. 4, we define the axis $\vec{A}$, parallel to the axis of the future hob cutter:

$$\vec{A} = cos\,\omega \cdot \vec{j} - sin\,\omega \cdot \vec{k}\ . \tag{10}$$

The size ω of the angular parameter, is determined provided that the helix which belongs to helical movement $(\vec{V}, p)$, on the cylinder R_{rp}, admits as the tangent to the cylindrical flank of the female rotor generating rack, the unit vector, t, see Fig. 3.

We define,

$$\omega = \beta_d - \omega_s, \tag{11}$$

where:

- ω is the tilt angle of the hob cutter axis, in relation to the front plane of the female rotor (left helix),

$$\omega = arcsin\left[\frac{L_p}{2\pi R_{r_s}}\right]; \tag{12}$$

-β_d - the inclination angle of the helix of the female rotor on cylinder of radius R_{rp};

-ω_s - gradient angle of the helix of radius R_{rs} belonging to the hob cutter,

$$\omega_s = arcsin\left[\frac{R_{r_p}}{R_{r_s}} \cdot \frac{cos\,\beta_d}{z_{lobs}}\right]. \tag{13}$$

Note: Obviously, for a straight hob cutter, the relationship (15) is modified.

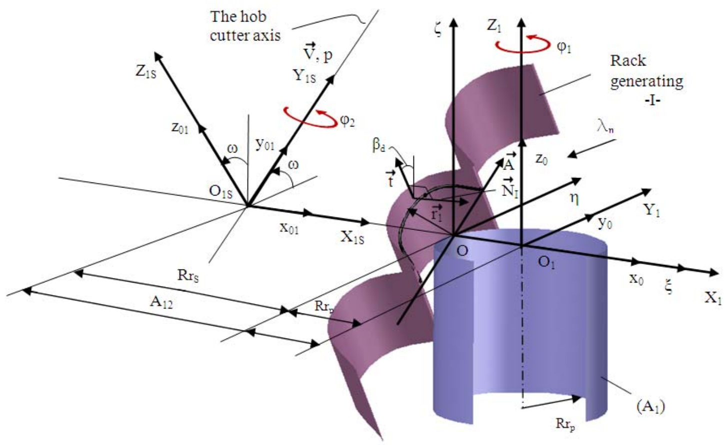

Fig. 3. Method of the decomposition helical movement for the female rotor

The condition for determining the characteristic curve is, see (2),

$$\begin{vmatrix} N_{x_0} & N_{y_0} & N_{z_0} \\ x_0(\lambda,t)+a & y_0(\lambda,t)+t\,sin\,\beta_d & tcos\beta_d \\ 0 & cos\omega & sin\omega \end{vmatrix} = 0\,. \tag{14}$$

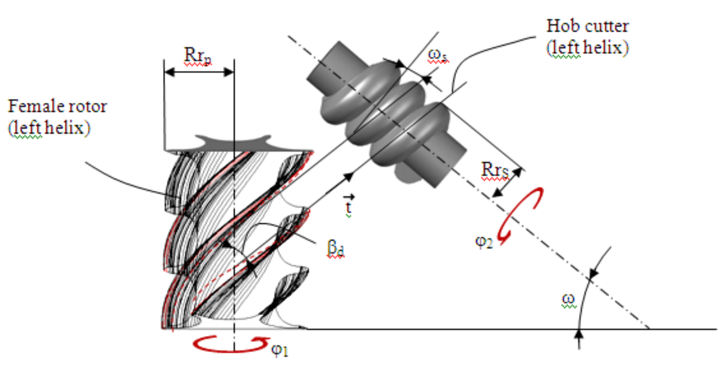

Fig. 4. Axis position of hob cutter

The set of values of the parameters l_η and t, determines the matrix ,

$$C_I = \left\| x_{0_i} \quad y_{0_i} \quad z_{0_i} \right\|^T, \quad i=1.....n \tag{15}$$

The curve C_I, known in numerical form, represents the curve of tangency between the surface I and peripheral surface of the hob cutter (hob cutter generated by enveloping the profile Σ).

The characteristic curve, translated into $X_{2S}Y_{2S}Z_{2S}$, attached to the hob cutter, with the transformation:

$$\begin{Vmatrix} X_{2S} \\ Y_{2S} \\ Z_{2S} \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & cos\omega & sin\omega \\ 0 & sin\omega & cos\omega \end{Vmatrix} \begin{Vmatrix} x_{0_i} + \left[R_{r_p} + R_{r_s} \right] \\ y_{0_i} \\ z_{0_i} \end{Vmatrix}, \tag{16}$$

yields the expression of the peripheral helical surface of the hob cutter,

$$\Pi \left| \begin{aligned} X_{2S} &= X_{2S}\left[x_{0_i}, y_{0_i}, z_{0_i}, \varphi_2 \right]; \\ Y_{2S} &= Y_{2S}\left[x_{0_i}, y_{0_i}, z_{0_i}, \varphi_2 \right]; \\ Z_{2S} &= Z_{2S}\left[x_{0_i}, y_{0_i}, z_{0_i}, \varphi_2 \right]. \end{aligned} \right. \qquad i = 1,2,\ldots,n. \tag{17}$$

Associating to equations (19), the condition:

$$Z_{2S} = 0, \tag{20}$$

the axial section of the peripheral surface, helical shaped, mutually enveloping the female rotor is obtained.

5. Product software for profiling primary peripheral surface by enveloping with point contact

• *Introducing profile piece*

The application allows to configure the profile of generated piece, the profile consists of several basic profiles (straight segment, circle arc, involute arc, epicycloids arc, meshing profiles). Tangent condition between profile components of the piece profiles a priori is solved by the user. It also allowed to introduce incremental profiles, of contact or tangency, to modelling the discontinuities resulting on hob cutter profile.

Using the toolbar "select profiles to add", you can choose the type of elementary profile add to profile piece in a window similar to that in Fig. 5. If arcs, user inserts the ends coordinates and the size of the circle radius to which it belongs (radius being "positive" or "negative" to control the concavity / convexity of the arc of a circle); if evolvent arc, the user inserts initial point arc, the size of the basic radii, between the inner and outer of the arc, respectively. Also the direction in which evolvent arc is drawn: counterclockwise or clockwise can be controlled.

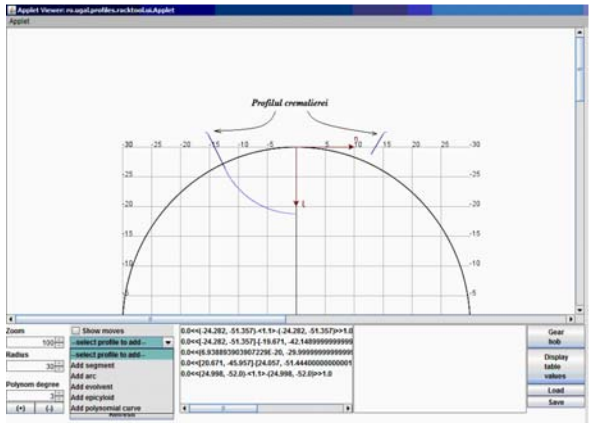

Fig. 5. Applet – Rack profile generation

For the epicycloids arc, the user enters the coordinates of the initial point of the arc, the size of the radii of the mobile circle (ruler) and of the fixed circle (base), roll angle and size.

Finally, the application allows the introduction of discrete profiles by coordinate file (csv), then approximated by subscribers program Bézier form of polynomials, whose degree can be chosen from the toolbar.

The user can edit parameters of a already added profile, or inspect (coordinates extracted from) an existing profile in the program.

• *Primary peripheral surface profiling of a hob cutter*

The program allows to determine the discontinuities on the surface rack (transversal of that profile), (Fig. 6), to be replaced with two Bézier polynomials (Fig. 6).

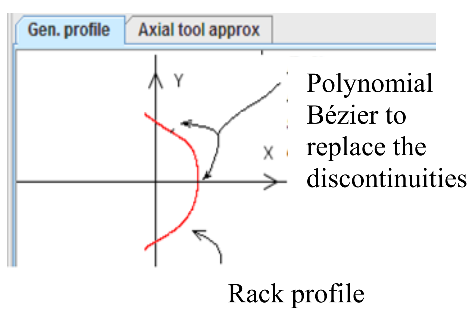

Fig. 6. Applet : rack profile after replacing the discontinuities

In the applet, it presents models of the primary peripheral surfaces of hob cutters to generate the female rotor (Fig.7), from the construction of the helical compressor. The program allows exporting profiles axial sections of the hob cutters coordinates in the file type *csv* (comma separated values), which can be further processed to achieve and control processing on a numerical command machine.

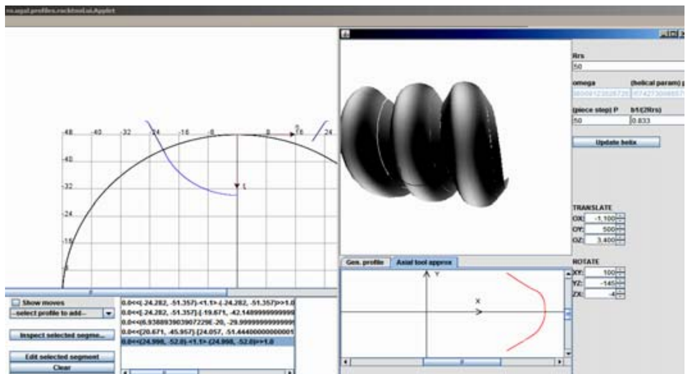

Fig. 7. Applet- profiling of the primary surfaces of the hob cutters, for the female rotor

Also, the peripheral surface models of the hob cutter, characteristic curves are drawn (and the surface contact curve between rack flanks primary peripheral of the hob cutter), details of which can be exported in the same way as files *csv.*

6. Numerical examples

The paper presents the profiling of the hob cutter for a composite profile. The equations (3)…(10) describe the components of generating rack for the chosen profile (see Fig. 1).

First application (screw compressor, ratio 4/6)

Table 1

The constructive dates of the generating rack, (Fig. 1)

R_0 *[mm]*	r_0 *[mm]*	u_{max} *[mm]*	ψ_{max} *[^{0}]*	υ_{max} *[^{0}]*	υ_{1max} *[^{0}]*	u_{1max} *[mm]*	L_p *[mm]*	Rr_2 *[mm]*	c_0 *[mm]*
22.000	1.100	10.30	63.40	63.40	58.285	6.451	50.265	48.00	4.00

Fig. 8 represents the solid model of the hob cutter for the female rotor, the rack and the characteristic curve.

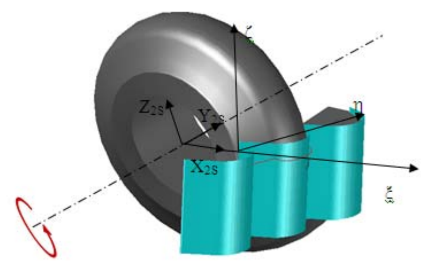

Fig. 8. The solid model of the hob cutter for the
female rotor, the rack and the characteristic curve

In Table 2 the coordinates of axial section of the hob cutter peripheral surface are described.

Table 2

The axial section coordinates of the hob cutter peripheral surface

	X_{1S}*[mm]*	Y_{1S}*[mm]*			X_{1S}*[mm]*	Y_{1S}*[mm]*
1	76.00005	-24.84933	...	691	76.0076	24.46793
2	76.00012	-24.84312	...	692	76.0063	24.48611
3	76.00042	-24.82196	...	693	76.00487	24.49183
4	76.00084	-24.81578	...	694	76.00383	24.51004
5	76.00148	-24.79446	...	695	76.00275	24.51563
6	76.00225	-24.78835	...	696	76.00206	24.53233
7	76.00323	-24.76715	...	697	76.00131	24.53815

Fig. 9 represents the axial section profile of the hob cutter.

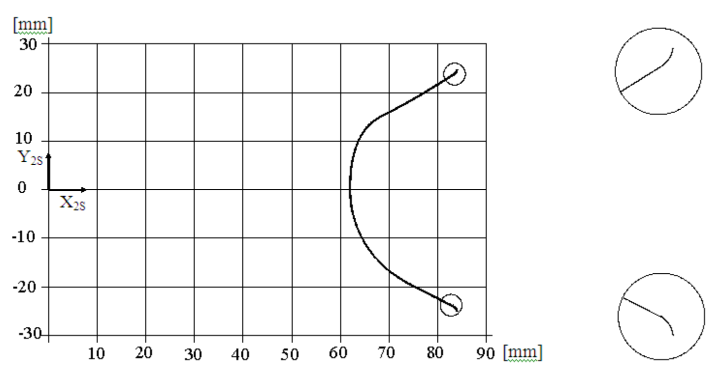

Fig. 9. The axial section profile of the hob cutter

Second application (screw compressor, ratio 3/5)

Table 3

The constructive dates of the generating rack (Fig. 1)

R_0 [mm]	r_0 [mm]	u_{max} [mm]	ψ_{max} [^{0}]	υ_{max} [^{0}]	υ_{1max} [^{0}]	u_{1max} [mm]	L_p [mm]	c_0 [mm]	Rr_2 [mm]
22.000	2.000	7.045	70.300	70.300	35.053	7.774	62.832	4.000	50.000

Fig. 10 represents the solid model of the hob cutter for the female rotor.

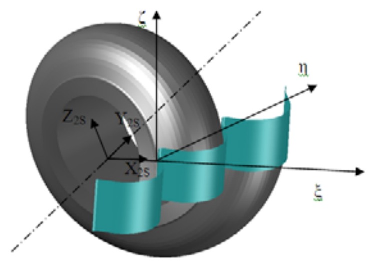

Fig. 10. The solid model of the hob cutter for the female rotor

Fig. 11 represents the axial section profile of the hob cutter for the female rotor.

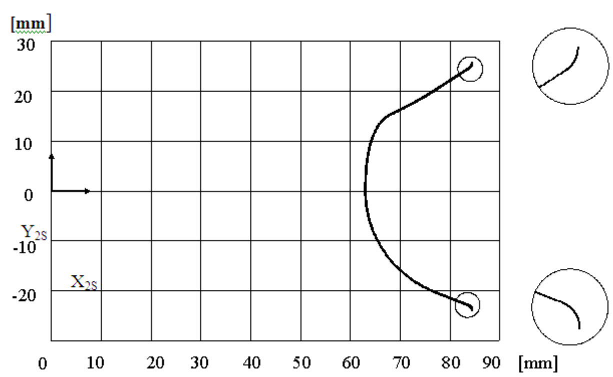

Fig. 11 . The axial section profile of the hob cutter for the female rotor

7. Conclusions

1. The helical surfaces bordering the components rotors of the screw compressors are complex surfaces (cylindrical helical surfaces and constant pitch). Generating this kind of surfaces is done with a specific rack, whose transverse profile is determinate on technological and functional considerations necessary to obtain an optimum yield. The female rotor can be expressed by simple analytical forms, or approximation of Bézier polynomials.

2. The hob cutter profiles are generated on the properties of the composition and decomposition of the helical motion, using the intermediate surfaces method. The proposed approach is characterized by a substantial reduction effort of the analytical calculation.

3. The solid models 3D of the tools peripheral surfaces were also represented.

B I B L I O G R A P H Y

[1] *F. L. Litvin,* 1989, "Theory of Gearing", NASA RP-1212 (AVSCOM 88-C-035), Washington, D.C.

[2] *R.Houston, D. Mavripilis, F. Oswald, Y.S. Liu,* 1994, "A basis for solid modeling of gear teeth whith application in design manufacture", Mech. Mach. Theory 29 (5):710-723;

[3] *N.Oancea,* 2004*,* "Methode numérique pour l'étude des surfaces enveloppes", Mech. Mach. Theory 31 (7): 957-972;

[4] *N.Oancea,* Surface generation through winding, Volume II, Complementary Theorems, 2004, "Dunărea de Jos"University publishing house, ISBN 973-627-106-4, ISBN 973-627-170-6;

[5] *S.P.Radzevich,* 2008, « Kinematic geometry of surface machining", CRC, Boca Raton, ISBN 978-1-4200-6340;

[6] *V.Teodor,* 2010, "Contribution to the elaboration of a method for profiling tools-tools which generate by enwrapping", Lambert Academic Publishing, ISBN 978-3-9433-8261-8;

[7] *R. P.Rodin,* 1990, "Osnovy proektirovania rezhushich instrumentov (Basics of desing of cutting tools) Vishcha Shkola, Kiev;

[8] *I.Veliko, N. Gentcho,* 1998, "Profiling of rotation tools for forming of helical surfaces", Int.J. Mach Tools; manuf.38:125-1148;

[9] *V. S. Liukshin,* 1968, "Teoria vintovih poverhnostei, Mashinostroenie "(Theory of helical surfaces, Mechanical Engineering), Moskva;

[10] *N.Stosic, I. Smith, A. Kovacevic* "Screw Compressors – Mathematical Modelling and Performance Calculation". Springer, feb. 2005;

[11] *N Stosic, I. K Smith, and A Kovacevic, 2002:* Optimization of Screw Compressor Design, XVI International Compressor Engineering Conference at Purdue, July 2002.