ANALYSIS OF THE INFLUENCE OF MEASUREMENT FOR THE CONTROL CIRCUIT OF GRAVITY GRADIOMETER ON GRAVITY GRADIENT MEASUREMENT

Xuewu QIAN[1,3], Tingzhou NING[2,3*]

In order to study the influence of the measurement and control circuit system on gravity gradient components, the analysis started from the amplification gain and initial phase angle of the control circuit. The impact of gain discrepancies and initial phase angle errors in the control circuit on gravity gradients were analyzed, leading to certain conclusions. The adder and voltage amplifier gain errors in the control circuit have a significant effect on the gradient measurement. The amplification gain of the control circuit system has a greater impact on larger gradient components and a smaller impact on smaller gradient components. The initial phase angle error and the amplification gain of the measurement and control system have opposite effects on the gradient components. Finally, a method for achieving precise positioning of the initial phase angle using trigger pulse was proposed. Through experiments with the control system, when the sampling rate is 10Hz, the maximum gradient error was 0.1Eu, meeting the experimental requirements.

Keywords: Gravity Gradiometer, Phase Angle, Control Circuit, Gradient Component, Trigger Pulse

1. Introduction

Gravity gradient measurement boasts a history spanning more than a century [1-2]. These gradients effectively depict the intricate attributes of the Earth's external gravitational field, influenced by varying density formations within the planet. This type of measurement holds substantial importance across diverse domains, including Earth science, geology, energy exploration, and inertial-aided navigation. Notably, commercially operational instruments include BHP Billiton's FALCON^{TM} Partial Tensor Airborne Gradiometer, along with Full Tensor Gradiometers by Bell Geospace and ARKeX [3-5]. Meanwhile, the evolution

[1] Prof., School of Electronic Engineering, Jinan Vocational College, Jinan 250103, China, e-mail: sdqxwu@163.com
[2] School of Mechanical Manufacturing, Jinan Vocational College, Jinan 250103, China, e-mail: ning-tingzhou@163.com
[3] Jinan Key Laboratory of 5G + Advanced Control Technology, Jinan Vocational College, Jinan 250103, China.
* represents the corresponding author.

continues with ongoing development of superconducting and cold atom gravity gradiometer instrument (GGI).

Due to the extremely faint nature of gravity gradient signals, achieving highly precise measurements poses a formidable challenge due to a range of factors such as internal instrument noise, environmental temperature, accelerometer parameters, and installation errors. Reference [6] addresses accelerometer noise reduction, while reference [7] explores error compensation techniques for gravity gradiometers. Reference [8] derive error equations and matching methods from the perspective of accelerometer scale factors and installation errors.

In real-world gravity gradiometer systems, the precision of motor control, stability of the mechanical structure, and the initial phase angle significantly impact the measurement accuracy of the gravity gradiometer [9-10]. These factors dictate whether the gravity gradient can be accurately measured and form the foundation for analyzing the impact of other errors on the gravity gradiometer. This paper predominantly analyzes the effects of gain errors in the amplification circuit and errors in the initial phase angle on the performance of the gravity gradiometer.

2. Measurement Principle of Rotational Accelerometer Gravity Gradiometer

The Rotational Accelerometer Gravity Gradiometer utilizes four exceptionally accurate accelerometers, symmetrically and orthogonally arranged on a rotating disk, as depicted in Fig.1. These accelerometers are strategically positioned so that the distance from their centers to the disk's center remains uniform. The sensitive axes of the accelerometers align tangentially with the disk's edge, featuring opposing directions for each corresponding pair of sensitive axes.

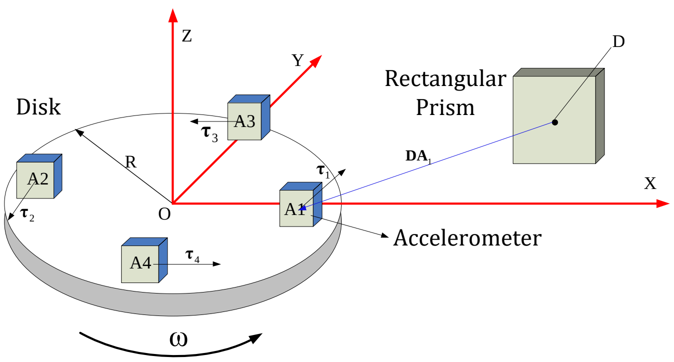

Fig. 1. Schematic Diagram of Rotational Accelerometer Gravity Gradiometer Measurement

During operation, the gravity gradiometer modulates the gravity gradient signal by maintaining a consistent angular rate ω for the disk's rotation. With a solitary disk configuration, it achieves partial tensor gravity gradient measurement (such as FALCON^{TM}). On the other hand, a full tensor gravity gradiometer utilizes three disks arranged in an orthogonal umbrella-shaped configuration, allowing for comprehensive measurement of the complete tensor of gravity gradients.

A gradiometer coordinate system, denoted as O-XYZ, is established on the disk. Here, the origin O is positioned at the disk's center. The Z-axis coincides with the disk's rotation axis, while the center of mass D of the rectangular prism resides on the X-axis. The edges of the rectangular prism are in alignment with the axes of the gradiometer coordinate system.

According to the universal law of gravitation, the acceleration component a1 of the mass body of the rectangular prism on the input axis of accelerometer $\mathbf{A}_i$ can be calculated using the following equation:

$$a_i = \frac{GM}{\left|\mathbf{DA}_i\right|^3}\mathbf{DA}_i \cdot \boldsymbol{\tau}_i \tag{1}$$

Where, G is the universal gravitational constant, M is the mass of the rectangular prism, $\mathbf{DA}_i$ is the vector from the center of mass of the rectangular prism to the center of mass of the accelerometer $\mathbf{A}_i$, and $\boldsymbol{\tau}_i$ is the unit vector in the direction of the input axis of accelerometer $\mathbf{A}_i$.

In the ideal case, the measurement equation for the output signal of a gravity gradiometer is:

$$(a_1 + a_2) - (a_3 + a_4) = -2R\left[\left(\Gamma_{yy} - \Gamma_{xx}\right)\sin 2(\omega t + \theta) + 2\Gamma_{xy}\cos 2(\omega t + \theta)\right] \tag{2}$$

Where, R is the distance from the center of mass of the accelerometer to the center of the disk. θ is the initial phase angle. (Γ_{yy} - Γ_{xx}) and Γ_{xy} is the gravity gradient component. ω is the angular rate at which the disk is rotating.

The gravity gradient component (Γ_{yy} - Γ_{xx}) and Γ_{xy} can be obtained using modulation and demodulation methods. This involves multiplying the gravity gradient signal with the corresponding trigonometric functions (sin and cos) and then performing periodic integration. It can be calculated using the following equation:

$$\begin{cases} \left(\Gamma_{yy} - \Gamma_{xx}\right) = \int_{nT}^{mT}\left[(a_1 + a_2) - (a_3 + a_4)\right]\sin 2(\omega t + \theta)dt \\ \Gamma_{xy} = \int_{nT}^{mT}\left[(a_1 + a_2) - (a_3 + a_4)\right]\cos 2(\omega t + \theta)dt \end{cases} \tag{3}$$

3. Impact of Amplifier Gain in Gravity Gradiometry

The accelerometer output signals are in the form of current signals, which need to be converted to voltage signals through an I/V conversion amplifier. The gain of the conversion amplifier is denoted as KAi (i=1,2,3,4). The respective signals from two accelerometers are then added using an adder amplifier with a gain of KVi (i=1,2). Subsequently, the signals are subtracted using a voltage amplifier with a gain of KS. Finally, the signals are collected using a digital multimeter (DMM). The schematic diagram of the measurement and control system's gain amplification and the Hardware-in-the-Loop Simulation Platform of GGI are shown in Fig.2 and Fig.3 respectively.

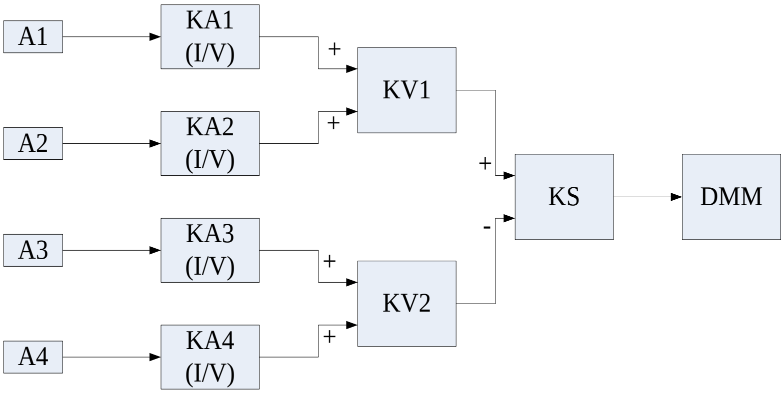

Fig. 2. Schematic Diagram of Gain Amplification in the Measurement and Control System

Fig.3 Hardware-in-the-Loop Simulation Platform of GGI

In practical measurement and control systems, factors such as device variations and environmental differences can result in variations in the gain factors KAi (i=1,2,3,4) of the I/V conversion amplifier and the gain factors KVi (i=1,2) of the adder amplifier. These variations can also affect the noise suppression performance, leading to significant differences between the analyzed gravity gradient values and the true values, and even hinder the achievement of accurate gravity gradient measurements. Therefore, analyzing the impact of gain variations in the amplification circuits of the measurement and control system on the gravity gradient is of significant practical importance.

Table 1.

Amplification Gain Variations for Gradient Error of 0.1Eu(θ =0°)

KA1/KA2	KA3/KA4	KV1/KV2	KS	Γ_{xy}(Eu)	Γ_{yy}-Γ_{xx}(Eu)	Error (Eu)
10^8	10^8	6	2	0	-335.01	0
(1±0.0006）×10^8	(1±0.0006）×10^8	6	2	0	-335.01±0.1	0.1
10^8	10^8	6±0.00175	2	0	-335.01±0.1	0.1
10^8	10^8	6	2±0.00058	0	-335.01±0.1	0.1

Table 2.

Amplification Gain Variations for Gradient Error of 0.1Eu(θ =30°)

KA1/KA2	KA3/KA4	KV1/KV2	KS	Γ_{xy}(Eu)	Γ_{yy}-Γ_{xx}(Eu)	Error (Eu)
10^8	10^8	6	2	146.02	-170.50	0
(1±0.0012）×10^8	(1±0.0012）×10^8	6	2	146.02± 0.09	-170.50±0.1	0.1
10^8	10^8	6±0.0035	2	146.02± 0.09	-170.50±0.1	0.1
10^8	10^8	6	2±0.00058	146.02± 0.09	-170.50±0.1	0.1

Assuming the distance from the accelerometer center to the disk center is R=0.5m, the density of the rectangular prism is 18200kg/m^3, and the length of each side of the rectangular prism is 0.3m. The distance from the rectangular prism center to the disk center is 0.72m. The rotation period of the disk is T=20s, the acceleration due to gravity is g=9.8m/s^2, and the accelerometer scale factor is K_I =32mA/g. The amplification gains of the measurement and control system are set as follows: the I/V converter has an actual gain of KA=10^8, the adder has an actual gain of KV=6, the voltage amplifier has an actual gain of KS=2, and the DMM sampling rate is 10Hz. When the initial phase angle is zero and the maximum gradient error is 0.1Eu, the analysis of the variations in the amplification gains of the measurement and control system is shown in Table 1. When the initial phase angle is 30° and the maximum gradient error is 0.1Eu, the analysis of the variations in the amplification gains of the measurement and control system is shown in Table 2.

The analysis reveals that when the initial phase angle is 0°, a maximum gain difference of 0.06% for the I/V converter, 0.029% for the summing amplifier, or 0.029% for the voltage amplifier can result in a gradient component (Γ_{yy} - Γ_{xx}) error of 0.1Eu, while having minimal impact on the Γ_{xy} gradient component. On the other hand, when the initial phase angle is 30°, a maximum gain difference of 0.12% for the I/V converter, 0.058% for the summing amplifier, or 0.058% for the voltage amplifier can lead to a (Γ_{yy} - Γ_{xx}) gradient error of 0.1Eu and a Γ_{xy} gradient error of 0.09Eu. The gain differences in the summing amplifier and voltage amplifier have a significant impact on the gradients, with larger gain differences affecting larger gradient values more prominently and smaller gradient values to a lesser extent.

4. Impact of Initial phase angle of the measurement and control system in Gravity Gradiometry

When demodulating the gradient signals in the measurement and control system, it is necessary to accurately determine the initial phase angle. Only when the initial phase angle is correctly positioned, the demodulated gradient values will approach the true values. Therefore, the determination of the initial phase angle plays a crucial role in achieving accurate gradient demodulation.

Assuming the initial phase angle is θ and the demodulation phase angle error is $\Delta\theta$, with the accelerometer's center-to-disk distance being R=0.5m, the rectangular prism density as 18200kg/m^3, the dimensions of the rectangular prism as 0.3m for each side, the distance between the rectangular prism's center and the disk center as 0.72m, the disk rotation period as T=20s, the acceleration due to gravity as g=9.8m/s^2, and the scale factor of the accelerometer as K_I = 32mA/g. According to equations (2) and (3), the influence of the initial phase angle error on the gravity gradient can be derived. The expressions for the gradient errors induced by the phase angle error $\Delta\theta$ are as follows:

$$\begin{cases} \Delta(\Gamma_{yy}-\Gamma_{xx}) = -K_I R\left[(1-\cos(2\Delta\theta))(\Gamma_{yy}-\Gamma_{xx}) - 2\sin(2\Delta\theta)\Gamma_{xy}\right]\cdot T \\ \Delta\Gamma_{xy} = K_I R\left[(1+\sin(2\Delta\theta))(\Gamma_{yy}-\Gamma_{xx}) - 2\cos(2\Delta\theta)\Gamma_{xy}\right]\cdot T \end{cases} \tag{3}$$

After the signal from the signal source is converted into analog accelerometer signals, it needs to undergo signal conditioning through the measurement and control amplification circuit before being collected by the

DMM. Due to the signal processing through multiple instruments, there will inevitably be a certain time delay in the signal. Based on testing, the delay time is approximately 50ms, which means that the DMM starts data acquisition after a 50ms delay to accurately demodulate the signal. The DMM sampling frequency is 10Hz, and the maximum allowable initial phase angle error that can be adjusted is 1.8°. The influence of the initial phase angle error on the gravity gradient analysis is conducted under a 1.8° phase angle error. The impact of the initial phase angle and its error on the gravity gradient at 0° and 30° is shown in Fig.4 and Fig.5, respectively.

From Fig.4 and Fig.5, it can be observed that as the initial phase angle error increases, the influence on the gravity gradient component errors (Γ_{yy}-Γ_{xx}) and Γ_{xy} also increases. However, larger gradient values are less affected by the error, while smaller gradient values are more affected. When the initial phase angle θ=0°, gradient component Γ_{xy} is most affected by the initial phase angle error. When the phase angle error is 0.018°, a gradient error of 0.1Eu is produced for the gradient component Γ_{xy}. Therefore, in order to achieve a gradient resolution above 0.1Eu, the initial phase angle error should not exceed 0.018°.

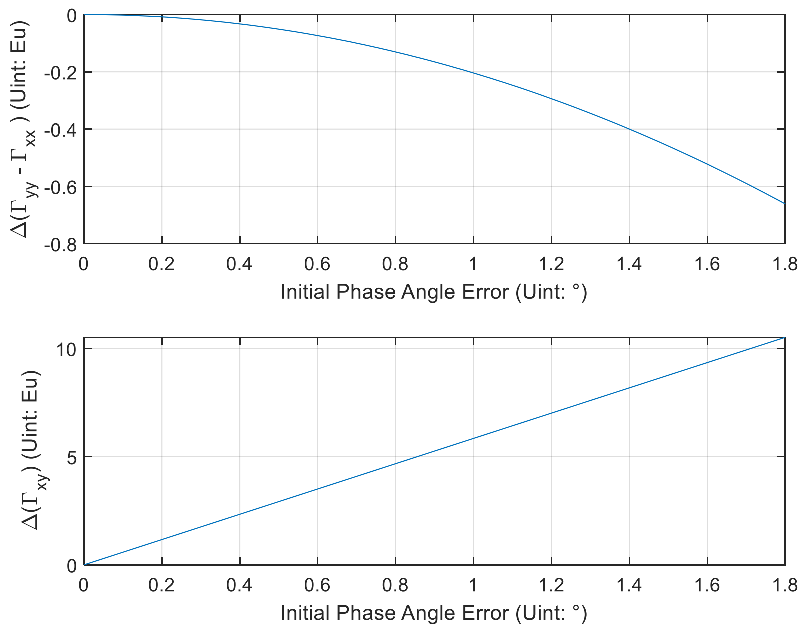

Fig. 4. Influence of Initial Phase Angle Error on Gravity Gradiometry(θ =0°)

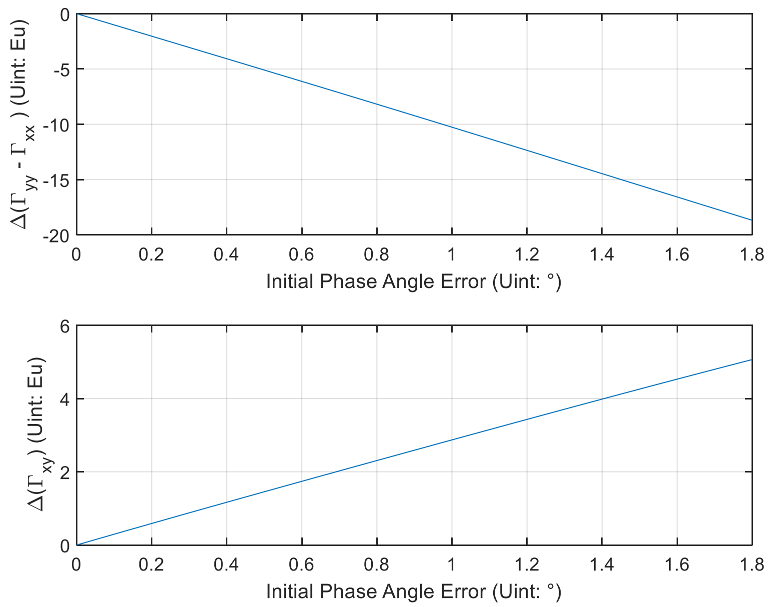

Fig. 5. Influence of Initial Phase Angle Error on Gravity Gradiometry(θ =30°)

Fig.6 illustrates the effect of the initial phase angle on the difference between the decoded gradient values and the ideal values during the rotation of the disk. When the decoding phase angle error is 1.8°, the variation of the initial phase angle from 0 to 360° is shown.

From Fig.6, it can be observed that the difference between the decoded gradient values and the ideal values varies periodically with different initial phase angles. When the distance from the centroid of the rectangular body to the center of the disk is 0.72m, the maximum gradient error generated is 21.04Eu for (Γ_{yy}-Γ_{xx}) and 10.52Eu for Γ_{xy}.

In the context of the gravity gradient measurement and control system, the initial phase is established with flexibility through a combination of high-precision programmable instruments. This enables accurate determination of the initial phase angle, facilitated by a pulse-triggering technique. Experimental validation has demonstrated the effectiveness of this approach, particularly when employing a DMM sampling rate of 10Hz, resulting in a maximum gradient error well within 0.1Eu. This outcome satisfies the stipulated requirements for experimental testing.

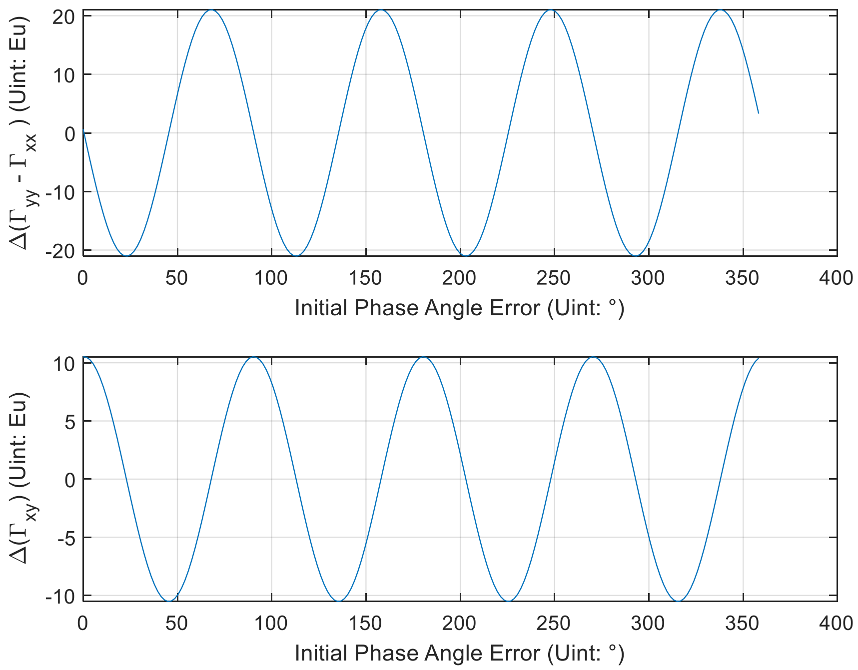

Fig. 6. Influence of Initial Phase Angle on Gradient

Given that the gravity gradient instrument operates as a precision device, it is sensitive to factors like environmental temperature, ground vibrations, and alterations in its surroundings. Consequently, when calibrating the gradient instrument, it's crucial to maintain consistent conditions, including stable temperature, platform stability, and minimal changes in the environmental surroundings.

5. Conclusions

The precision of gravity gradient measurement hinges on the effectiveness of the measurement and control circuit. In this study, we zeroed in on the amplification circuit gain and the initial phase angle to dissect the effects of gain disparities and phase angle errors on gravity gradient demodulation. Our analysis revealed that the amplification gains within the addition circuit and voltage amplifier wielded considerable influence over the gradient. The impact of the amplification circuit's gain was more pronounced on larger gradient components and relatively subdued on smaller ones. Conversely, initial phase angle errors affected smaller gradient components more significantly while exerting a lesser impact on larger ones.

The simulation outcomes demonstrated that both the amplification gain and the initial phase angle error in the measurement and control circuit significantly affected gradient values. Ensuring consistent amplification gain within the control circuit and precise determination of the initial phase angle necessitate subsequent rounds of experimental testing, analysis, and in-depth research. The research on the relationship between initial phase angle and gravity gradient measurement accuracy is of great significance for the development of gravity gradiometers and the improvement of measurement precision.

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