INFLUENCE OF THE SUSPENSION DAMPING ON RIDE COMFORT OF PASSENGER RAILWAY VEHICLES

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În lucrare se analizează influenţa amortizării suspensiei unui vagon de călători asupra confortului mersului, evaluat prin indicele Wz. Pentru aceasta s-a considerat modelul complet al unui vehicul cu două etaje de suspensie şi cutie flexibilă. Aplicând analiza modală, s-a dat o nouă formă ecuaţiilor de mişcare care descriu mişcările simetrice şi antisimetrice ale vehiculului şi modurile lor de excitare. Mărirea amortizării suspensiei secundare poate avea efecte contrare asupra confortului în funcţie de poziţia de-a lungul cutiei şi de viteză. De aceea, problema alegerii acestei amortizări necesită o soluţie de compromis. Amortizarea suspensiei primare imbunătăţeşte confortul dar creşterea ei este limitată de suprasarcinile dinamice roată-şină.

The paper herein analyzes the influence of the suspension damping of a passenger car upon the ride comfort, evaluated via the Wz index. Therefore, the complete model of a vehicle with two suspension levels and a flexible carbody has been taken into consideration. Upon applying the modal analysis, a new form has been provided for the movement equations that describe the symmetrical and anti-symmetrical movements of the vehicle and their modes of excitation. An increase in the damping of the secondary suspension may trigger a decrease of the comfort, in dependence with the position along the carbody and the velocity. This is the reason why the issue of selecting this damping requires a solution of compromise. The primary suspension damping enhances the comfort, but its increase is limited by the wheel/rail dynamic overloads.

Keywords: railway vehicle, ride comfort, damping, random vibration

1. Introduction

The ride comfort is a complex concept, representing an essential element in the analysis of the railway vehicles dynamics and needs to be considered when modelling and evaluating their behaviour. Besides other factors, the ride comfort primarily depends on the vibration behaviour to which the vehicle is subjected [1].

The vibration of railway vehicles, both vertically and horizontally, derives from the track irregularities and need to be mentioned that the two types of vibrations are decoupled due to the construction symmetries [2]. The vertical

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vibrations – bounce and pitch vibrations –impact the rolling quality, safety, ride comfort and track quality.

The rise in the velocity will trigger the increase in the vibration behaviour at the carbody level, with a negative effect upon the ride comfort, therefore it needs to improve the vehicle construction, by strengthening the carbody structural rigidity [3], or the optimisation of the suspension parameters [4, 5]. When the suspension system performance is not satisfactory any longer, the solution could be the active suspension [6, 7]. Despite of the favourable results, the active suspension is not a widespread operational solution, due to the fact that the price of implementing and maintenance of this system is too high versus the benefits.

In order to evaluate the ride comfort, there are various international standards, namely ISO 2631 [8], BS 6841 [9], Index Sperling Ride [10, 11], ENV-12999 [12] and UIC 513 [13] – generally speaking, they assess the vibration level in terms of comfort based on the frequency-weighted acceleration. Among them, a simple and widely used method is the Sperling's, *Index Sperling Ride,* which stands out [3, 14-16] by the fact that its implementation leads in a number with a precise signification that may be easily interpreted, in dependence of the action of different elements in the vehicle vibrant system.

This paper deals with an issue not talked about much before, i.e. the influence of the suspension damping upon ride comfort evaluated by the Wz index. The complete model of a passenger car has been considered, including the carbody bending vibrations. Upon implementing the technique of modal analysis, the equations of motion are processed differently, intuitively, which points out that the vehicle movement may be decomposed into symmetrical and anti-symmetrical decoupled movements and the excitation modes are shown for each of them. The damping influence upon the vehicle frequency response is analyzed. And the impact of suspension damping upon the ride comfort is seen in terms of velocity, bogie axle base and carbody rigidity. The geometric filtering effect is under study, an effect due to the distance between wheelsets and its influence upon the comfort index, not dealt with before [3, 17].

2. The mechanical model and the movement equations

The case of a two-floor suspension railway vehicle that travels on a constant speed V on a track with random irregularities is considered (Fig. 1). The vehicle carbody of a length L is modelled via an Euler-Bernoulli beam of constant section and uniformly distributed mass, with the bending module EI, mass on length unit m and damping coefficient μ. The displacement of a beam section is $w(x, t)$, where t is time.

Here, only the carbody bending modes have been considered for both generality and simplicity. When a specific structure is studied and the carbody global and local mode shapes are taken into account, the FEM carbody model has to be used.
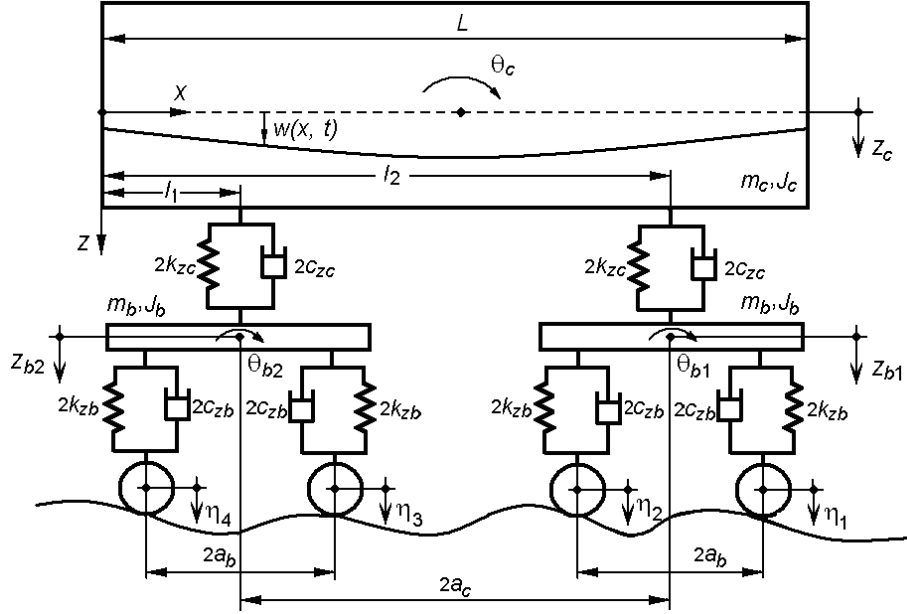
Fig. 1. The vehicle mechanical model.

The hypothesis of rigid track is adopted because the track rigidity is much higher than the one of the vehicle suspension and the frequencies of wheelsets on the track are much higher than the vehicle's. Therefore, the vertical wheelset displacement equals the corresponding irregularity.

The bogies suspended masses are considered two-degree freedom rigid bodies, namely the bounce movement z_{bi} and pitch θ_{bi}, with $i = 1, 2$. The mass of a bogie is m_b and its inertia moment $J_b = m_b i_b^2$, with i_b – the bogie gyration radius. The suspension levels are modelled via Kelvin-Voigt systems. The elastic constants are k_{zb}, k_{zc}, and the damping ones c_{zb}, c_{zc}.

The movement equations are:
- for bending the carbody

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \mu I\frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + m\frac{\partial^2 w(x,t)}{\partial t^2} = \sum_{i=1}^{2} F_i \delta(x - l_i), \qquad (1)$$

where $\delta(.)$ is Dirac's delta function, and F_i represents the force due to the i secondary bogie suspension

$$F_i = -2c_{zc}\left(\frac{\partial w(l_i,t)}{\partial t} - \dot{z}_{bi}\right) - 2k_{zc}\left(w(l_i,t) - z_{bi}\right), \qquad (2)$$

that acts at the distance l_i from the carbody end.

- for bounce of bogies

$$m_b \ddot{z}_{b1,2} + 2c_{zb}(2\dot{z}_{b1,2} - \dot{\eta}_{1,3} - \dot{\eta}_{2,4}) + 2k_{zb}(2z_{b1,2} - \eta_{1,3} - \eta_{2,4}) +$$

$$+ 2c_{zc}\left[\dot{z}_{b1,2} - \frac{\partial w(l_{1,2},t)}{\partial t}\right] + 2k_{zc}[z_{b1,2} - w(l_{1,2},t)] = 0 ; \tag{3}$$

- for pitch of bogies

$$I_b \ddot{\theta}_{b1,2} + 2c_{zb}a_b(2a_b\dot{\theta}_{b1,2} - \dot{\eta}_{1,3} + \dot{\eta}_{2,4}) +$$

$$+ 2k_{zb}a_b(2a_b\theta_{b1,2} - \eta_{1,3} + \eta_{2,4}) = 0 \qquad , \tag{4}$$

where a_b, a_c are the axle bases of bogies and carbody and η_i with $i = 1,..,4$, the irregularities against the wheelset i.

It may be noticed that the pitch movements of bogies are decoupled from the other movements of the vehicle.

The movement equations with partial derivates may be turned into equations with ordinary derivates by implementing the method of modal analysis. For this, the rigid and bending modes of carbody are considered as

$$w(x,t) = z_c(t) + \left(\frac{L}{2} - x\right)\theta_c(t) + \sum_{i=2}^{\infty} X_i(x)T_i(t), \tag{5}$$

where $z_c(t)$ and $\theta_c(t)$ represent the carbody vibration rigid modes, namely the bounce and pitch. $T_i(t)$ is time-dependent coordinate, and $X_i(x)$ is the eigenfunction of vibration mode i at bending

$$X_i(x) = \sin\beta_i x + \sinh\beta_i x - \frac{\sin\beta_i L - \sinh\beta_i L}{\cos\beta_i L - \cosh\beta_i L}(\cos\beta_i x - \cosh\beta_i x), \tag{6}$$

with $\beta_i = \sqrt{\omega_i^2 m/(EI)}$ and $\cos\beta_i L \cosh\beta_i L - 1 = 0$, where ω_i is the natural angular frequency of the vibration mode i.

Looking at the two first bending modes only, symmetrical and anti-symmetrical, the vehicle vibration is described by a set of 8 equations, among which 6 are coupled, and other two are independent (pitch of bogies). An appropriate choice of the coordinates and a correct processing of the equations system will give the decomposition of the set with 6 movement equations, coupled in two independent sets of 3 equations each.

The two systems describe the carbody symmetrical and anti-symmetrical movements. If we have

$$q_1^+ = z_c \; ; \quad q_1^- = \theta_c \; ; \quad q_2^+ = T_2 \; ; \quad q_2^- = T_3 \; ;$$

$$q_3^+ = \frac{1}{2}(z_{b1} + z_{b2}); \quad q_3^- = \frac{1}{2}(z_{b1} - z_{b2}); \tag{7}$$

$$X_2(l_1) = X_2(l_2) = \varepsilon^+ \; ; \quad X_3(l_1) = -X_3(l_2) = \varepsilon^-$$

then the equations of the symmetrical movements are obtained

$$m_c \ddot{q}_1^+ + 4c_{zc}(\dot{q}_1^+ + \varepsilon^+ \dot{q}_2^+ - \dot{q}_3^+) + 4k_{zc}(q_1^+ + \varepsilon^+ q_2^+ - q_3^+) = 0; \tag{8}$$

$$m_{m2} \ddot{q}_2^+ + c_{m2} \dot{q}_2^+ + k_{m2} q_2^+ + 4c_{zc}\varepsilon^+ [\dot{q}_1^+ + \varepsilon^+ \dot{q}_2^+ - \dot{q}_3^+] + \\ + 4k_{zc}\varepsilon^+ [q_1^+ + \varepsilon^+ q_2^+ - q_3^+] = 0; \tag{9}$$

$$m_b \ddot{q}_3^+ + 4c_{zb}(\dot{q}_3^+ - \dot{\eta}^+) + 4k_{zb}(q_3^+ - \eta^+) + \\ + 4c_{zc}(\dot{q}_3^+ - \dot{q}_1^+ - \varepsilon^+ \dot{q}_2^+) + 4k_{zc}(q_3^+ - q_1^+ - \varepsilon^+ q_2^+) = 0; \tag{10}$$

along with the anti-symmetrical equations

$$J_c \ddot{q}_1^- + 4c_{zc}a_c(a_c \dot{q}_1^- - \varepsilon^- \dot{q}_2^- + \dot{q}_3^-) + 4k_{zc}a_c(a_c q_1^- - \varepsilon^- q_2^- + q_3^-) = 0; \tag{11}$$

$$m_{m3} \ddot{q}_2^- + c_{m3} \dot{q}_2^- + k_{m3} q_2^- + 4c_{zc}\varepsilon^- [-\dot{q}_1^- + \varepsilon^- \dot{q}_2^- - \dot{q}_3^-] + \\ + 4k_{zc}\varepsilon^- [-q_1^- + \varepsilon^- q_2^- - q_3^-] = 0; \tag{12}$$

$$m_b \ddot{q}_3^- + 4c_{zb}(\dot{q}_3^- - \dot{\eta}^-) + 4k_{zb}(q_3^- - \eta^-) + \\ + 4c_{zc}(\dot{q}_3^- - a_c \dot{q}_1^- - \varepsilon^- \dot{q}_2^-) + 4k_{zc}(q_3^- - a_c q_1^- - \varepsilon^- q_2^-) = 0; \tag{13}$$

where m_c and $J_c = m_c i_c^2$ are the carbody mass and inertia moment with i_c its gyration radius and $m_{m2,3}$, $c_{m2,3}$ and $k_{m2,3}$ are the masses, damping and stiffness of the second and third mode

$$m_{m2,3} = m \int_0^L X_{2,3}^2(x)\mathrm{d}x;$$

$$k_{m2,3} = EI \int_0^L \left(\frac{\mathrm{d}^2 X_{2,3}(x)}{\mathrm{d}x^2} \right)^2 \mathrm{d}x; \quad c_{m2,3} = \mu I \int_0^L \left(\frac{\mathrm{d}^2 X_{2,3}(x)}{\mathrm{d}x^2} \right)^2 \mathrm{d}x. \tag{14}$$

The below equations include the excitation mode of the anti-symmetrical symmetrical movements

$$4\eta_1^+ = \eta_1 + \eta_2 + \eta_3 + \eta_4 \, ; \; 4\eta^- = \eta_1 + \eta_2 - \eta_3 - \eta_4 \, . \tag{15}$$

The parameters of the carbody bending vibration are the modal angular frequency and the damping ratio

$$\omega_{m2,3} = \sqrt{\frac{k_{m2,3}}{m_{m2,3}}} \, , \; \zeta_{m2,3} = \frac{c_{m2,3}}{2\sqrt{k_{m2,3} m_{m2,3}}} \, . \tag{16}$$

In order to facilitate the analysis of vibrations, the damping ratio of the suspension levels considered uncoupled, as below

$$\zeta_{b,c} = \frac{4c_{b,c}}{2\sqrt{4k_{b,c} m_{b,c}}} \, . \tag{17}$$

In the following section, the movement equations will be used to evaluate the comfort of a passenger car. Thus, the carbody frequency response is calculated, taking into account the vibration harmonic behaviour where the track irregularities have a sinusoidal shape with the wavelength l and the amplitude η_0

$$\eta_{1,2} = \eta_0 \cos\frac{\omega}{V}(Vt \pm a_b + a_c) \, , \; \eta_{3,4} = \eta_0 \cos\frac{\omega}{V}(Vt \pm a_b - a_c) \, . \tag{18}$$

where $\omega = 2\pi V/l$ is the angular frequency.

The vibrations of carbody are excited by the track irregularities against the wheelsets, and the plan of bogie axles has both a bounce and pitch movement. Since the bogies pitch movement is decoupled from the carbody's, the symmetrical and anti-symmetrical modes of carbody vibration (vehicle) will be determined by the symmetrical and anti-symmetrical axles bounce movements, presented in figure 2.
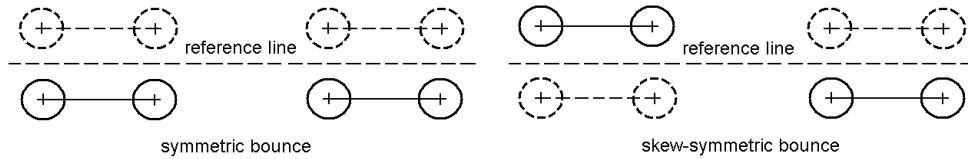

Fig. 2. Excitation modes of wheelsets

The frequency features of the two excitation modes depend by the axle base in bogie and carbody, as well as by velocity. Their forms for the symmetrical and anti-symmetrical modes are as follows

$$\overline{H}^{+}(\omega) = \cos\frac{a_b\omega}{V}\cos\frac{a_c\omega}{V}; \quad \overline{H}^{-}(\omega) = i\cos\frac{a_b\omega}{V}\sin\frac{a_c\omega}{V}, \tag{19}$$

where ω is the angle frequency and $i^2 = -1$.

The excitation modes will bring a series of maximum and minimum values depending on the velocity and may attenuate or not the resonance peaks of the carbody frequency response. For instance, for the frequencies $f = (2n+1)V/4a_b$ with $n = 0, 1, ...$, the wheelsets' bounce is zero and, thus, the carbody bounce and pitch are not excited at these frequencies – the geometric filtering effect of the axle base. The distance between bogies derives the geometric filtering effect for the symmetrical bounce of wheelsets at frequencies $f=(2n+1)V/4a_c$ and for the anti-symmetrical bounce mode at frequencies $f = nV/2a_c$.

Figure 3 shows the frequency feature of the two excitation modes at velocities of 180 and 240 km/h, for $a_c = 19$ m and $a_b = 2.56$ m. Since the distance between bogies is much longer than the one between the wheelsets in a bogie, the frequency features will display itself in the shape of a carrying wavelength, corresponding to the geometric filtering effect of the distance between bogies and modulated in amplitude by the effect of geometric filtering of the distance between wheelsets. The frequency of the geometric filtering effect increases along with the velocity of vehicle. For example, at 180 km/h, the frequency of the geometric filtering effect of axles bounce due to the bogie axle base is of 9.76 Hz, and 13 Hz at 240 km/h. The filtering due to the carbody axle base introduces the first minimum value at frequency of 1.31 Hz for the symmetrical bounce of axles at 180 km/h and of 1.75 Hz for 240 km/h. For the anti-symmetrical bounce, the first minimum shows at 2.63 Hz if velocity is 180 km/h and at 3.5 Hz for 240 km/h. Should one of the geometric filtering frequencies coincides with one of the natural vehicle frequencies, at a certain velocity, then the intensity of the vibration behaviour lowers. Still, at high velocities, the filtering frequency of the axles bounce exceeds the range of the natural vehicle frequencies and this filtering effect exerts a much smaller impact.
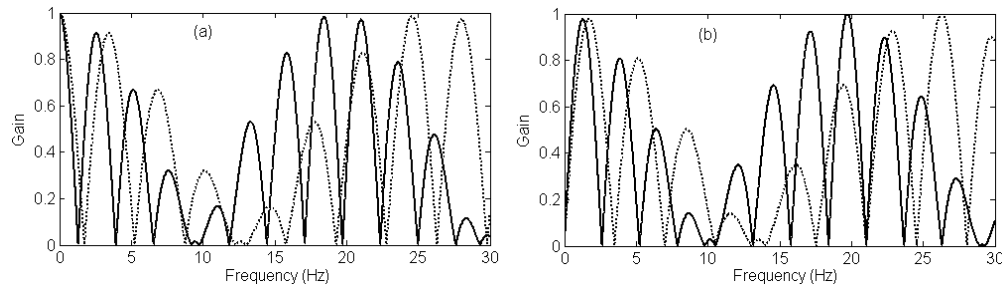

Fig. 3. The frequency features of the vehicle excitation modes:
(a) symmetrical mode; (b) anti-symmetrical mode: ——, $V = 180$ km/h; $\cdots$, $V = 240$ km/h.

The frequency response in a point x of carbody is as below

$$\overline{H}_c(x,\omega) = \overline{H}_{Z_c}(\omega) + \left(\frac{L}{2} - x\right)\overline{H}_{\theta_c}(\omega) + \sum_{i=2}^{3} X_i(x)\overline{H}_{T_i}(\omega),$$
(20)

where $\overline{H}_{z_c}(\omega)$, $\overline{H}_{\theta_c}(\omega)$, $\overline{H}_{T_{2,3}}(\omega)$ are the frequency response corresponding to the vibration modes z_c, θ_c and $T_{2,3}$.

3. The evaluation of comfort compared to the vertical vibrations

3.1. The spectrum of power density of irregularities

In order to evaluate the comfort at vertical vibrations, there will be taken into account the track irregularities as being random and stationary. For the power spectral density of such irregularities, the ORE recommended form is looked at

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)},$$
(21)

where Ω is the wave number, $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m, and $A = 4.032 \cdot 10^{-7}$ rad m or $A = 1.080 \cdot 10^{-6}$ rad m, depending on the track quality.

The track irregularities become an excitation factor for a vehicle that travels at speed V and this is the reason why the power spectral density of the track irregularities need to be expressed as a function of the angle frequency $\omega = V\Omega$, and $G(\omega) = S(\omega/V)/V$ respectively. As a result, the rel. (21) will trigger

$$G(\omega) = \frac{A\Omega_c^2 V^3}{[\omega^2 + (V\Omega_c)^2][\omega^2 + (V\Omega_r)^2]}.$$
(22)

Starting from the carbody frequency response $\overline{H}_c(x,\omega)$ and the power spectral density of track irregularities, the acceleration power spectral density may be calculated at a point located at distance x from the carbody referential

$$G_c(x,\omega) = \omega^4 G(\omega)\left|\overline{H}_c(x,\omega)\right|^2.$$
(23)

Further, based on the acceleration power spectral density, we will have the Wz comfort index, as seen below. An interest will be taken in the acceleration power spectral density in two points, namely at the center of carbody and above the rear bogie.

3.2. Ride index comfort Wz

The Wz index of a vehicle reflects the vehicle ability to maintain the vibrations within the limits that will provide comfort or merchandise integrity (the ride quality the goods wagons).

Wz is a rms value of the frequency-weighted accelerations assessed for definite time limits or definite sections in the track. The mathematical expression, introduced by *Sperling*, is as follows

$$Wz = 4.42(a_{wrms})^{0.3}, \tag{24}$$

where a_{wrms} is the rms value of frequency-weighted acceleration $a_w(t)$ în m/s².

In order to calculate the total index Wz for a continuous spectrum, at a certain carbody point, we have the following formula

$$Wz = \left(2 \int_{0.4}^{30} G_c(f) B^2(f) \mathrm{d}f \right)^{(1/6.67)}, \tag{25}$$

where $G_c(f)$ is the power spectral density of acceleration in cm/s²/Hz, $B(f)$ is a factor of acceleration evaluation and f is the frequency of vibrations.

For the vertical comfort, it has to be

$$B(f) = 0.588 \sqrt{\frac{1.911f^2 + (0.25f^2)^2}{(1 - 0.277f^2)^2 + (1.563f - 0.0368f^3)^2}}. \tag{26}$$

The relation between the comfort index and the sensitivity to vibrations is assessed on a 1 to 4 scale, where Wz = 4 when the vibrations level has a harmful impact upon the human body at a prolonged exposure.

Table 1

Carbody mass	$m_c = 34320$ kg
Bending module	$EI = 3.2 \cdot 10^9$ Nm²
Carbody length	$L = 26.4$ m
Carbody axle base	$2a_c = 19$ m
Carbody gyration radius	$i_c = 7.6$ m
Carbody modal damping ratio	$\zeta_{m2,3} = 0.015$
Vertical rigidity of secondary suspension	$4k_{zc} = 2.4$ MN/m
Vertical damping of secondary suspension	$4c_{zc} = 68.88$ kNs/m
Bogie mass	$m_b = 3200$ kg
Bogie gyration radius	$i_b = 0.8$ m
Bogie axle base	$2a_b = 2.56$ m
Vertical rigidity of primary suspension	$4k_{zb} = 4.4$ MN/m
Vertical damping of primary suspension	$4c_{zb} = 52.21$ kNs/m

4. Numerical application

Based on the model and method above, this section will feature the results of numerical simulation regarding the damping influence upon the frequency response of a carbody on a passenger car, as well as upon the ride comfort (evaluated by the Wz index). The model parameters are shown in table 1.

Figure 4 shows the influence of secondary suspension damping upon each carbody vibration mode. There are also included the frequency responses of carbody movement 240 km/h, for ζ_b = 0.22. The peaks of resonance frequencies of symmetrical modes are noticed – low bounce at 1.17 Hz and symmetrical bending (with 2 nodes) at 8.1 Hz, plus the resonance frequencies of anti-symmetrical modes: low pitch frequency at 1.47 Hz and anti-symmetrical carbody bending (with 3 nodes) at 22.10 Hz. Likewise, a series of anti-resonance frequencies is to be noticed, as due to the geometric filtering effect.

While increasing the secondary suspension damping, there is the tendency to reduce the carbody response at low bounce and low pitch frequency. On the other hand, the increase in damping leads to an intensification of the carbody response within the frequency range of the bending resonance. In fact, at higher frequencies, the damping increase will yield an intensification of vibration behaviour.
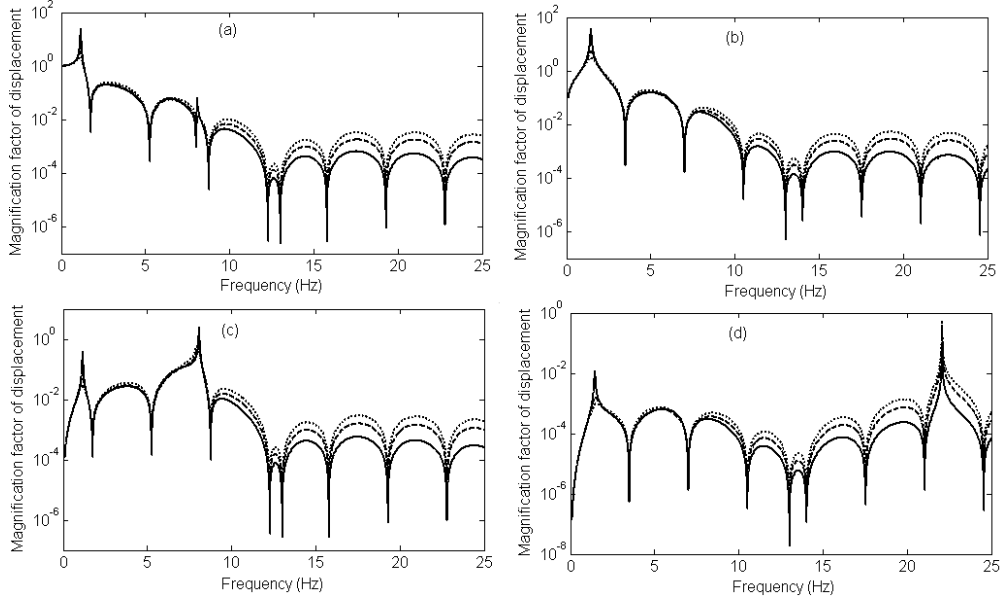

Fig. 4. Influence of secondary suspension damping upon carbody vibration modes: (a) carbody bounce; (b) carbody pitch; (c) symmetrical bending; (d) anti-symmetrical bending;
——, $\zeta_c = 0$; − − −,$\zeta_c = 0.1$; · · · ·,$\zeta_c = 0.2$.

In figure 5, we see the acceleration response at speed 240 km/h for the undamped case (fig. 5, a) and for the reference values of damping ratios, namely $\zeta_b = 0.22$ and $\zeta_c = 0.12$ (fig. 5, b). For the undamped case, besides the peaks of resonance frequency above, there are those for high bounce at 6.65 Hz and high pitch at 6.71 Hz.

At carbody centre, its response is due to only the symmetrical modes of vibration, bounce and symmetrical bending. A higher damping will trigger a lowering in the response at the carbody centre at the low bounce resonance frequency only, whereas this response is amplified in the frequency range of high bounce and bending resonance.

Above bogies, vibration comprises all four vibration modes, and it results into an intensification of the vibration behaviour. The damping increase amplifies this result, except for the bounce and pitch frequency range, where the carbody response is lowered. Intensification in the vibration behaviour may also derive from the fact that the geometrical filtering effect is very much lowered, due to the position above bogie.
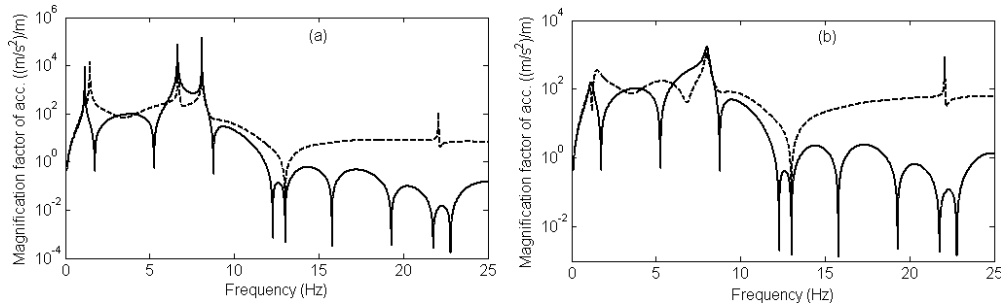

Fig. 5. Influence of damping upon carbody response: (a) $\zeta_b = 0$, $\zeta_c = 0$; (b) $\zeta_b = 0.22$, $\zeta_c = 0.12$;
——, at carbody centre; − − −, above the rear bogie.

The issues of the carbody frequency response are basic properties of vehicles, which do not depend on the size of excitation coming from the track. They will influence the vehicle vibration behaviour while travelling over the track irregularities and, therefore, the comfort (see below).

The geometric filtering effect due to the vehicle axle base impacts the Wz index and acts selectively in dependence with velocity and point position along the carbody (see fig 6). The reference value $\zeta_b = 0.22$ has been considered for the primary suspension vertical damping during the numerical simulations.

The value of the Wz index changes along with velocity increase and its evolution is different compared to the position along carbody. Above bogie, we notice a tendency of Wz index going up with velocity – the carbody centre is a different story, as a series of maximum and minimum values appears, due to the geometric filtering effect.
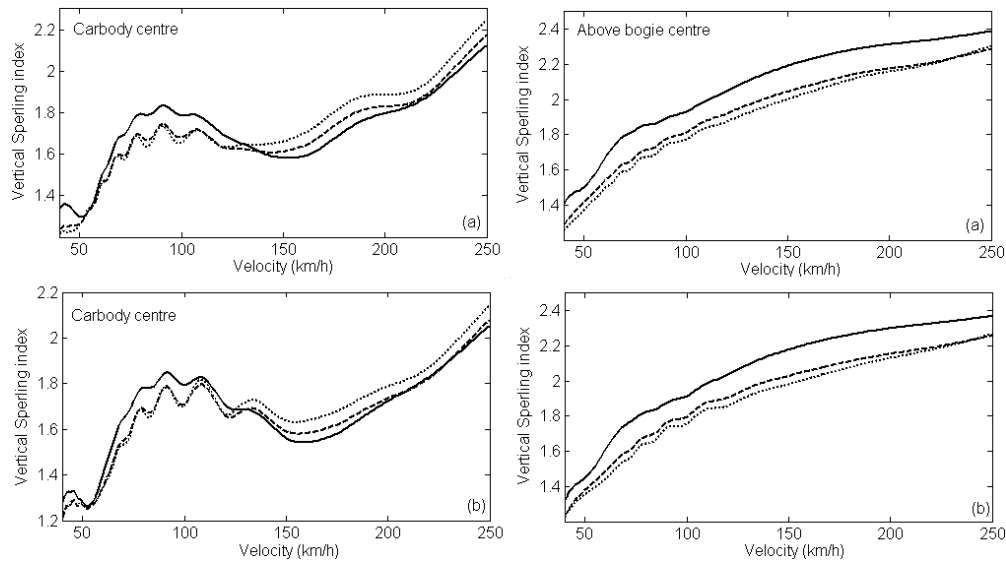
Fig. 6. Influence of velocity and vehicle axle base upon index Wz:
(a) $2a_b$ = 2.56 m, $2a_c$ = 19 m; (b) $2a_b$ = 3 m, $2a_c$ = 19 m;
——, ζ_c = 0.1; − − −,ζ_c = 0.2; · · · ·,ζ_c = 0.3.

The maximum values apply to the condition where the geometric filtering effect is close to a natural frequency of the vehicle. Should the filtering effect frequency (the characteristic is zero) coincides with one of the vehicle natural frequencies (mostly the dominant one), then the Wz index is minimum. Looking at figure 6, *a,* it may be noticed that such areas of maximum and minimum values occur up to 220 km/h, where the wheelset bounce filtering frequency exceeds the range of the vehicle natural frequencies and the filtering effect is weaker.

An increase in the secondary suspension damping ratio has different effects in dependence with the position along the carbody and velocity. At carbody centre, it will lead to lowering the resonance amplitude for low velocities, whereas the vibration intensifies at higher velocities, thus increasing the Wz index. Above bogie, the increase of ζ_c has a favourable impact upon comfort, irrespective of speed.

The increase in the bogie axle base (fig. 6, *b*) generally triggers a lowering in vibrations level. There are exceptions, namely at carbody centre, where the increase of a_b has a contrary effect upon the Wz comfort index for velocities of 90 – 140 km/h. The velocity introduces a selective filtering similarly with the axle bases. For big bogie axle bases, filtering lobes height rises and filtering has a lower effect at the vehicle resonance frequencies. This fact explains that, at a 120 km/h velocity and a secondary suspension damping ratio 0.2, the increase of bogie axle base from 2.56 to 3 m will lead to a Wz from 1.63 to 1.65. On the contrary, the same change of axle base improves the vibratory comfort at high velocities.

At 240 km/h and the same damping ratio $\zeta_c = 0.2$, there is a lower comfort index Wz by 4%, at carbody centre and above bogie by about 2 %.

The secondary suspension damping greatly influences the ride comfort, as seen in figure 7. In fact, there is a value for damping that minimizes the carbody vibration level in any point of it. The explanation is that, on the one hand, this damping limits the vibration at resonance frequencies at small damping values and, on the other hand, when damping is high, the system dynamic rigidity rises, thus the vibrations behaviour is more intense.

The analysis of diagrams shows that index Wz has a minimum value corresponding to a certain damping value, which depends on the position along carbody (centre or above bogie) and on velocity. It may be noticed that comfort index Wz becomes minimum at carbody centre for small values of damping ratio. Increasing damping to a definite value means a general comfort improvement. Beyond this limit, damping has contrary effects, enhances comfort above bogie and worsens it at carbody centre. The idea is to minimize the Wz value at carbody critical point (where vibrations level is higher) to obtain the best damping. Diagrams show that the critical point is above bogie. For instance, at 120 km/h, damping ratio that minimizes the index Wz in this point is $\zeta_c = 0.36$, 0.3 at $V = 180$ km/h and 0.23 for 240 km/h.
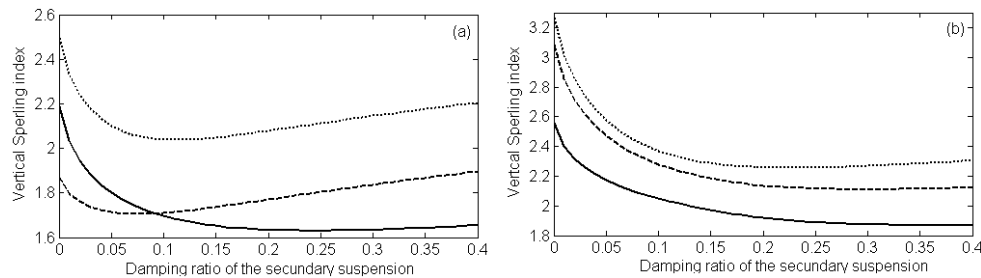

Fig. 7. Influence of secondary suspension damping upon index Wz:
(a) carbody centre; (b) above rear bogie;
———, $V = 120$ km/h; − − −, $V = 180$ km/h;; · · · ·, $V = 240$ km/h.

So far, the focus has been placed on how the secondary suspension damping ratio influences the vibrations behaviour of a vehicle and, therefore, the comfort evaluation index. Even though the primary suspension damping ratio cannot be raised as much as desired due to the magnitude of wheel-rail contact dynamic forces, it is still interesting to analyse to what extent this parameter influences the carbody vibrations behaviour.

In figure 8, the index Wz in calculated for 120, 180 and 240 km/h. For the usual domain of primary suspension damping values, it may be noticed that an increased damping reduces the vibrations level and the index Wz, where this lowering is uniform in both points under study. It should be highlighted that there

is also a damping value for this case that minimizes the carbody vibrations level in any point, but this value is extremely high and therefore has no practical interest. For instance, at 240 km/h, the minimum value of Wz at carbody centre is obtained for $\zeta_b = 2.25$, and above bogie for $\zeta_b = 1.66$.

Upon comparing the influence of damping of two-level suspension upon comfort, it is evident that primary suspension damping has a small influence. Indeed, the bogie mass comes between the primary suspension and carbody, and whose inertia reduces the damping efficiency of the primary suspension.
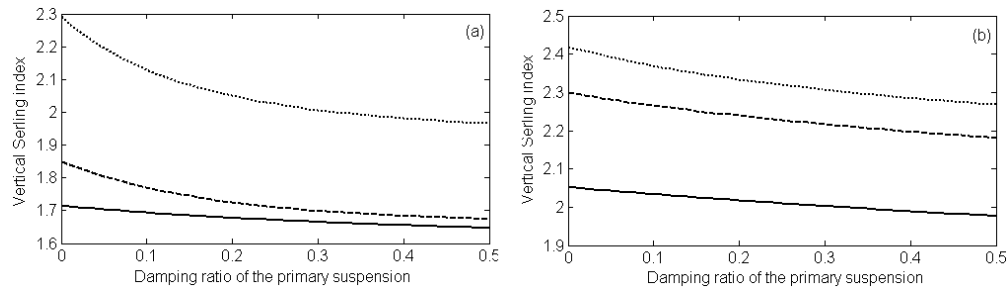

Fig. 8. Influence of primary suspension damping ratio upon index Wz:
(a) carbody centre; (b) above rear bogie;
——, $V = 120$ km/h; − − −, $V = 180$ km/h;; · · · ·, $V = 240$ km/h.

Finally, a last issue to be looked at refers to the influence of the carbody bending rigidity, along with the secondary suspension damping ratio upon comfort at vertical vibrations. Figure 9 presents the values of index Wz calculated for three situations, elastic carbody (bending frequency of 6 Hz and 8.1 Hz namely – reference value) and rigid carbody.
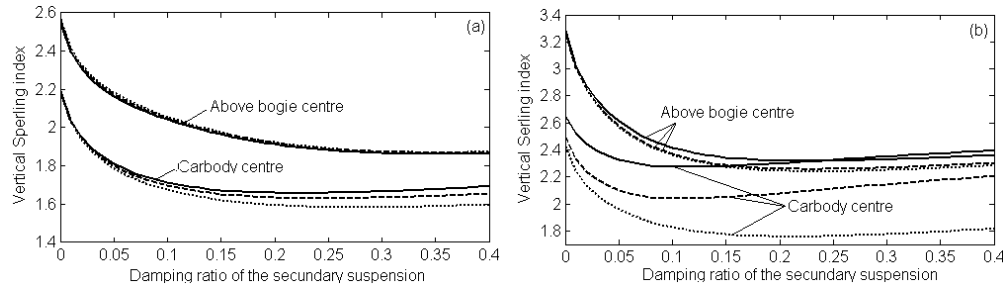

Fig. 9. Influence of carbody flexibility upon index Wz: (a) $V = 120$ km/h; (b) $V = 240$ km/h;
——, flexible carbody (bending frequency 6 Hz); − − −, flexible carbody
(bending frequency 8.1 Hz); · · · ·, rigid carbody.

Both diagrams show the advantage of a rigid cardbody versus elastic carbody, at the centre and mainly at high velocities. For the damping ratio considered reference $\zeta_c = 0.15$ at 240 km/h, we will have the same values for Wz:

2.27 – elastic carbody (bending frequency 6 Hz), 2.05 – for elastic carbody (bending frequency of 8.1 Hz) and 1.77 – for rigid carbody. In the above bogie point, the use of rigid carbody brings an improvement in vibratory comfort at only high velocities. For example, at 240 km/h, decreasing of index Wz is circa 3%, if comparison is made with the elastic carbody with a bending frequency of 6 Hz and 0.5% if the carbody has the bending frequency of 8.1 Hz.

It is important to notice that the vibrations level is higher above bogie at 120 km/h, irrespective of carbody elasticity characteristics; for higher velocities, the critical point may migrate from a flexible carbody end to centre while the secondary suspension damping increases.

5. Conclusions

The suspension damping plays an essential role in terms of the carbody vibrations behaviour and the ride comfort. The selection of damping ratio is a sensitive issue when a too small damping may compromise the vehicle dynamic performance, and a too high damping leads to an increase in the system dynamic rigidity and, therefore, to an intensification of the vibration behaviour.

The model of a passenger car with two-level suspension has been considered, where the carbody is modelled via an Euler-Bernoulli beam and connected by Kelvin-Voigt systems by the two rigid bogies. Upon applying the modal analysis, the symmetrical and anti-symmetrical decoupled movements and their excitation modes have been pointed out.

The analysis of the vibrations behavior due to the track random irregularities has proved that the level of vibrations increases along with the velocity and it is influenced by the geometric filtering effect given by the vehicle axle bases.

A solution for improving the vibrations behavior based on the increase in the suspension floors damping has been considered limited. On the one hand, it is a known fact that the primary suspension damping ratio cannot be increased too much, because of the limitation imposed on by the size of wheel-rail contact dynamic forces. On the other hand, raising the secondary suspension damping ratio has different effects in dependence of the position along the carbody and velocity. At carbody centre, it acts for lowering the amplitude at resonance within the small velocities range, while higher velocities trigger the amplification of vibrations behaviour and then to comfort worsening. Above bogie, damping increase has favourable effects upon comfort, irrespective of velocity.

The position of the critical point from the vibrations perspective depends on the velocity. Generally speaking, the critical point is above bogie. There is also the possibility that the critical point at high velocities is found at carbody centre when this one shows great elasticity at bending.

In a nutshell, the secondary suspension damping cannot be established but only as a compromise. The selection of the damping ratio is made by starting with the idea to have the best comfort level at the carbody critical point.

Other solutions that may constitute alternatives for improving the comfort of passenger cars at high velocities refer to using rigid carbodies – difficult to achieve in many circumstances – or to increasing the bogie axle base, provided that the vehicle dynamic behaviour is not tampered with during curve travelling.

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