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THERMODYNAMIC TRIPOLES: A FRAMEWORK FOR STUDYING AND OPTIMIZING IRREVERSIBLE MACHINES

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Inspired by dipoles from the Electrical Circuits Theory, the Thermodynamic Tripoles *are an original approach providing a common conceptual framework to various branches of Irreversible Engineering Thermodynamics, thus allowing them to cooperate in evaluating various irreversibilities. We find that a generic thermal machine can be described with two characteristic functions, while certain special cases (brakes, heat exchangers and reversible machines) need only one function. As a proof of validity, a formula for the mechanical power is found and then used to recover the known Curzon-Ahlborn result regarding endoreversible machines.*

Keywords: Finite Speed Thermodynamics, Finite Time Thermodynamics, Thermal Machines Optimization.

1. Introduction

Various branches of the Irreversible Engineering Thermodynamics developed independently, each focusing on certain types of irreversibilities. Examples include Finite Speed Thermodynamics (FST: [1], [2], [3], [4], [5]) and Finite Time Thermodynamics (FTT: [6], [7], [8], [9]). Each of them has its own concepts, tools and methods. The *Thermodynamic Tripoles* framework is an original approach in which concepts specific (at least) to FST and FTT can be easily expressed, allowing us to study their interaction and explore possible ways of integration into a single broader theory. Inspired by electrical dipoles, a *thermodynamic tripole* describes a thermodynamic system through a number of functions relating the energy flows of the system to the thermodynamic forces acting upon it. We will find that in the general case two such functions are needed, while certain degenerate cases (brakes, heat exchangers and reversible machines) need only one. We will apply the thermodynamic tripoles theory to an endoreversible Carnot machine to recover the known Curzon-Ahlborn result [6].

2. Definition

We call *thermodynamic tripole* a thermodynamic system Ω which exchanges energy with its environment in at most three ways: 1) heat Q_H

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exchanged at temperature T_H, 2) heat Q_L exchanged at temperature T_L, and 3) work W, so that after a time τ its internal energy U and entropy S are unchanged. We will call the time duration τ the *cycle duration*.

A tripole is represented graphically as shown in Fig. 1.

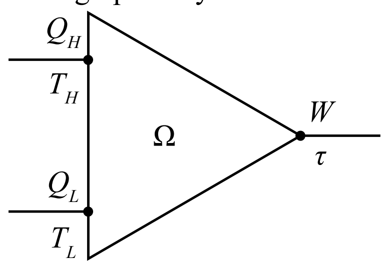

Fig. 1. The schematic representation of a tripole

For a given tripole, we consider that the temperatures T_H and T_L and the cycle duration τ determine uniquely the three energy exchanges Q_H, Q_L and W. This dependency is expressed by the following three functions:

$$\begin{cases} Q_H = \varphi_H(T_H, T_L, \tau) \\ Q_L = \varphi_L(T_H, T_L, \tau) \\ W = \varphi_W(T_H, T_L, \tau) \end{cases} \tag{2.1}$$

These functions characterize completely the tripole from a thermodynamic viewpoint. Based on these, we define the following functions:

$$\begin{cases} f_H(T_H, T_L, \tau) = \dfrac{\varphi_H(T_H, T_L, \tau)}{T_H \tau} \\ f_L(T_H, T_L, \tau) = -\dfrac{\varphi_L(T_H, T_L, \tau)}{T_L \tau} \\ f_W(T_H, T_L, \tau) = \dfrac{\varphi_W(T_H, T_L, \tau)}{\left(T_H - T_L\right)\tau} \end{cases} \tag{2.2}$$

With these, the energies exchanged by the tripole are:

$$\begin{cases} Q_H = T_H \tau \cdot f_H(T_H, T_L, \tau) \\ Q_L = -T_L \tau \cdot f_L(T_H, T_L, \tau) \\ W = \left(T_H - T_L\right)\tau \cdot f_W(T_H, T_L, \tau) \end{cases} \tag{2.3}$$

The *average energy fluxes* (averaged over the cycle duration) are:

$$\begin{cases} \bar{\dot{Q}}_H = T_H \cdot f_H(T_H, T_L, \tau) \\ \bar{\dot{Q}}_L = -T_L \cdot f_L(T_H, T_L, \tau) \\ \bar{\dot{W}} = \left(T_H - T_L\right) \cdot f_W(T_H, T_L, \tau) \end{cases} \tag{2.4}$$

3. The Thermodynamics Laws

Let us apply the Thermodynamics Laws to the tripole.

The tripole's internal energy being the same at the beginning and at the end of the cycle, it follows that the internal energy variation is zero:

$$\Delta U = 0 \tag{3.1}$$

From the First Law it follows that:

$$W = Q_H + Q_L, \tag{3.2}$$

which means:

$$f_W(T_H, T_L, \tau) = \frac{T_H}{T_H - T_L} f_H(T_H, T_L, \tau) - \frac{T_L}{T_H - T_L} f_L(T_H, T_L, \tau) \tag{3.3}$$

This equation shows that if we know the functions f_H and f_L, then the function f_W is determined. This is the reason why we can characterize the tripole using only the two functions f_H and f_L, which we will call the tripole's *characteristic functions*.

The tripole's entropy being the same at the beginning and at the end of the cycle, the entropy variation is zero:

$$\Delta S = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} + S_{gen} = 0 \tag{3.4}$$

S_{gen} is the entropy generated inside the tripole, which is nonnegative (the Second Law guarantees this):

$$S_{gen} = -\left(\frac{Q_H}{T_H} + \frac{Q_L}{T_L}\right) \geq 0, \tag{3.5}$$

which means:

$$f_H(T_H, T_L, \tau) \leq f_L(T_H, T_L, \tau) \tag{3.6}$$

We conclude that any thermodynamic tripole Ω (obeying the Thermodynamics Laws) can be completely described by two functions f_H and f_L which satisfy the inequality (3.6). We will consider that the tripole <u>is</u> such a pair of functions:

$$\boxed{\Omega = (f_H, f_L), \quad f_H : \mathbb{R}_+^3 \to \mathbb{R}, \quad f_L : \mathbb{R}_+^3 \to \mathbb{R}, \quad f_H \leq f_L} \tag{3.7}$$

Knowing the characteristic functions, we can find the average energy fluxes exchanged by the tripole with the environment in any situation (i.e., for any given parameters T_H, T_L and τ):

$$\begin{cases} \bar{\dot{Q}}_H = T_H \cdot f_H(T_H, T_L, \tau) \\ \bar{\dot{Q}}_L = -T_L \cdot f_L(T_H, T_L, \tau) \\ \bar{\dot{W}} = T_H \cdot f_H(T_H, T_L, \tau) - T_L \cdot f_L(T_H, T_L, \tau) \end{cases} \tag{3.8}$$

4. Special cases

4.1 Brake

Let us suppose that one of the heat fluxes is zero – e.g. let's take $\bar{\dot{Q}}_L = 0$. From the definition of the tripole it follows that the function f_L is zero. But in this case the inequality (3.6) says that:

$$f_H(T_H, T_L, \tau) \le 0, \tag{4.1}$$

which means that the heat flux $\bar{\dot{Q}}_H$ is nonpositive:

$$\bar{\dot{Q}}_H = T_H \cdot f_H(T_H, T_L, \tau) \le 0, \tag{4.2}$$

and the average power will also be nonpositive:

$$\bar{\dot{W}} = \bar{\dot{Q}}_H + \bar{\dot{Q}}_L = \bar{\dot{Q}}_H + 0 \le 0 \tag{4.3}$$

If the other heat flux ($\bar{\dot{Q}}_H$) is zero, from (3.6) it follows that f_L is nonnegative, so the flux $\bar{\dot{Q}}_L$ is nonpositive:

$$\bar{\dot{Q}}_L = -T_L \cdot f_L(T_H, T_L, \tau) \le 0, \tag{4.4}$$

which means the power will also be nonpositive:

$$\bar{\dot{W}} = \bar{\dot{Q}}_H + \bar{\dot{Q}}_L = 0 + \bar{\dot{Q}}_L \le 0 \tag{4.5}$$

In conclusion, a monothermal tripole (i.e., exchanging heat with only one external system) can only receive work – which it will transform entirely in heat released into the environment. This is a classical consequence of the Second Law, which confirms the validity of the mathematical model described here.

We call this kind of monothermal tripole a *brake* and we represent it graphically as in Fig. 2. Brakes have only one characteristic function (the other one being equal to zero). Although any of the two functions f_H and f_L can be nonzero (as we have seen above), we convene to always make $f_H = 0$ and keep f_L as the single characteristic function of a brake:

$$\Omega_{brake} = \left(0,\ f(T_L, \tau)\right) \tag{4.6}$$

$$\begin{cases} \bar{\dot{Q}}_H = 0 \\ \bar{\dot{Q}}_L = -T_L \cdot f(T_L, \tau) \\ \bar{\dot{W}} = \bar{\dot{Q}}_L \end{cases} \tag{4.7}$$

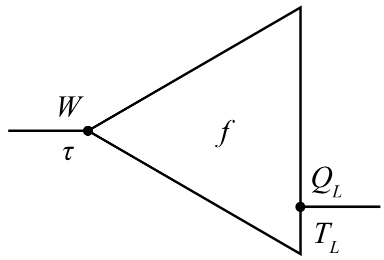

Fig. 2. A brake tripole

Brakes can be used to model various irreversibility sources. For example, friction losses directly proportional to speed can be modeled with a brake having the characteristic function:

$$f(T_L, \tau) = \frac{1}{T_L} \cdot k \cdot \underbrace{\left(\frac{L}{\tau}\right)}_{\text{speed } w}, \tag{4.8}$$

where k [J/m] is a constant, L is a characteristic size of the system (e.g., the piston's stroke), and $L/\tau = w$ is the average speed. This brake receives mechanical work and converts it entirely into heat, which is released into the environment:

$$\begin{cases} \bar{\dot{Q}}_H = 0 \\ \bar{\dot{Q}}_L = -kw \\ \bar{\dot{W}} = -kw \end{cases} \tag{4.9}$$

4.2. Heat exchanger

If the tripole doesn't exchange mechanical work with its environment, we have a heat exchanger. In this case the two characteristic functions f_H and f_L are additionally constrained by the "zero power" condition:

$$\bar{\dot{W}} = T_H \cdot f_H(T_H, T_L, \tau) - T_L \cdot (T_H, T_L, \tau) = 0 \tag{4.10}$$

$$f_L(T_H, T_L, \tau) = \frac{T_H}{T_L} f_H(T_H, T_L, \tau) \tag{4.11}$$

One of the characteristic functions can be derived from the other. Consequently, such a tripole can be characterized using only one function:

$$\Omega_{heat\ exchanger} = (f(T_H, T_L, \tau),\ T_H / T_L \cdot f(T_H, T_L, \tau)), \tag{4.12}$$

which determines the energy fluxes:

$$\begin{cases} \bar{\dot{Q}}_H = T_H \cdot f(T_H, T_L, \tau) \\ \bar{\dot{Q}}_L = -\bar{\dot{Q}}_H \\ \bar{\dot{W}} = 0 \end{cases} \tag{4.13}$$

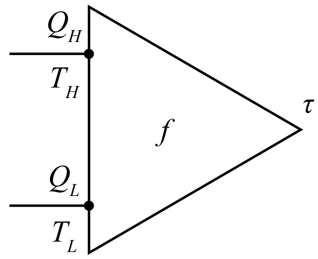

Fig. 3. A heat exchanger tripole

4.2.1. Limited duration heat transfer

Let us consider a heat exchanger tripole which exchanges a constant heat flux $\dot{Q}_H$ during the time period $\tau_{exchange} < \tau$, and exchanges no heat during the rest of the cycle. The heat exchanged during a full cycle will be:

$$Q_H = \dot{Q}_H \tau_{exchange} \tag{4.14}$$

The characteristic function of this heat exchanger is:

$$f(T_H, T_L, \tau) = \frac{Q_H}{T_H \tau} = \frac{\dot{Q}_H}{T_H} \cdot \frac{\tau_{exchange}}{\tau} \tag{4.15}$$

and the average heat flux follows:

$$\bar{\dot{Q}}_H = T_H \cdot f(T_H, T_L, \tau) = \dot{Q}_H \frac{\tau_{exchange}}{\tau} \tag{4.16}$$

4.2.2. Newtonian heat transfer

The Newtonian heat transfer means a linear dependency between the temperature difference and the heat flux:

$$\dot{Q}_H = K(T_H, T_L, \tau) \cdot \left(T_H - T_L\right), \tag{4.17}$$

where K is the thermal conductance (which may depend on the temperatures and on the cycle duration).

Substituting this into equations (4.15) and (4.16), we obtain the characteristic function of the Newtonian heat exchanger:

$$f(T_H, T_L, \tau) = K(T_H, T_L, \tau) \cdot \left(1 - \frac{T_L}{T_H}\right) \frac{\tau_{exchange}}{\tau} \tag{4.18}$$

and its average heat flux:

$$\bar{\dot{Q}}_H = K(T_H, T_L, \tau) \cdot \left(T_H - T_L\right) \frac{\tau_{exchange}}{\tau} \tag{4.19}$$

4.3. Reversible tripole

If in equation (3.5) expressing The Second Law we take equality, we have a reversible tripole (S_{gen} = 0). From (3.6) it follows that the functions f_H and f_L become equal. This means that a reversible tripole is described by a single characteristic function:

$$\Omega_{reversible} = (f(T_H, T_L, \tau),\ f(T_H, T_L, \tau)), \tag{4.20}$$

with the energy fluxes given by:

$$\begin{cases} \bar{\dot{Q}}_H = T_H \cdot f(T_H, T_L, \tau) \\ \bar{\dot{Q}}_L = -T_L \cdot f(T_H, T_L, \tau) \\ \bar{\dot{W}} = \left(T_H - T_L\right) \cdot f(T_H, T_L, \tau) \end{cases} \tag{4.21}$$

We see that the efficiency of this tripole (viewed as a motor) is given by the Carnot formula:

$$\eta = \frac{\bar{\dot{W}}}{\bar{\dot{Q}}_H} = 1 - \frac{T_L}{T_H} \tag{4.22}$$

This result should not be surprising: indeed, if the tripole contains a gas, the setup described in the definition leads to a Carnot cycle (or equivalent – Stirling with perfect heat regeneration).

Important remark: As opposed to an irreversible tripole, which has two distinct characteristic functions f_H and f_L, the reversible tripole can be described with only one characteristic function f. Unfortunately, this creates confusion when one studies first the reversible case and then wants to advance to the irreversible case: the habit of describing the system with only one function, acquired while studying the reversible case, leads to an attempt to describe also the irreversible system with one function – which is incorrect and leads to contradictions.

<u>4.3.1. Reversible Carnot machine</u>

Let us consider that the tripole is a reversible Carnot machine with one cylinder containing a perfect gas. During the isothermal transformations we consider constant instantaneous heat fluxes, equal to $\dot{Q}_H$ and $\dot{Q}_L$, respectively.

<u>The isothermals:</u>

By integrating The First Law on the isothermal expansion, we find that the mechanical work is equal to the heat received from the hot source:

$$W_H = Q_H = mRT_H \ln(V_2 / V_1) \tag{4.23}$$

If this heat is receive in constant flux, the time needs to be:

$$\tau_H = \frac{Q_H}{\dot{Q}_H} = \frac{mRT_H}{\dot{Q}_H} \ln\left(V_2 / V_1\right) \tag{4.24}$$

Similarly, for the cold isothermal (with negative $\dot{Q}_L$):

$$W_L = Q_L = mRT_L \ln(V_4 / V_3) \tag{4.25}$$

$$\tau_L = \frac{mRT_L}{\dot{Q}_L} \ln\left(V_4 / V_3\right) \tag{4.26}$$

<u>The adiabats:</u>

Let us denote:

$$\theta = \frac{T_H}{T_L} \tag{4.27}$$

By integrating The First Law on the adiabatic expansion, we get:

$$\frac{V_3}{V_2} = \theta^{\frac{1}{\gamma-1}} \tag{4.28}$$

The stroke is:

$$L_{23} = \frac{V_3 - V_2}{A_p} = \frac{V_2}{A_p}\left(\theta^{\frac{1}{\gamma-1}} - 1\right) \tag{4.29}$$

If the piston travels this distance with the average speed $\bar{w}_{23}$, the duration must be:

$$\tau_{23} = \frac{L_{23}}{\bar{w}_{23}} = \frac{V_2}{A_p \bar{w}_{23}}\left(\theta^{\frac{1}{\gamma-1}} - 1\right) \tag{4.30}$$

The mechanical work is:

$$W_{23} = \frac{mR\left(T_H - T_L\right)}{\gamma - 1} \tag{4.31}$$

Similarly, for the adiabatic compression (with the negative speed $\bar{w}_{41}$):

$$\frac{V_4}{V_1} = \theta^{\frac{1}{\gamma-1}} \tag{4.32}$$

$$\tau_{41} = -\frac{V_1}{A_p \bar{w}_{41}}\left(\theta^{\frac{1}{\gamma-1}} - 1\right) \tag{4.33}$$

$$W_{41} = -\frac{mR\left(T_H - T_L\right)}{\gamma - 1}, \tag{4.34}$$

which we see is the opposite of the mechanical work on the adiabatic expansion.

<u>The cycle:</u>

We can compute the cycle duration:

$$\tau = \tau_H + \tau_L + \tau_{23} + \tau_{41} \tag{4.35}$$

$$\tau = \frac{mRT_H}{\dot{Q}_H}\ln\left(V_2/V_1\right) + \frac{mRT_L}{\dot{Q}_L}\ln\left(V_4/V_3\right) + \\ + \frac{V_2}{A_p\overline{w}_{23}}\left(\theta^{\frac{1}{\gamma-1}} - 1\right) - \frac{V_1}{A_p\overline{w}_{41}}\left(\theta^{\frac{1}{\gamma-1}} - 1\right) \tag{4.36}$$

We define the total compression ratio (L_{max} is the total length of the cylinder, and L_{min} is the length of the "dead space"):

$$\lambda = \frac{L_{max}}{L_{min}} = \frac{V_{max}}{V_{min}} = \frac{V_3}{V_1}, \tag{4.37}$$

so that after a few calculations we can write the cycle duration:

$$\tau = \left(\frac{T_H}{\dot{Q}_H} - \frac{T_L}{\dot{Q}_L}\right) mR\ln\left(\frac{\lambda}{\theta^{\frac{1}{\gamma-1}}}\right) + \frac{L_{max}}{\overline{w}_{23}}\left(\frac{\lambda}{\theta^{\frac{1}{\gamma-1}}} - \frac{\overline{w}_{23}}{\overline{w}_{41}}\right)\left(\frac{\theta^{\frac{1}{\gamma-1}}}{\lambda} - \frac{1}{\lambda}\right) \tag{4.38}$$

We see that the cycle duration is determined by the temperatures, which makes the characteristic function of the tripole become a function of the temperatures alone. From the first equation of the system (4.21) we express the characteristic function:

$$f(T_H,T_L) = \frac{Q_H}{T_H\tau(T_H,T_L)} = \frac{mRT_H\ln(V_2/V_1)}{T_H\tau(T_H,T_L)} = \frac{mR\ln\left(\frac{\lambda}{\theta^{\frac{1}{\gamma-1}}}\right)}{\tau(T_H,T_L)} = \\ = \frac{1}{\left(\frac{T_H}{\dot{Q}_H} - \frac{T_L}{\dot{Q}_L}\right) + \frac{L_{max}}{mR\overline{w}_{23}} \frac{\left(\frac{\lambda}{\theta^{\frac{1}{\gamma-1}}} - \frac{\overline{w}_{23}}{\overline{w}_{41}}\right)\left(\frac{\theta^{\frac{1}{\gamma-1}}}{\lambda} - \frac{1}{\lambda}\right)}{\ln\left(\frac{\lambda}{\theta^{\frac{1}{\gamma-1}}}\right)}} \tag{4.39}$$

The average energy fluxes are:

$$\begin{cases} \bar{\dot{Q}}_H = T_H \cdot f(T_H,T_L) \\ \bar{\dot{Q}}_L = -T_L \cdot f(T_H,T_L) \\ \bar{\dot{W}} = \left(T_H - T_L\right)\cdot f(T_H,T_L) \end{cases} \tag{4.40}$$

<u>Very fast adiabats:</u>

We note that the average power is decreased by the second term in the denominator of (4.39), which is always positive. Because of this, let us consider that we can increase the adiabats speeds enough that that term becomes negligible. The characteristic function becomes:

$$f(T_H, T_L) = \frac{1}{\frac{T_H}{\dot{Q}_H} - \frac{T_L}{\dot{Q}_L}} \tag{4.41}$$

and the energy fluxes can be written simply:

$$\begin{cases} \bar{\dot{Q}}_H = \dfrac{T_H}{\frac{T_H}{\dot{Q}_H} - \frac{T_L}{\dot{Q}_L}} \\ \bar{\dot{Q}}_L = \dfrac{-T_L}{\frac{T_H}{\dot{Q}_H} - \frac{T_L}{\dot{Q}_L}} \\ \bar{\dot{W}} = \dfrac{T_H - T_L}{\frac{T_H}{\dot{Q}_H} - \frac{T_L}{\dot{Q}_L}} \end{cases} \tag{4.42}$$

Optimizing the temperatures:

Assuming Newtonian heat transfer from a source with temperature T_{SH} and to a sink with temperature T_{SL}, the power can be written by simply plugging in the Newtonian heat transfer formulae (4.17) for the instantaneous heat flux into the average mechanical power formula from (4.42), obtaining:

$$\bar{\dot{W}} = \frac{T_H - T_L}{\frac{T_H}{K_H (T_{SH} - T_H)} - \frac{T_L}{K_L (T_{SL} - T_L)}} \tag{4.43}$$

In order to obtain the maximum power, we nullify both partial derivatives with respect to the temperatures. After calculations, we get the optimum point:

$$\frac{T_L^*}{T_H^*} = \sqrt{\frac{T_{SL}}{T_{SH}}} \tag{4.44}$$

$$T_L^* = \sqrt{T_{SL}} \frac{\sqrt{T_{SH}}\sqrt{K_H} + \sqrt{T_{SL}}\sqrt{K_L}}{\sqrt{K_H} + \sqrt{K_L}} \tag{4.45}$$

$$T_H^* = \sqrt{T_{SH}} \frac{\sqrt{T_{SH}}\sqrt{K_H} + \sqrt{T_{SL}}\sqrt{K_L}}{\sqrt{K_H} + \sqrt{K_L}} \tag{4.46}$$

$$\bar{\dot{W}}_{max} = \frac{\left(\sqrt{T_{SH}} - \sqrt{T_{SL}}\right)^2}{\left(\frac{1}{\sqrt{K_L}} + \frac{1}{\sqrt{K_H}}\right)^2} \tag{4.47}$$

This is the known Curzon-Ahlborn result – which confirms that the formulae proven in this section are valid.

Remark: Although the results of this section were obtained for a piston-cylinder machine, they are also valid for continuous flux machines – we can consider that the tripole is a kilogram of gas exchanging heat and work with parts of the machine as it travels cyclically through it; at the end we just have to multiply all the fluxes with the number of kilograms of gas found in the machine.

5. Conclusions

Inspired by the notion of electrical dipoles, we propose a new framework for studying thermal machines: the thermodynamic tripoles. Any thermodynamic system is described through two characteristic functions in the general case (or just one function in some special, degenerate cases). These functions allow us to model conveniently both the internal irreversibilities (caused by the finite speed of the machine) and the external irreversibilities (caused by heat exchange in finite time).

When only external irreversibilities are taken into account (and the adiabats are very fast), a formula is derived for the average mechanical power:

$$\bar{\dot{W}} = \frac{T_H - T_L}{\frac{T_H}{\dot{Q}_H} - \frac{T_L}{\dot{Q}_L}} \tag{5.1}$$

Using this formula, the known Curzon-Ahlborn result for endoreversible machines follows naturally.

Even though here we detailed the endoreversible case only, essentially the same procedure can be used to compute the average power when the machine also has internal irreversibilities – which can be given by FST. This makes the tripoles framework a valuable tool for optimizing thermal machines in a relevant way for practical engineering situations.

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