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EFFECT OF DISTINCT CONDUCTIVE AND THERMODYNAMIC TEMPERATURES ON THE REFLECTION OF PLANE WAVES IN MICROPOLAR ELASTIC HALF-SPACE

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The present investigation is concerned with wave propagation in micropolar thermoelastic solid half space with distinct conductive and thermodynamic temperatures. Reflection of plane waves incident obliquely at the free surface of micropolar generalized thermoelastic solid half space with two temperature is investigated. Amplitude ratios various reflected waves are obtained in closed form and it is found that these are function of angle of incidence, frequency and are affected by the micropolar thermoelastic properties of the medium. Effect of two temperatures is shown on these amplitude ratios for a specific model. Results of some earlier workers have also been deduced from the present investigation as a special case.

Keywords: Micropolar thermoelastic solid, Conductive and thermodynamic temperatures, Elastic waves, Reflection coefficient, Transmission coefficient.

1. Introduction

Thermoelasticity with two temperature is one of the non-classical theories of thermoelasticity of elastic solids. The main difference of this theory with respect to the classical one is the thermal dependence. The theory of heat conduction in a deformable body, formulated by Chen and Gurtin [1], Chen , Gurtin and William[2, 3] depends on two different temperatures, the conductive temperature Φ and thermodynamic temperature T. Boley and Tolins[4], Warren and Chen [5] investigated the wave propagation in the two temperature theory of thermoelasticity.Youssef [6], Puri and Jordan [7] studied the propagation of plane waves in thermoelastic medium with two temperature model. Youssef, Al-Lehaibi [8] and Youssef, Al-Harby [9] and Magana, Quintanilla [10] investigated various problems on the basis of two temperature thermoelasticity with relaxation time. Mukhopadhyay, Kumar [11] studied thermoelastic interaction on two temperature generalized thermoelasticity in an infinite medium with a cylindrical cavity. Recently, Roushan, Santwana [12] and Kaushal, Sharma Kumar [13] studied the propagation of waves in generalized thermoelastic continua with two

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temperatures. Kaushal, Kumar and Miglani [14] studied the wave propagation in temperature rate dependent thermoelasticity with two temperatures.
The comprehensive review on the micropolar elasticity was given by Eringen [15-17] and Nowacki [18]. Touchert, Claus Armin [19] also derived the basic equations of the linear theory of micropolar coupled thermoelasticity.
Dost, Taborrok [20] present a generalized Green and Lindsay theory. Chandrasekharaiah [21] developed a heat flux dependent micropolar thermoelsticity. Boschi and Iesan [22] presented micropolar thermoelasticity that permits the transmission of heat as thermal waves at finite speed.
Parfitt and Eringen [23] and obtained the expressions for amplitude ratios of different reflected waves in a closed form. Kumar and Singh [24], [25] studied the problems of reflection of plane waves from the flat boundary of a micropolar generalized thermoelastic with stretch and without stretch respectively.Tomar, Kumar and Kaushik [26] obtained the reflection coefficients in micropolar elastic half-space with stretch. Kumar [27] investigated the reflection coefficient in micropolar viscoelastic generalized half-space.

Kumar and Sharma [28] obtained the amplitude ratios from the stress free boundary in a micropolar thermoelastic half space without energy dissipation. Hisa and Cheng [29] and Hisa, Chiu, Su and Chen [30] investigated propagation of longitudinal and transverse waves in elastic micropolar porous media. Singh[31], Kumar and Rupender [32, 33] investigated wave propagation at the free surface of magneto thermo-microstretch elastic solid.

Marin investigated some theorems in micropolar thermoelastic materials [36-38]. Marin [39] presented some results in nonlinear micropolar thermoelastic bodies with voids. Lagrange identity method for microstretch thermoelastic material was studied by Marin [40]. Recently Marin [41] investigated some weak solutions in elasticity of dipolar bodies with stretch.

In this paper, we study the problem of reflection of plane waves at the free surface of micropolar generalized thermoelastic solid half space with two temperatures. Effect of two temperatures is depicted graphically on the amplitude ratios for incidence of various plane waves, that is, Longitudinal displacement wave (LD wave), Thermal wave (T wave), Coupled transverse wave (CD-I wave and CD-II wave).

2. Basic equations

The field equations in an isotropic, homogeneous, micropolar elastic body in the context of generalized theory of thermoelasticity with two temperatures, without body forces, body couples and heat sources, [35], are given by

$$(\lambda + 2\mu + \kappa)\nabla(\nabla.\vec{u}) - (\mu + \kappa)\nabla \times (\nabla \times \vec{u}) + \kappa\left(\nabla \times \vec{\phi}\right) - \nu\nabla T = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \qquad (1)$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+\kappa\nabla\times\vec{u}-2\kappa\vec{\phi}=\rho\hat{j}\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{2}$$

$$K^*\nabla^2\Phi=\rho c^*\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+\nu\Phi_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u}), \tag{3}$$

where $T=(1-a\nabla^2)\Phi$ and the constitutive relations are

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+\kappa(u_{j,i}-\varepsilon_{ijr}\phi_r)-\nu(1-a\nabla^2)\Phi\delta_{ij}, \tag{4}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}, \quad i,j,r=1,2,3 \tag{5}$$

where λ and μ are Lame's constants. κ, α, β and γ are micropolar constants, ρ is the density, $\hat{j}$ is the microinertia, $\vec{u}$ is the displacement vector, $\vec{\phi}$ is the microrotation vector, t_{ij} are the components of the stress tensor, m_{ij} are the components of couple stress tensor, T is the temperature change, Φ is the conductive temperature, Φ_0 is the reference temperature, K^* is the thermal conductivity, c^* is the specific heat at constant strain, τ_0 is the relaxation time, a is the two temperature parameter, δ_{ij} is the Kronecker delta, ϵ_{ijr} is the alternating symbol, $\nu=(3\lambda+2\mu+\kappa)\alpha_T$, where α_T is the coefficient of linear thermal expansion.

3 Formulation of the problem

We consider a homogeneous, isotropic, micropolar, generalized thermoelastic solid half space with two temperatures .The rectangular Cartesian co-ordinate system $Ox_1x_2x_3$ having origin on the surface x_3=0 with x_3-axis pointing vertically downward into the half space.

We consider two dimensional problem in x_1x_3-plane, so that the displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ are taken as

$$\vec{u}=(u_1(x_1,x_3),0,u_3(x_1,x_3)), \qquad \vec{\phi}=(0,\phi_2(x_1,x_3),0) \tag{6}$$

For convenience, the following non dimensional quantities are introduced

$$x_1'=\frac{\omega^*x_1}{c_1},\ x_3'=\frac{\omega^*x_3}{c_1},\ u_1'=\frac{\rho\omega^*c_1}{\nu T_0}u_1,\ u_3'=\frac{\rho\omega^*c_1}{\nu T_0}u_3,\quad \phi_2'==\frac{\rho {c_1}^2}{\nu T_0}\phi_2,$$

$$t'=\omega^*t,\ \tau_1'=\omega^*\tau_1, \Phi'=\frac{\Phi}{\Phi_0}, t_{ij}'=\frac{1}{\nu T_0}t_{ij}, m_{ij}'=\frac{\omega^*}{c_1\nu T_0}m_{ij},\ \tau_0'=\omega^*\tau_0 \tag{7}$$

where $\omega^*=\frac{\rho c^* c_1^2}{K^*}$, $c_1^2=\frac{\lambda+2\mu+\kappa}{\rho}$

The displacement components $u_1\ and\ u_3$ are related to the potential functions ϕ and ψ as

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3}, \qquad u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1} \tag{8}$$

Using equation (8) in equations (1)-(3) and with the aid of equations (6) and (7); (after suppressing the primes), we obtain

$$\nabla^2\phi-(1-a\nabla^2)\Phi-\frac{\partial^2\phi}{\partial t^2}=0, \tag{9}$$

$$\nabla^2\psi - a_1\phi_2 - a_2\frac{\partial^2\psi}{\partial t^2} = 0\,, \tag{10}$$

$$\nabla^2\phi_2 - a_3\nabla^2\psi - a_4\phi_2 - a_5\frac{\partial^2\phi_2}{\partial t^2} = 0\,, \tag{11}$$

$$\nabla^2\Phi = a_6\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}(1 - a\nabla^2)\right)\Phi + a_7\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\Phi\,, \tag{12}$$

where

$$a_1 = \frac{\kappa}{\mu+\kappa}, a_2 = \frac{\rho c_1^2}{\mu+\kappa}, a_3 = \frac{\kappa c_1^2}{\gamma\omega^{*2}}, a_4 = 2\,a_3\,, a_5 = \frac{\rho\hat{j}c_1^2}{\gamma}, a_6 = \frac{\rho c^* c_1^2 T_0}{K^*\omega^*\Phi_0}, \qquad a_7 = \frac{\nu c_1^2}{K^*\omega^*}$$

and $\nabla^2 = \frac{\partial^2}{\partial {x_1}^2} + \frac{\partial^2}{\partial {x_3}^2}$.

4. Boundary Conditions

The boundary conditions at the free surface x_3=0 are the vanishing of normal stress, tangential stress, tangential couple stress and temperature gradient. Mathematically these can be written as

$$T_{33} = 0\,, \quad T_{31} = 0, \quad m_{32} = 0, \quad \frac{\partial T}{\partial x_3} = 0 \tag{13}$$

5. Reflection

We consider a Longitudinal displacement wave (LD-wave) or Thermal wave (T-wave) or Coupled transverse and microrotational waves (CD-I wave and CD-II wave) is incident at the plane x_3=0 and making an angle θ_0 normal to the surface. Corresponding to each incident wave, we get reflected LD-wave, T-wave, CD-I and CD-II waves as shown in Fig.1.

In order to solve the equations (9)-(12), we assume the solutions of the form

$$\{\phi, T, \Phi, \psi, \phi_2\} = \{\tilde{\phi}, \tilde{T}, \tilde{\Phi}, \tilde{\psi}, \widetilde{\phi_2}\}\exp\,[\iota\{k(x_1\sin\theta - x_3\cos\theta) - \omega t\}] \tag{14}$$

where k is the wave number and ω is the angular frequency and $\tilde{\phi}, \tilde{T}, \tilde{\Phi}, \tilde{\psi}, \widetilde{\phi_2}$ are arbitrary constants. Making use of equation (14) in equations (9)-(12), after simplification yield

$$V^4 + D_1V^2 + E_1 = 0\,, \tag{15}$$

$$V^4 + D_2V^2 + E_2 = 0\,, \tag{16}$$

where

$$D_1 = \left(\frac{a_1a_3}{\omega^2 a_2} - 1\right)\frac{1}{\left(a_5 - \frac{a_4}{\omega^2}\right)} - \frac{1}{a_2}\,, \qquad E_1 = \frac{1}{\left(a_5 - \frac{a_4}{\omega^2}\right)a_2}\,,$$

$$D_2 = \frac{1 + \left(a - \frac{1}{\omega^2}\right)a_6\omega^4\left(\frac{\iota}{\omega} + \tau_0\right) + a_7\omega^2\left(\frac{\iota}{\omega} + \tau_0\right)}{\omega^2 a_6\left(\frac{\iota}{\omega} + \tau_0\right) - 1}\,, E_2 = \frac{a\omega^2\left[a_7\left(\frac{\iota}{\omega} + \tau_0\right) - a_6\left(\frac{\iota}{\omega} + \tau_0\right)\right]}{a_6\left(\frac{\iota}{\omega} + \tau_0\right) - \frac{1}{\omega^2}}$$

Equation (15) and (16) are quadratic in V^2, therefore the roots of these equations give four values of V^2. Corresponding to each value of V^2 in equation

(15), there exist two types of waves in decreasing order of their velocities, namely a LD-wave, T-wave. Similarly corresponding to each value of V^2 in equation (16), there exist two types of waves, namely a CD-I wave, CD-II wave. Let V_1 , V_2 are the velocities of reflected LD-wave, T-wave and V_3, V_4 are the velocities of reflected CD-I wave, CD-II wave.

In view of equation (14), the appropriate solutions of equations (9)-(12) are assumed of the form

$$\{\phi, \Phi\} = \sum_{i=1}^{2}\{1\,, f_i\}\, S_{0i}[\exp\,[\iota\{k_i(x_1 \sin\theta_{0i} - x_3 \cos\theta_{0i}) - \omega_i t\}] + P_i]\,, \quad (17)$$

$$\{\psi, \phi_2\} = \sum_{j=3}^{4}\{1\,, f_j\}\, T_{0j}\, exp[\iota\{k_j(x_1 \sin\theta_{0j} - x_3 \cos\theta_{0j}) - \omega_j t\} + P_j]\,, \quad (18)$$

where

$$P_i = S_i \exp[\iota\{k_i(x_1 \sin\theta_{0i} + x_3 \cos\theta_{0i}) - \omega_i t\}],$$

$$P_j = T_j exp[\iota\{k_j(x_1 \sin\theta_{0j} + x_3 \cos\theta_{0j}) - \omega_j t\}],$$

and S_{0i} are the amplitudes of incident (LD-wave, T-wave) and T_{0j} are the amplitudes of incident (CD-I, CD-II) waves respectively. S_i are the amplitudes of reflected (LD-wave, T-wave) and T_j are the amplitudes of reflected (CD-I, CD-II) waves respectively.

In order to satisfy the boundary conditions, we use the extension of Snell's law

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3} = \frac{\sin\theta_4}{V_4} \quad (19)$$

where $k_1V_1 = k_2V_2 = k_3V_3 = k_4V_4 = \omega$, at x_3=0. (20)

Making use of the values of ϕ, ψ, Φ and ϕ_2 from (17) and (18) in boundary conditions (13) and with the aid of equations (4)-(8) and using the equations (19) and (20), we obtain a system of four non-homogeneous equations which can be written as

$$\sum_{j=1}^{4} a_{ij} Z_j = Y_i\,; (i = 1, 2, 3, 4), \quad (21)$$

where

$$a_{1i} = \left(d_1 + d_2\left(1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{V_i^2}{\omega^2} + (1 - \tau_1\iota\omega)\left(1 + a\frac{V_i^2}{\omega^2}\right)f_i,$$

$$a_{1j} = \frac{V_j^3}{\omega^2 \mathrm{V}_0}\sin\theta_0\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0},\ a_{2i} = -(2d_4 + d_5)\frac{V_i^3}{\omega^2 \mathrm{V}_0}\sin\theta_0\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0},$$

$$a_{2j} = \left(2d_4\frac{V_j^2}{\omega^2}\left(1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0\right) - d_5\frac{V_j^4}{\omega^2 V_0^2}\sin^2\theta_0\right) - d_5 f_j,$$

$$a_{3i} = 0\,,\ a_{3j} = \frac{V_j}{\omega}\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0}\, f_j\,, a_{4i} = \iota\frac{V_i}{\omega}\left(1 + a\frac{V_i^2}{\omega^2}\right)\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}\, f_i\,,\quad a_{4j} = 0$$

(i =1, 2 and j =3, 4) and

$$Z_1 = \frac{S_1}{A^*},\ Z_2 = \frac{S_2}{A^*},\ Z_3 = \frac{T_3}{A^*},\ Z_4 = \frac{T_4}{A^*}\,, \quad (22)$$

(1) For incident LD-wave:

$A^* = S_{01}\,,\ S_{02} = T_{03} = T_{04} = 0, Y_1 = -a_{11}\,,\ Y_2 = a_{21}\,, Y_3 = a_{31} = 0\,, Y_4 = a_{41},$

(2) For incident T-wave:

$A^* = S_{02}\,, S_{01} = T_{03} = T_{04} = 0\,, Y_1 = -a_{12}\,, Y_2 = a_{22}\,, Y_3 = a_{32} = 0\,, Y_4 = a_{42},$

(3) For incident CD-I wave:
$A^* = T_{03}, S_{01} = S_{02} = T_{04} = 0, Y_1 = a_{13}, Y_2 = -a_{23}, \; Y_3 = a_{33}, \; Y_4 = a_{43} = 0,$

(4) For incident CD-II wave:
$A^* = T_{04}, S_{01} = S_{02} = T_{03} = 0, Y_1 = a_{14}, Y_2 = -a_{24}, \; Y_3 = a_{34}, \; Y_4 = a_{44} = 0,$

where $Z_1, \; Z_2, \; Z_3, \; Z_4$ are the complex amplitude ratios of reflected LD-wave, T-wave and coupled CD-I, CD-II waves.

6. Particular cases

In the absence of two temperature effect, we obtain the amplitude ratios at the free surface of micropolar generalized thermoelastic solid half space as

$$\sum_{j=1}^{4} a_{ij} Z_j = Y_i \; ; (i = 1, 2, 3, 4),$$

where the values of a_{ij} are given by

$$a_{1i} = \left(d_1 + d_2\left(1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{V_i^2}{\omega^2} + (1 - \tau_1 \iota\omega) f_i \,, \;\; a_{1j} = d_2 \frac{V_j^3}{\omega^2 V_0} \sin\theta_0 \sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0},$$

$$a_{2i} = -(2d_4 + d_5)\frac{V_i^3}{\omega^2 V_0}\sin\theta_0 \sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0},$$

$$a_{2j} = \left(2d_4 \frac{V_j^2}{\omega^2}\left(1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0\right) - \left(d_5 \frac{V_j^2}{\omega^2 V_0^2}\sin^2\theta_0\right)\right) - d_5 f_j,$$

$$a_{3i} = 0\,, \;\; a_{3j} = \iota \frac{V_j}{\omega}\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0}\, f_j \;, a_{4i} = \iota \frac{V_i}{\omega}\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}\, f_i \,, \quad a_{4j} = 0,$$

and

$$Z_1 = \frac{S_1}{A^*}, \;\; Z_2 = \frac{S_2}{A^*}, \;\; Z_3 = \frac{T_3}{A^*}, \;\; Z_4 = \frac{T_4}{A^*} \qquad , \tag{23}$$

where $Z_1, \; Z_2, \; Z_3, \; Z_4$ are the amplitude ratios of reflected LD-wave, T-wave and coupled CD-I, CD-II waves .

The above results are in agreement with those obtained by Singh and Kumar [25]by changing the dimensionless quantities into physical quantities.

7. Numerical results and discussion

For numerical computations, we take the following values of relevant parameters. Following [35], the values of micropolar constants are taken as:
$\lambda = 9.4 \times 10^{10} Nm^{-2}, \qquad \mu = 4.0 \times 10^{10} Nm^{-2}, \qquad \kappa = 1.0 \times 10^{10} Nm^{-2},$
$\gamma = 7.79 \times 10^{-10} N, \qquad \hat{j} = 2 \times 10^{-20}\, m^2, \qquad \rho = 1.74 \times 10^3 Kgm^{-3}$
and thermal parameters are taken as:
$T_0 = 0.298\, K, \qquad \Phi_0 = 0.295\, K, \qquad \nu = 2.68 \times 10^6 Nm^{-2}K^{-1},$
$c^* = 1.04 \times 10^3 Jkg^{-1}K^{-1}, \;\; K^* = 1.7 \times 10^2 Jm^{-1}s^{-1}K^{-1}, \omega = 1, \tau_0 = 0.02$

In Figs. 2-13, we represent the solid line for micropolar generalized thermoelastic solid (*a*=0), small dashes line for micropolar generalized thermoelastic solid with two temperature (*a*=0.3), and large dashes line for micropolar generalized thermoelastic solid with two temperature(*a*=0.9).

7.1. LD-Wave Incident

Fig. 2 shows that the amplitude ratios $|Z_1|$ increases monotonically with the angle of incidence for all values of a and attaining the maximum value 1 at the grazing incidence.

Fig. 3 shows that the amplitude ratio $|Z_2|$ attains the maximum value 1 near normal incidence and then it decreases monotonically with the angle of incidence for all values of a. The value of $|Z_2|$ converges to minimum value at the grazing incidence for all values of a.

In Fig. 4 , the values of amplitude ratio $|Z_3|$ increases monotonically when $0 < \theta_0 < 43^\circ$ and as θ_0 increases further, it decreases for all values of a. The values of $|Z_3|$ remain more for $a = 0$ in comparison to $\mathrm{a} = 0.3$ and 0.9 depicting the effect of two temperatures.

Fig. 5 depicts that the values of amplitude ratio $|Z_4|$ increases monotonically for $0 < \theta_0 < 45^\circ$ and decrease monotonically as θ_0 increase for all values of a. The values of $|Z_4|$ remain more for $a = 0.9$ in comparison to $\mathrm{a} = 0$ and 0.3.

7.2. T-Wave Incident

Fig. 6 depicts that the values of amplitude ratio $|Z_1|$ begin with the value 1 near normal incidence then it decreases monotonically with the angle of incidence and attaining the minimum value at the grazing incidence for all values of a.

Fig. 7 shows that the values of amplitude ratio $|Z_2|$ increases monotonically for all values of a with the angle of incidence and attaining the maximum value 1 at the grazing incidence.

Fig. 8 depicts that the values of amplitude ratio $|Z_3|$ for $\mathrm{a} = 0.3$ and $\mathrm{a}= 0.9$ increase in the range $0 < \theta_0 < 50^\circ$ and then decrease as θ_0 increases further whereas for $a = 0$, the value of $|Z_3|$ increase in the interval $0 < \theta_0 < 55^\circ$ and then decrease sharply for further range.

Fig.9 shows that as θ_0 lies between $0 < \theta_0 < 30^\circ$, value of $|Z_4|$ increase for all values of a and as θ_0 increase further, the values of $|Z_4|$ oscillate for all a and attaining the minimum value at the grazing incidence.

7.3. CD-I Wave Incident

Fig. 10 depicts that the values of amplitude ratio $|Z_1|$ increase monotonically for all values of a, as θ_0 lies between $0 < \theta_0 < 45^\circ$ and then decreases attaining its minimum value near the grazing incidence. It is evident that as a increase, the amplitude ratio decrease depicting the effect of two temperature.

Fig. 11 shows that the values of amplitude ratio $|Z_2|$ first increases monotonically for all values of a as θ_0 increases and then decrease as θ_0 increase for all values of a . The maximum value of $|Z_2|$ is attained for $a = 0$ in the range $55 < \theta_0 < 65^\circ$.

Fig. 12 depicts that the values of amplitude ratio $|Z_3|$ for all values of a. The values of $|Z_3|$ for $a = 0$ are more in comparison to $a = 0.3$ and $a = 0.9$ near the normal incidence, then it increases as angle of incidence increases and it attains its maximum value 1 at the grazing incidence.

In Fig.13, the value of amplitude ratio $|Z_4|$ is maximum near the normal incidence , then it decreases monotonically with angle of incidence and attains its minimum value at grazing incidence for all values of a. The values of $|Z_4|$ are more for $a = 0.9$ in comparison to $a = 0$ and $a = 0.3$.

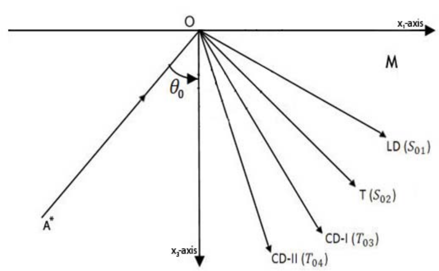

Fig. 1 Geometry of the problem

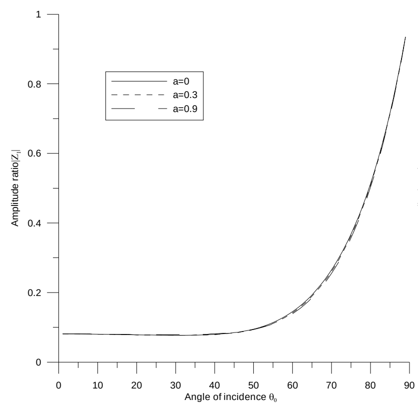

Fig.2

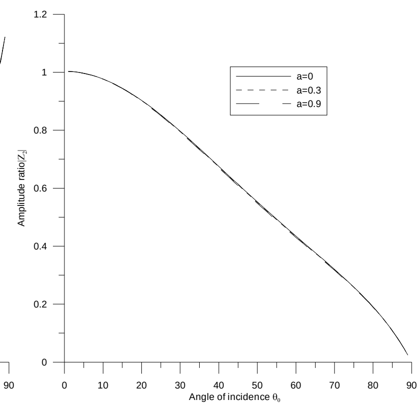

Fig.3

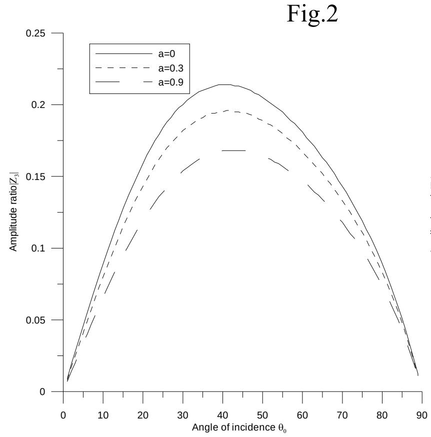

Fig. 4

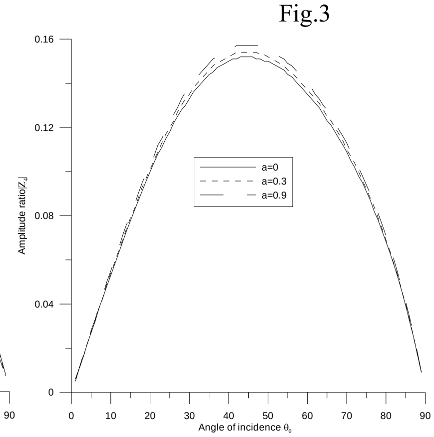

Fig. 5

Figs.2-5. Variations of amplitude ratios with the angle of incidence for LD-Wave

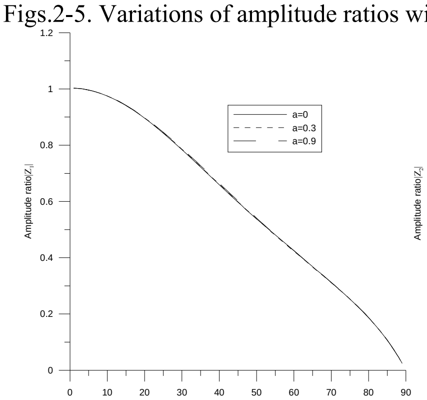

Fig. 6

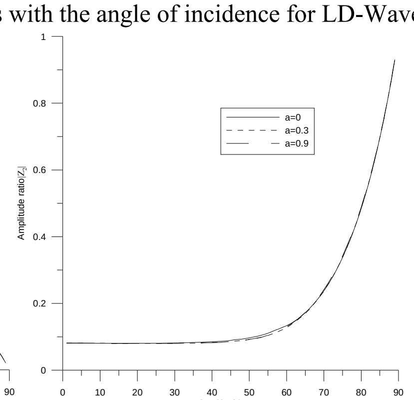

Fig. 7

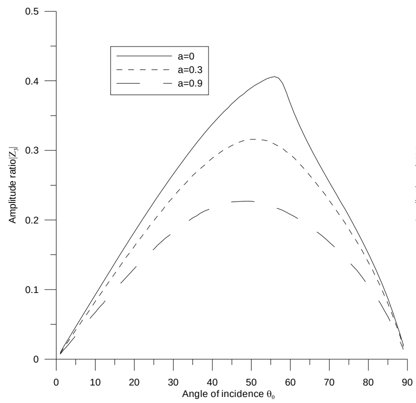

Fig. 8

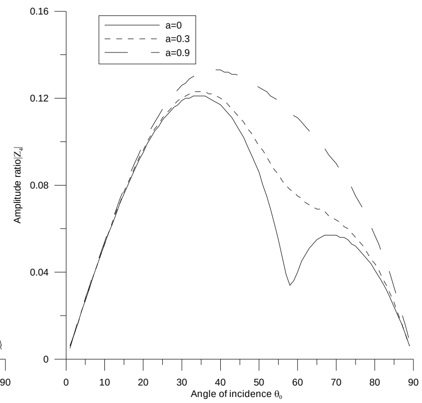

Fig. 9

Figs.6-9. Variations of amplitude ratios with the angle of incidence for T-Wave

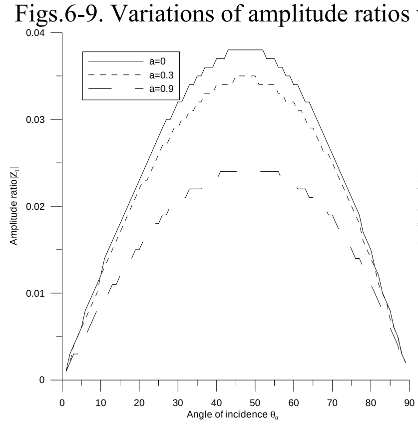

Fig. 10

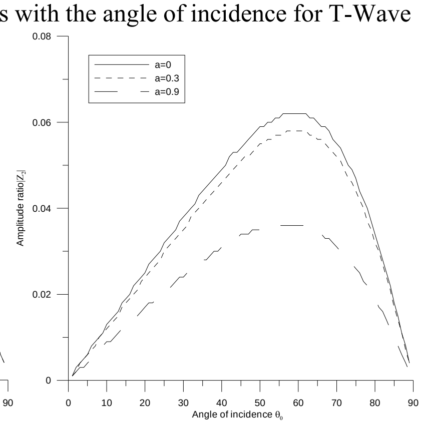

Fig. 11

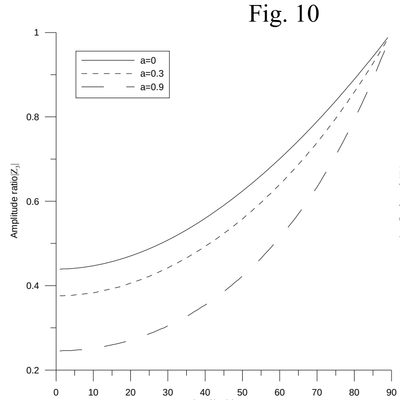

Fig. 12

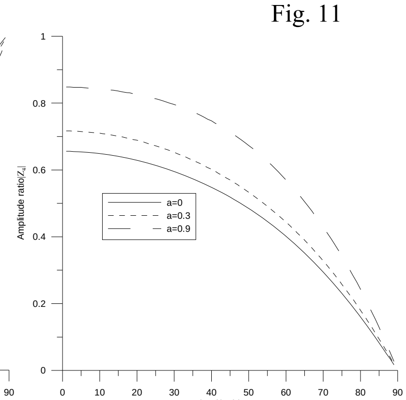

Fig. 13

Figs.10-13. Variations of amplitude ratios with the angle of incidence for CD-I Wave

8. Conclusion

Effect of two temperatures has significant impact on the amplitude ratios. It is depicted from the figures that the behavior and trend of variation of amplitude ratios is same for all the values of a. Also it is observed that the values of $|Z_i|; 1 \leq i \leq 3$, decreases with increase in the values of a that shows the effect of two temperatures, whereas the values of $|Z_4|$ increase with increase in the values of a. The research work is supposed to be useful in further studies, for both theoretical and observational wave propagation in more realistic models of the micropolar thermoelastic solid present in the earth's interior.

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