CENTRIFUGAL COMPRESSORS SURGE SIMULATION

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Various theoretical results were determined modifying the closing time of a centrifugal compressor evacuation valve. Hereby, it can be observed that, for short closing times, the system evolutions are similar with those found for the instantaneous vane's closure. While the closing time increases, the system behavior grows toward a normal one especially if the closing time overpasses too much the period of an potential periodic process. We determine some small oscillations at the beginning of the process, after which the flow is slowly decreasing to zero, without anything spectacular. In the presented variants it can be observed that in the first moment, the computed knots number and, obvious, the analyze time, have a great influence on the system oscillations description. A large number of knots results when the flow oscillations number tends to infinite. It is obvious that the values of this vane's closing time could not go down below particular values, because the analysis would lose the relevance in these cases. Not the same thing could be said about the knots number, which is recommended to be as high as possible. An interesting analysis of this phenomenon could be made studying the compressor system behavior in the case of a finite evacuation vane's closing time. Elaborating different theoretical models described in literature, the authors present an original mathematical model of simulation for the centrifugal compressors surge.

Keywords: centrifugal compressor, evacuation valve, flow oscillations, closing time

1. Introduction

Until now, only few generalities were defined in the specialized literature. Starting from these general equations, an analytical model was established for studying the kinetic compressors surge (axial and centrifugal). The analytical simulation was performed using this model.

A flow characteristic was acquired on a bench through a compressor test, which contains the variation for the adiabatic efficiency compression process curves, together with the compression degree variation ones.

2. Geometrical data of the compression system

One can consider the principle schema of a compressor system, used effectively in a centrifugal compressor test, like in Fig. 1, [1], [2].

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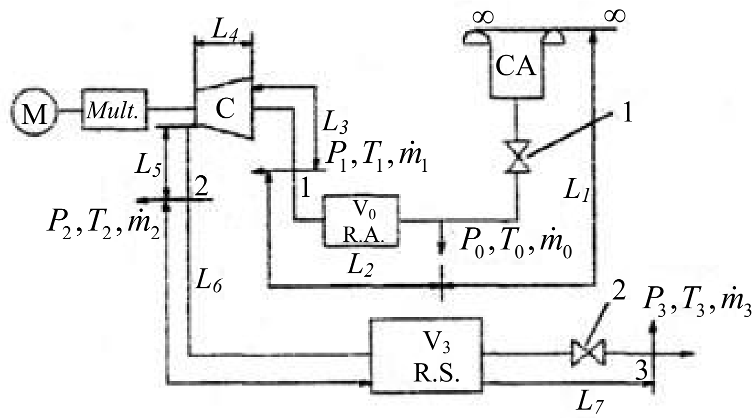

Fig. 1. The principle schema of a compressor system

The main components of the installation were marked in the figure

- M , engine for compressor drive;
- C , centrifugal compressor;
- $C.A.$, air intake duct;
- $R.A.$, air intake reservoir from the intake chamber;
- $R.S.$, air outlet reservoir;
- 1, emission valve;
- 2 , evacuation valve;
- L_l , the length of the connection pipes;
- P_l, T_l, the air pressure and temperature in the compressor system characteristic sections.

Taking into account that the analytic expressions of those curves are necessary in the next computations, we tabular extracted the coordinates of few essential points for the distributions of π_c, η_c, based on the available images. The data are contained in Table no. 1 and Table no. 2.

Table no. 1

$\dot{m}_{red}$	-1	0	1	2	3
π_c	2.8	2.5	2.5	2.6	2.8

Table no. 2

$\dot{m}_{red}$	-1	0	1	2	3
η_c	0.4	0.56	0.7	0.7	0.7

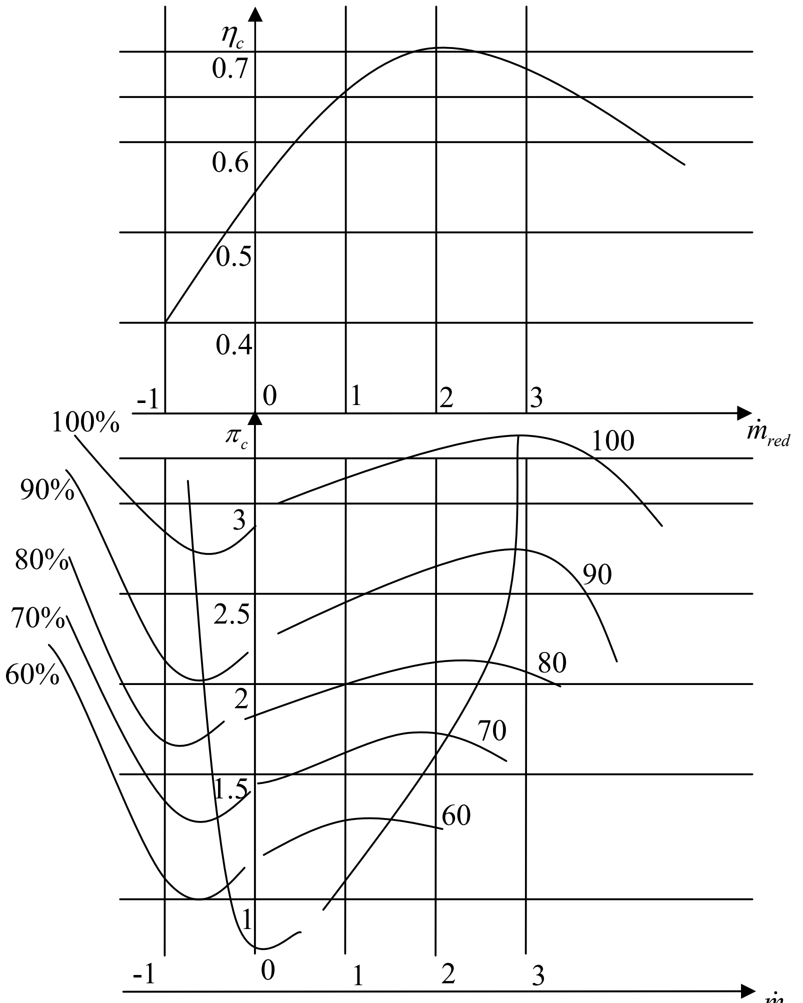

Fig. 2. The distribution of π_c and η_c function of $\dot{m}_{red}$

Hereinafter, few geometrical data are presented, which are used in the next computations

$$V_0 = 9.40 \ m^3$$

$$A_0 = 2.685 \ m^2$$

For the two curves approximation, a polynomial function will be used in the form, [3]

$$y = a_0 H_0 + \frac{a_1 H_1}{1!} + \frac{a_2 H_2}{2!} + \frac{a_3 H_3}{3!} + \ldots \quad (1)$$

where

$$H_0 = 1$$
$$H_1 = \frac{x - x_0}{h}$$
$$H_2 = \frac{(x - x_0)(x - x_1)}{h^2}$$
$$H_3 = \frac{(x - x_0)(x - x_1)(x - x_2)}{h^3}$$
$$h = \frac{x_{n-1} - x_0}{n-1}$$
$$a_0 = y_0$$
$$a_1 = y_1 - y_0 = \Delta^1 y$$
$$a_2 = \Delta^2 y$$

Practically, the tables with differences are presented below: Table no. 3 for the compression degree and Table no. 4 for the efficiency.

Table no. 3

	x_0	x_1	x_2	x_3	x_4
$\dot{m}_{red}$	-1	0	1	2	3
$y(\pi_c)$	2.8	2.5	2.5	2.7	2.35
$\Delta^1 y$	-0.3	0	0.15	-0.3	
$\Delta^2 y$	0.3	0.15	-0.45		
$\Delta^3 y$	-0.15	-0.6			
$\Delta^4 y$	-0.45				
$\Delta^5 y$					

Table no. 4

	x_0	x_1	x_2	x_3	x_4
$\dot{m}_{red}$	-1	0	1	2	3
$\eta_c = y$	0.4	0.56	0.7	0.7	0.7
$\Delta^1 y$	0.16	0.09	0.05	0	
$\Delta^2 y$	-0.07	-0.04	-0.05		
$\Delta^3 y$	0.03	-0.01			
$\Delta^4 y$	-0.04				

Based on those values, the coefficients a_1 of the two polynomials which approximate the compressor performance at $\overline{n} = 0.9$ are determined, [4].

Replacing and elaborating those polynomials, we get the expressions of the compression degree

$$\pi_c = 2.5 - 0.1625\dot{m}_{red} + 0.16875\dot{m}_{red}^2 + 0.0125\dot{m}_{red}^3 - 0.01875\dot{m}_{red}^4 \quad (2)$$

respectively for efficiency

$$\eta_c = 0.56 + 0.11667\dot{m}_{red} - 3.3334\cdot 10^{-2}\dot{m}_{red}^2 + 8.3334\cdot 10^{-3}\dot{m}_{red}^3 - 1.6667\cdot 10^{-3}\dot{m}_{red}^4$$

both equations being valid for $\dot{m}_{red} \in [0,2]$.

3. The coefficients computation from the fundamental equations

Given the geometrical and gasodynamical characteristics, one may pass at the equations coefficients computation which defines the surge process or, generally, the non stationary flow through the compression system, [5].

$$R_1 = L_1 / A_1 = 45.86 \quad (3)$$

$$R_2 = L_2 / A_2 = 29.06$$

$$S_0 = kRT_0 / V_0$$

where

$T_0 = 288K$, $V_0 = 9.4m^3$, $S_0 = 12310.37$, $C = 1.0135\cdot 10^5$,

$k_0 = 16051.94$, $k = 77730$, $B = 37.07$, $B^{'} = 57.734$,

$B^{''} = 0.6725$, $S_3 = 37.07\cdot T_3(y)$

Regarding the other quantities which appear

$D = 6.473$, $Q = 0.0209$, $P = 0.303$, $M = 0.0209$, $J = 6.473$,

$M^{'} = 0.0418$, $M^{'} \approx M$

The following relations may be written

$$\pi^{'} = 0.1625 + 0.3375\dot{m}_{red} + 0.3375\dot{m}_{red}^2 - 0.075\dot{m}_{red}^3 \quad (4)$$

$$T_3(y) = \frac{288 + \left(288 - 0.0209y^2\right)\left(\pi^{0.286} - 1\right)}{I}$$

4. The problems determination

4.1. The complete problem

The differential complete system which characterizes the non stationary flow in the compression system was established in the anterior paragraphs. This system will be particularized in the concrete case of the discussed installation, [6].

If we consider

$$F(y) = \int f(y)dy + c \text{ , where } c = \textit{constant of integration} \tag{5}$$

$$G(z,y) = -k + k\left(\frac{y}{z}\right)$$

$$H(x,y) = -q(x,y) - q(x,y)\left(\frac{y}{x}\right)$$

$$E(x,y,z) = h_1(y)(y-z) + h_2(y)(y-x)$$

where

$$\dot{m}_{red} = y$$

$$f(y) = \frac{-c\pi'}{\pi R_1 + R_2}$$

$$q(x,y) = B''\sqrt{p_{33}^2 + BT_3(y)x}$$

$$h_1(y) = \frac{\pi\rho_0}{\theta R_1 + R_2}$$

$$h_2(y) = \frac{S_3(y)}{\theta R_1 + R_2}$$

then, the system may be written

$$\begin{cases} \dot{x} = H(x,y) \\ \dot{y} = u - F(y) \\ \dot{z} = G(z,y) \\ \dot{u} = -E(x,y,z) \end{cases} \tag{6}$$

with the initial conditions at $t < 0$, at the stopping moment

$$\dot{y}_0 = \dot{m}_{red_0} = 3kg/s$$

$$z_0 = 3$$

$$x_0 = 3$$

$$u_0 = F(y_0)$$

respectively at the starting moment

$$y_0 = x_0 = z_0 = u_0 = 0$$

Replacing the constant terms, the variables from the system get the following forms

$$f(y) = -2210 \frac{\pi^{'}}{\pi + 0.634} \quad (7)$$

$$g(x, y) = 0.6725\sqrt{p_{33}^4 + 57.734 \cdot T_3(y)x}$$

where

$$T_3(y) = 288 + \left(288 - 0.0209y^2\right)\frac{\left(\pi^{0.286} - 1\right)}{I}$$

$$I(y) = 0.56 + 0.11667y + 0.03334y^2 + 0.008334y^3 - 0.001667y^4$$

$$h_1(y) = 268.443 \frac{\pi}{\pi + 0.634}$$

$$h_2(y) = 0.803 \frac{T_3(y)}{\pi + 0.634}$$

and where

$$S_3(y) = 37.07 \cdot T_3(y)$$

$$G(z, y) = -77730\left(1 - \frac{y}{z}\right)$$

$$E(x, y, z) = h_1(y)(y - z) + h_2(y)(y - x)$$

$$H(x, y) = -q(x, y)\left(1 - \frac{y}{x}\right)$$

4.2. The problem in the case of the complete closure of the outlet valve and the inlet accomplishment from an unlimited space

In this case, the system becomes, [7]

$$\begin{cases} \dot{y} = u_1 - F_1(y) \\ \dot{u}_1 = -E_1(y) \end{cases} \quad (8)$$

where

$$F_1(y) = -c\frac{\pi}{R_2} = -1.0135 \cdot 10^5 \frac{\pi}{29.06} = -3487.6\pi$$

and where the function

$$E_1(y) = g_1(y)y$$

where

$$g_1(y)=\frac{S_3(y)}{R_2}=1.276\cdot T_3(y)$$

where

$$T_3(y)=288+\left(288-0.0209y^2\right)\frac{\pi^{0.286}-1}{I}$$

with the initial conditions

$$y_0=3$$

$$u_{10}=F_1(y_0)$$

4.3. The simplified problem of the evacuation valve instantaneous closure

In this case, the equivalent differential system becomes

$$\begin{cases} y^{'}=u_2-\alpha\left(\dfrac{y^3}{3-y}\right) \\ u_2^{'}=-y \end{cases} \tag{9}$$

with the initial conditions y_0, u_{20}.

Considering that

$$y^{'}=\frac{\dot{y}}{\omega_0}=\frac{\ddot{m}_{red}}{\omega_0}$$

$$\alpha=\frac{ca_1}{R_2\omega_0}$$

where a_1 would be the coefficient of y in the approximation by a three degree parabola of the compression degree π_c, this means

$$\pi_c=a_0+a_1y+a_2y^2+a_3y^3$$

Taking only the part for π_c, with $y>0$, one acquires $a_1=0.35$. Therefore, $\alpha=1220.664/\omega_0$. Regarding the pulsation ω_0, it may be established using the Horvath relation, [2]:

$$\omega_0=\sqrt{\frac{S_3}{R_2}}=23.448\ rad/s \tag{10}$$

At this pulsation corresponds a frequency $\nu=3.74\ Hz$ and a period of the process $T=0.2674\ s$.

Therefore, replacing $\alpha = 51.97$
the system may be written

$$\begin{cases} y' = u_2 - 51.97\left(\dfrac{y^3}{3-y}\right) \\ u'_2 = -y \end{cases} \tag{11}$$

with the conditions $y_0, u_{20} = F_1\left(y_0\right)$.

The previous system may lead to the Van der Pol equation

$$y'' + 51.97\left(y^2 - 1\right)y' + y = 0 \tag{12}$$

5. The main compression system variables computation

Resolving the system, one gets the functions $y\left(t\right), z\left(t\right), x\left(t\right)$. These allow establishing the variation laws in time for the air pressures and temperatures in the main sections, [8]

$$p_0 = p_\infty - \left(\frac{K_s}{2}\right)\left(\frac{z^2}{A_0^2}\right) \tag{13}$$

$$p_1 = p_0 - 45.86\dot{y}$$

$$p_2 = p_1 \pi\left(y\right)$$

$$p_3 = \sqrt{p_{33}^2 + K_d x^2}$$

where $K_d = 29.7 \cdot T_3\left(y\right)$, $K_s = 1.145$

Therefore

$$p_0 = p_\infty - 0.08z^2 \tag{14}$$

$$p_3 = \sqrt{p_{33}^2 + 29.7 \cdot T_3\left(y\right)x^2}$$

The temperatures are given by the relations

$$T_1 = 288 - 0.0209y^2$$

$$T_2 = T_1\left[1 + \left(\pi^{0.286} - 1\right)/I\right]$$

$$T_3 = T_2 + 0.0209y^2$$

6. The surge problem solution

Consequently to the anterior chapter, one will attempt to establish the analytical solution of the differential equations system. Firstly, one resolves the simplest problem, which leads to the Van der Pol equation, [9].

$$\begin{cases} \dfrac{dx}{dt} = y - \alpha\left(\dfrac{x^3}{3-x}\right) \\ \dfrac{dy}{dt} = -x \end{cases} \quad (15)$$

in Cartesian coordinates.

The limit conditions are written

$$x_0 = x(0), y_0 = y(0)$$

and the constant term $\alpha = 51.97 \cdot 10^{-5}$.

Generally, a Van der Pol equation has the form, [10]

$$x'' + k\left(x^2 - 1\right)x' + x = B \sin \omega t \quad (16)$$

where $k > 0$ and ω and B are constants.

6.1. The Van der Pol equation study

In the case of the complete evacuation vane closure, one gets a Van der Pol type differential equation

$$y'' + 51.97 \cdot 10^{-5}\left(y^2 - 1\right)y' + y = 0 \quad (17)$$

$$y' = \frac{dy}{dx} = \frac{1}{\omega_0}\frac{dy}{dt} = \frac{y}{\omega_0}$$

The equation may be generated starting from the first degree differential equation system

$$\begin{cases} y' = u_2 - \alpha\left[\left(y^3/3\right) - y\right] \\ u_2' = -y \end{cases} \quad (18)$$

with the initial conditions

$$y_0 u_\infty = \alpha\left(\frac{y_0^3}{3} - y_0\right)$$

$$y'(0) = 0, \ \alpha = 51.97 \cdot 10^{-5}$$

One arrives at the equation known form from literature, if one changes the variable terms notations [11].

$$y \to x, \ u_2 \to y, \ T \to t.$$

It may be written

$$\begin{cases} \dfrac{dx}{dt} = y - \alpha\left[\left(x^3/3\right) - x\right] \\ \dfrac{dy}{dt} = -x \end{cases} \tag{19}$$

with the initial conditions

$$\begin{cases} x(0) = x_0 = 3 \\ y(0) = y_0 = 6\alpha = 311.82 \cdot 10^{-5} \end{cases}$$

$$\begin{cases} x(0) = 0.5 \\ y(0) = -24.818 \cdot 10^{-5} \end{cases}$$

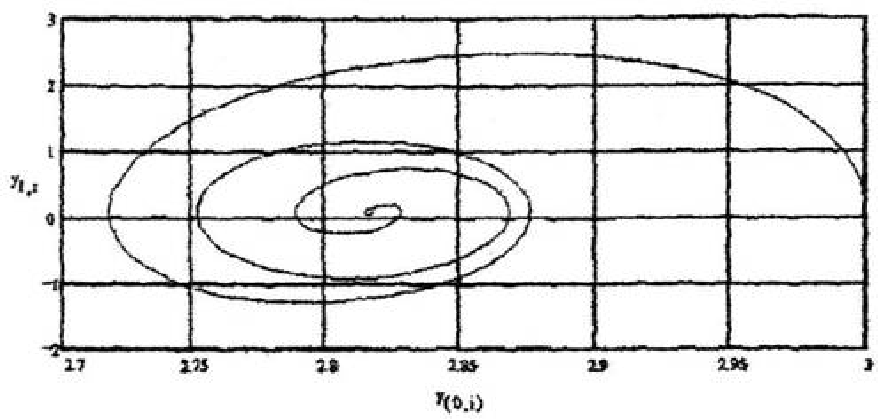

Fig. 3. Graphical representation of the Van der Pol equation solutions

6.2. The analytical method of the Taylor development

One notes the functions from the right member of the equations by

$$f(x, y) = y + 51.97 \cdot 10^{-5} x^3 \tag{20}$$

$$g(x, y) = -x$$

Developing the two unknown functions in Taylor series [5], one gets:

$$x = x(t) \tag{21}$$

$$y = y(t) \Rightarrow$$

$$x(t) = x(0) + \frac{t}{1!}x'(0) + \frac{t^2}{2!}x''(0) + \ldots$$

$$y(t)=y(0)+\frac{t}{1!}y^{'}(0)+\frac{t^2}{2!}y^{''}(0)+...$$

The coefficients compute is done taking into account that

$$\begin{cases} x(0)=x_0=3 \\ y(0)=y_0=6\alpha=311.82\cdot 10^{-5} \end{cases} \tag{22}$$

and replacing in the equations system

$$\begin{cases} x^{'}(0)=0 \\ y^{'}(0)=-3 \end{cases}$$

But

$$x^{''}=\frac{\partial f}{\partial x}x^{'}+\frac{\partial f}{\partial y}y^{'}$$

$$y^{''}=\frac{\partial g}{\partial x}x^{'}+\frac{\partial g}{\partial y}y^{'}$$

Taking into account that

$$\frac{\partial f}{\partial x}=51.97\cdot 10^{-5}-51.97\cdot 10^{-5}x^2 \tag{23}$$

$$\frac{\partial f}{\partial y}=1$$

$$\frac{\partial g}{\partial x}=-1$$

$$\frac{\partial g}{\partial y}=0$$

one gets

$$\begin{cases} x^{''}=\left(51.97\cdot 10^{-5}-51.97\cdot 10^{-5}x^2\right)x^{'}+y^{'} \\ y^{''}=-x^{'} \end{cases}$$

Therefore, the coefficients become

$$\begin{cases} x^{''}(0)=-3 \\ y^{''}(0)=0 \end{cases}$$

It passes at the three degree derivative

$$\begin{cases} x^{'''}=-2\cdot 51.97\cdot 10^{-5}xx^{'2}+51.97\cdot 10^{-5}\left(1-x^2\right)x^{''}+y^{''} \\ y^{'''}=-x^{''} \end{cases}$$

The coefficients become

$$\begin{cases} x'''(0) = 1.2473 \cdot 10^{-2} \\ y'''(0) = 3 \end{cases}$$

We are executing now the four degree derivative

$$\begin{cases} x^{IV} = -103.94 \cdot 10^{-5} - 311.82 \cdot 10^{-5} xx'x'' + 51.97 \cdot 10^{-5} \left(1 - x^2\right) x''' \\ y^{IV} = -x''' \end{cases}$$

Replacing, one acquires

$$\begin{cases} x^{IV} = -103.94 \cdot 10^{-5} \left(-8\right) \cdot 1247.28 + 3 = -518569.133 \cdot 10^{-5} \\ y^{IV} = -1247.28 \cdot 10^{-5} \end{cases} \tag{24}$$

It is computed the five degree derivative

$$\begin{cases} x^{V} = -103.94 \cdot 3 \cdot x'^2 \cdot x'' - 311.82 \cdot 10^{-5} \left[\left(x'^2 + xx' \right) x'' + xx'x''' \right] + \\ \qquad +51.97 \cdot 10^{-5} \left[-2xx'x''' + \left(1 - x^2\right) \right] x^{IV} + y^{IV} \\ y^{V} = -x^{IV} \end{cases}$$

Doing the computations, one acquires

$$\begin{cases} x^{V} = -311.82 \cdot 10^{-5} \cdot x'^2 \cdot x'' - 311.82 \cdot 10^{-5} \left[\left(x'^2 + xx' \right) x'' + xx'x''' \right] + \\ \qquad +311.82 \cdot 10^{-5} xx'x''' - 103.94 \cdot 10^{-5} \left(1 - x^2\right) x^{IV} + y^{IV} \\ y^{V} = -x^{IV} \end{cases}$$

$$\begin{cases} x^{V} = -623.64 \cdot 10^{-5} \cdot x'^2 \cdot x'' - 311.82 \cdot 10^{-5} xx''^2 + 103.94 \cdot 10^{-5} xx'x''' + \\ \qquad +51.97 \cdot 10^{-5} \left(1 - x^2\right) x^{IV} + y^{IV} \\ y^{V} = -x^{IV} \end{cases}$$

For $t = 0$ one acquires

$$\begin{cases} x^{V}(0) = 2.156 \cdot 10^{3} \\ y^{V}(0) = 5.1857 \end{cases}$$

The six degree derivative computation is done hereby

$$\begin{cases} x^{VI} = -623.64\cdot 10^{-5}\left(2x'x''^2 + x'^2x'''\right) - 311.82\cdot 10^{-5}\left(xx''^2 + xx''x'''\right) + \\ \qquad +103.94\cdot 10^{-5}\left[\left(x'^2 + xx''\right)x''' + xx'x^{IV}\right] + \\ \qquad +51.97\cdot 10^{-5}\left[2xx'x^{IV} + \left(1 - x^2\right)x^V\right] + y^V \\ y^{VI} = -x^V \end{cases}$$

At $t = 0$ it results

$$\begin{cases} x^{VI} = -9.052\cdot 10^{-5} \\ y^{VI} = -2.156\cdot 10^{3} \end{cases}$$

But, if we stop here with the coefficients computation, the two developments become

$$\begin{cases} x(t) = 3 + t(0) + \dfrac{t^2}{2}(-3) + \dfrac{t^3}{6}\left(1247.28\cdot 10^{-5}\right) + \dfrac{t^4}{24}\left(-518589.133\cdot 10^{-5}\right) + \\ \qquad \dfrac{t^5}{120}\left(2.156\cdot 10^{3}\right) + \dfrac{t^6}{720}\left(-9.052\cdot 10^{5}\right) \\ y(t) = 311.82\cdot 10^{-5} + t(-3) + \dfrac{t^2}{2}(0) + \dfrac{t^3}{6}(3) + \dfrac{t^4}{24}\left(-1247.28\cdot 10^{-5}\right) + \\ \qquad \dfrac{t^5}{120}(5.186) + \dfrac{t^6}{720}\left(-2.156\cdot 10^{3}\right) \end{cases}$$

Going back to the old variable terms or notations, one acquires

$$\begin{aligned} y(T) &= 3 - 1.5\cdot 10^{-5}T^2 + 207.8\cdot 10^{-5}T^3 - 21607.05T^4\cdot 10^{-5} + 1.7967\cdot 10T^5 \\ &\quad -1.918\cdot 10^{3}T^6 \\ u_2(T) &= 311.82\cdot 10^{-5} - 3\cdot 10^{-5}T + 0.5\cdot 10^{-5}T^3 - 51.97\cdot 10^{-5}T^4 + \\ &\quad +4321.4167\cdot 10^{-5}T^5 - 2.9945\cdot T^6 \end{aligned} \tag{25}$$

Taking into account that

$$T = \omega_0 t = 23.488t$$

by substitution, it results

$$\begin{aligned} y(t) &= 3 - 827.53\cdot 10^{-5}t^2 + 2.6937\cdot 10^{6}t^3 - 6.57627\cdot 10^{5}t^4 + 1.2844\cdot 10^{8}t^5 - \\ &\quad -3.22\cdot 10^{11}t^6 \\ u_1(t) &= 311.82\cdot 10^{-5} - 70.464\cdot 10^{-5}t + 275.843\cdot 10^{-5}t^2 - 158175\cdot 10^{2}t^4 + \\ &\quad +3.0893\cdot 10^{5}t^5 - 5.028\cdot 10^{8}t^6 \end{aligned}$$

where t is inserted in seconds.

If we accept t in hundredths of a second $[t_s]$, the functions become

$$y(t)=3-8.2753\cdot10^{-7}t_s^{\,2}+2.6937\cdot10^{6}t_s^{\,3}-6.57627\cdot10^{5}t_s^{\,4}+1.2844\cdot10^{8}t_s^{\,5}-$$
$$-3.22\cdot10^{11}t_s^{\,6}$$
$$(26)$$
$$u_2(t)=311.82\cdot10^{-5}-0.70464\cdot10^{-5}t_s+0.0275843\cdot10^{-5}t_s^{\,2}-1.58175\cdot10^{6}t^{4}+$$
$$+3.0893\cdot10^{5}t_s^{\,5}-56.28\cdot10^{-5}t_s^{\,6}$$

Once determined the function $y=f(t)$ of the air flow, the pressures and temperatures time dependent on the compression system may be immediately established, on the base of the anterior determined relations.

7. Conclusions

The mathematical model we appealed to in general conditions and which was singularized for some particular concrete situations lead us to Van der Pol type equation. Numerical simulations of this equation graphically generated a suggestive image of the surge phenomenon from which its cyclic behavior obviously results, as well as the factors which singularize the evolution or involution of this undesired aerodynamic phenomenon.

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